Introduction and Mathematical Concepts

In the movie *Spider-Man* many of the background scenes were created using animation techniques that rely on computers and mathematical concepts such as trigonometry and vectors. These mathematical tools will also be useful throughout this book in dealing with the laws of physics. (© Photofest)

Figure 1.1 The standard platinum–iridium meter bar. (Courtesy Bureau International des Poids et Mesures, France)

1.1 The Nature of Physics

The science of physics has developed out of the efforts of men and women to explain our physical environment. These efforts have been so successful that the laws of physics now encompass a remarkable variety of phenomena, including planetary orbits, radio and TV waves, magnetism, and lasers, to name just a few.

The exciting feature of physics is its capacity for predicting how nature will behave in one situation on the basis of experimental data obtained in another situation. Such predictions place physics at the heart of modern technology and, therefore, can have a tremendous impact on our lives. Rocketry and the development of space travel have their roots firmly planted in the physical laws of Galileo Galilei (1564–1642) and Isaac Newton (1642–1727). The transportation industry relies heavily on physics in the development of engines and the design of aerodynamic vehicles. Entire electronics and computer industries owe their existence to the invention of the transistor, which grew directly out of the laws of physics that describe the electrical behavior of solids. The telecommunications industry depends extensively on electromagnetic waves, whose existence was predicted by James Clerk Maxwell (1831–1879) in his theory of electricity and magnetism. The medical profession uses X-ray, ultrasonic, and magnetic resonance methods for obtaining images of the interior of the human body, and physics lies at the core of all these. Perhaps the most widespread impact in modern technology is that due to the laser. Fields ranging from space exploration to medicine benefit from this incredible device, which is a direct application of the principles of atomic physics.

Because physics is so fundamental, it is a required course for students in a wide range of major areas. We welcome you to the study of this fascinating topic. You will learn how to see the world through the "eyes" of physics and to reason as a physicist does. In the process, you will learn how to apply physics principles to a wide range of problems. We hope that you will come to recognize that physics has important things to say about your environment.

1.2 Units

Physics experiments involve the measurement of a variety of quantities, and a great deal of effort goes into making these measurements as accurate and reproducible as possible. The first step toward ensuring accuracy and reproducibility is defining the units in which the measurements are made.

In this text, we emphasize the system of units known as ***SI units,*** which stands for the French phrase "Le **S**ystème **I**nternational d'Unités." By international agreement, this system employs the ***meter*** (m) as the unit of length, the ***kilogram*** (kg) as the unit of mass, and the ***second*** (s) as the unit of time. Two other systems of units are also in use, however. The CGS system utilizes the centimeter (cm), the gram (g), and the second for length, mass, and time, respectively, and the BE or British Engineering system (the gravitational version) uses the foot (ft), the slug (sl), and the second. Table 1.1 summarizes the units used for length, mass, and time in the three systems.

Originally, the meter was defined in terms of the distance measured along the earth's surface between the north pole and the equator. Eventually, a more accurate measurement standard was needed, and by international agreement the meter became the distance between two marks on a bar of platinum–iridium alloy (see Figure 1.1) kept at a tempera-

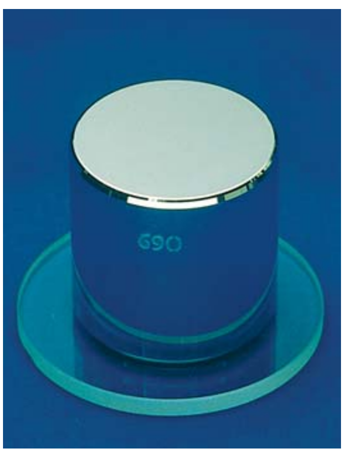

Figure 1.2 The standard platinum–iridium kilogram is kept at the International Bureau of Weights and Measures in Sèvres, France. (Courtesy Bureau International des Poids et Mesures, France)

Table 1.1 *Units of Measurement*

	System		
	SI	CGS	BE
Length	meter (m)	centimeter (cm)	foot (ft)
Mass	kilogram (kg)	gram (g)	slug (sl)
Time	second (s)	second (s)	second (s)

ture of 0 °C. Today, to meet further demands for increased accuracy, the meter is defined as the distance that light travels in a vacuum in a time of 1/299 792 458 second. This definition arises because the speed of light is a universal constant that is defined to be 299 792 458 m/s.

The definition of a kilogram as a unit of mass has also undergone changes over the years. As Chapter 4 discusses, the mass of an object indicates the tendency of the object to continue in motion with a constant velocity. Originally, the kilogram was expressed in terms of a specific amount of water. Today, one kilogram is defined to be the mass of a standard cylinder of platinum–iridium alloy, like the one in Figure 1.2.

As with the units for length and mass, the present definition of the second as a unit of time is different from the original definition. Originally, the second was defined according to the average time for the earth to rotate once about its axis, one day being set equal to 86 400 seconds. The earth's rotational motion was chosen because it is naturally repetitive, occurring over and over again. Today, we still use a naturally occurring repetitive phenomenon to define the second, but of a very different kind. We use the electromagnetic waves emitted by cesium-133 atoms in an atomic clock like that in Figure 1.3. One second is defined as the time needed for 9 192 631 770 wave cycles to occur.*

The units for length, mass, and time, along with a few other units that will arise later, are regarded as ***base*** SI units. The word "base" refers to the fact that these units are used along with various laws to define additional units for other important physical quantities, such as force and energy. The units for such other physical quantities are referred to as ***derived*** units, since they are combinations of the base units. Derived units will be introduced from time to time, as they arise naturally along with the related physical laws.

The value of a quantity in terms of base or derived units is sometimes a very large or very small number. In such cases, it is convenient to introduce larger or smaller units that are related to the normal units by multiples of ten. Table 1.2 summarizes the prefixes that are used to denote multiples of ten. For example, 1000 or 10^3 meters are referred to as 1 kilometer (km), and 0.001 or 10^{-3} meter is called 1 millimeter (mm). Similarly, 1000 grams and 0.001 gram are referred to as 1 kilogram (kg) and 1 milligram (mg), respectively. Appendix A contains a discussion of scientific notation and powers of ten, such as 10^3 and 10^{-3}.

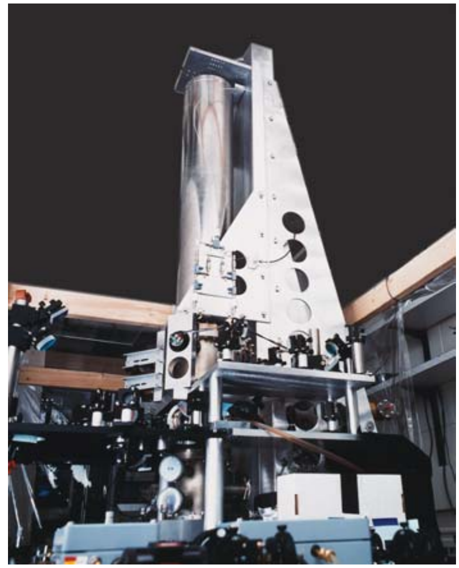

Figure 1.3 This atomic clock, the NIST-F1, is considered one of the world's most accurate clocks. It keeps time with an uncertainty of about one second in twenty million years. (© Geoffrey Wheeler)

1.3 The Role of Units in Problem Solving

THE CONVERSION OF UNITS

Since any quantity, such as length, can be measured in several different units, it is important to know how to convert from one unit to another. For instance, the foot can be used to express the distance between the two marks on the standard platinum–iridium meter bar. There are 3.281 feet in one meter, and this number can be used to convert from meters to feet, as the following example demonstrates.

Table 1.2 *Standard Prefixes Used to Denote Multiples of Ten*

Prefix	Symbol	Factor[a]
Tera	T	10^{12}
Giga[b]	G	10^9
Mega	M	10^6
Kilo	k	10^3
Hecto	h	10^2
Deka	da	10^1
Deci	d	10^{-1}
Centi	c	10^{-2}
Milli	m	10^{-3}
Micro	μ	10^{-6}
Nano	n	10^{-9}
Pico	p	10^{-12}
Femto	f	10^{-15}

[a] Appendix A contains a discussion of powers of ten and scientific notation.
[b] Pronounced jig′a.

Example 1 The World's Highest Waterfall

The highest waterfall in the world is Angel Falls in Venezuela, with a total drop of 979.0 m (see Figure 1.4). Express this drop in feet.

Reasoning When converting between units, we write down the units explicitly in the calculations and treat them like any algebraic quantity. In particular, we will take advantage of the following algebraic fact: Multiplying or dividing an equation by a factor of 1 does not alter an equation.

Solution Since 3.281 feet = 1 meter, it follows that (3.281 feet)/(1 meter) = 1. Using this factor of 1 to multiply the equation "Length = 979.0 meters," we find that

$$\text{Length} = (979.0\text{ m})(1) = (979.0\ \cancel{\text{meters}})\left(\frac{3.281\text{ feet}}{1\ \cancel{\text{meter}}}\right) = \boxed{3212\text{ feet}}$$

* See Chapter 16 for a discussion of waves in general and Chapter 24 for a discussion of electromagnetic waves in particular.

Figure 1.4 Angel Falls in Venezuela is the highest waterfall in the world. (© Kevin Schafer/The Image Bank/Getty Images)

The colored lines emphasize that the units of meters behave like any algebraic quantity and cancel when the multiplication is performed, leaving only the desired unit of feet to describe the answer. In this regard, note that 3.281 feet = 1 meter also implies that (1 meter)/(3.281 feet) = 1. However, we chose not to multiply by a factor of 1 in this form, because the units of meters would not have canceled.

A calculator gives the answer as 3212.099 feet. Standard procedures for significant figures, however, indicate that the answer should be rounded off to four significant figures, since the value of 979.0 meters is accurate to only four significant figures. In this regard, the "1 meter" in the denominator does not limit the significant figures of the answer, because this number is precisely one meter by definition of the conversion factor. Appendix B contains a review of significant figures.

In any conversion, if the units do not combine algebraically to give the desired result, the conversion has not been carried out properly. With this in mind, the next example stresses the importance of writing down the units and illustrates a typical situation in which several conversions are required.

Example 2 Interstate Speed Limit

Express the speed limit of 65 miles/hour in terms of meters/second.

Reasoning As in Example 1, it is important to write down the units explicitly in the calculations and treat them like any algebraic quantity. Here, two well-known relationships come into play—namely, 5280 feet = 1 mile and 3600 seconds = 1 hour. As a result, (5280 feet)/(1 mile) = 1 and (3600 seconds)/(1 hour) = 1. In our solution we will use the fact that multiplying and dividing by these factors of unity does not alter an equation.

Solution Multiplying and dividing by factors of unity, we find the speed limit in feet per second as shown below:

$$\text{Speed} = \left(65\,\frac{\text{miles}}{\text{hour}}\right)(1)(1) = \left(65\,\frac{\cancel{\text{miles}}}{\cancel{\text{hour}}}\right)\left(\frac{5280\text{ feet}}{1\,\cancel{\text{mile}}}\right)\left(\frac{1\,\cancel{\text{hour}}}{3600\text{ s}}\right) = 95\,\frac{\text{feet}}{\text{second}}$$

To convert feet into meters, we use the fact that (1 meter)/(3.281 feet) = 1:

$$\text{Speed} = \left(95\,\frac{\text{feet}}{\text{second}}\right)(1) = \left(95\,\frac{\cancel{\text{feet}}}{\text{second}}\right)\left(\frac{1\text{ meter}}{3.281\,\cancel{\text{feet}}}\right) = \boxed{29\,\frac{\text{meters}}{\text{second}}}$$

Problem solving insight

In addition to their role in guiding the use of conversion factors, units serve a useful purpose in solving problems. They can provide an internal check to eliminate errors, if they are carried along during each step of a calculation and treated like any algebraic factor. In particular, remember that ***only quantities with the same units can be added or subtracted.*** Thus, at one point in a calculation, if you find yourself adding 12 miles to 32 kilometers, stop and reconsider. Either miles must be converted into kilometers or kilometers must be converted into miles before the addition can be carried out.

A collection of useful conversion factors is given on the page facing the inside of the front cover. The reasoning strategy that we have followed in Examples 1 and 2 for converting between units is outlined as follows:

Reasoning Strategy

Converting Between Units

1. In all calculations, write down the units explicitly.
2. Treat all units as algebraic quantities. In particular, when identical units are divided, they are eliminated algebraically.
3. Use the conversion factors located on the page facing the inside of the front cover. Be guided by the fact that multiplying or dividing an equation by a factor of 1 does not alter the equation. For instance, the conversion factor of 3.281 feet = 1 meter might be applied in the form (3.281 feet)/(1 meter) = 1.

This factor of 1 would be used to multiply an equation such as "Length = 5.00 meters" in order to convert meters to feet.

4. Check to see that your calculations are correct by verifying that the units combine algebraically to give the desired unit for the answer. Only quantities with the same units can be added or subtracted.

DIMENSIONAL ANALYSIS

We have seen that many quantities are denoted by specifying both a number and a unit. For example, the distance to the nearest telephone may be 8 meters, or the speed of a car might be 25 meters/second. Each quantity, according to its physical nature, requires a certain *type* of unit. Distance must be measured in a length unit such as meters, feet, or miles, and a time unit will not do. Likewise, the speed of an object must be specified as a length unit divided by a time unit. In physics, the term ***dimension*** is used to refer to the physical nature of a quantity and the type of unit used to specify it. Distance has the dimension of length, which is symbolized as [L], while speed has the dimensions of length [L] divided by time [T], or [L/T]. Many physical quantities can be expressed in terms of a combination of fundamental dimensions such as length [L], time [T], and mass [M]. Later on, we will encounter certain other quantities, such as temperature, which are also fundamental. A fundamental quantity like temperature cannot be expressed as a combination of the dimensions of length, time, mass, or any other fundamental dimension.

Dimensional analysis is used to check mathematical relations for the consistency of their dimensions. As an illustration, consider a car that starts from rest and accelerates to a speed v in a time t. Suppose we wish to calculate the distance x traveled by the car but are not sure whether the correct relation is $x = \frac{1}{2}vt^2$ or $x = \frac{1}{2}vt$. We can decide by checking the quantities on both sides of the equals sign to see whether they have the same dimensions. If the dimensions are not the same, the relation is incorrect. For $x = \frac{1}{2}vt^2$, we use the dimensions for distance [L], time [T], and speed [L/T] in the following way:

$$x = \tfrac{1}{2}vt^2$$

Dimensions

$$[\mathrm{L}] \overset{?}{=} \left[\frac{\mathrm{L}}{\cancel{\mathrm{T}}}\right][\mathrm{T}]^{\cancel{2}} = [\mathrm{L}][\mathrm{T}]$$

Dimensions cancel just like algebraic quantities, and pure numerical factors like $\frac{1}{2}$ have no dimensions, so they can be ignored. The dimension on the left of the equals sign does not match those on the right, so the relation $x = \frac{1}{2}vt^2$ cannot be correct. On the other hand, applying dimensional analysis to $x = \frac{1}{2}vt$, we find that

Problem solving insight
You can check for errors that may have arisen during algebraic manipulations by doing a dimensional analysis on the final expression.

$$x = \tfrac{1}{2}vt$$

Dimensions

$$[\mathrm{L}] \overset{?}{=} \left[\frac{\mathrm{L}}{\cancel{\mathrm{T}}}\right][\cancel{\mathrm{T}}] = [\mathrm{L}]$$

The dimension on the left of the equals sign matches that on the right, so this relation is dimensionally correct. If we know that one of our two choices is the right one, then $x = \frac{1}{2}vt$ is it. In the absence of such knowledge, however, dimensional analysis cannot identify the correct relation. It can only identify which choices *may be* correct, since it does not account for numerical factors like $\frac{1}{2}$ or for the manner in which an equation was derived from physics principles.

1.4 Trigonometry

Scientists use mathematics to help them describe how the physical universe works, and trigonometry is an important branch of mathematics. Three trigonometric functions are utilized throughout this text. They are the sine, the cosine, and the tangent of the angle θ

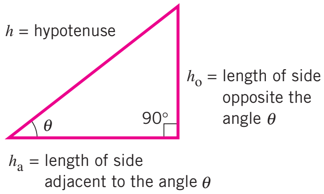

Figure 1.5 A right triangle.

(Greek theta), abbreviated as sin θ, cos θ, and tan θ, respectively. These functions are defined below in terms of the symbols given along with the right triangle in Figure 1.5.

■ DEFINITION OF SIN θ, COS θ, AND TAN θ

$$\sin\theta = \frac{h_o}{h} \tag{1.1}$$

$$\cos\theta = \frac{h_a}{h} \tag{1.2}$$

$$\tan\theta = \frac{h_o}{h_a} \tag{1.3}$$

h = length of the **hypotenuse** of a right triangle
h_o = length of the side **opposite** the angle θ
h_a = length of the side **adjacent** to the angle θ

The sine, cosine, and tangent of an angle are numbers without units, because each is the ratio of the lengths of two sides of a right triangle. Example 3 illustrates a typical application of Equation 1.3.

Example 3 Using Trigonometric Functions

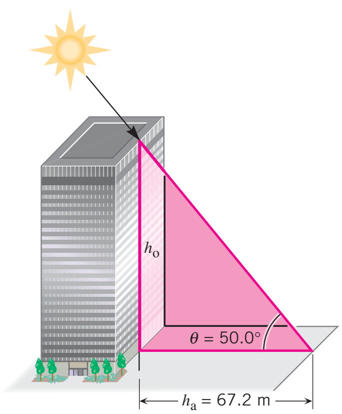

Figure 1.6 From a value for the angle θ and the length h_a of the shadow, the height h_o of the building can be found using trigonometry.

On a sunny day, a tall building casts a shadow that is 67.2 m long. The angle between the sun's rays and the ground is $\theta = 50.0°$, as Figure 1.6 shows. Determine the height of the building.

Reasoning We want to find the height of the building. Therefore, we begin with the colored right triangle in Figure 1.6 and identify the height as the length h_o of the side opposite the angle θ. The length of the shadow is the length h_a of the side that is adjacent to the angle θ. The ratio of the length of the opposite side to the length of the adjacent side is the tangent of the angle θ, which can be used to find the height of the building.

Solution We use the tangent function in the following way, with $\theta = 50.0°$ and $h_a = 67.2$ m:

$$\tan\theta = \frac{h_o}{h_a} \tag{1.3}$$

$$h_o = h_a \tan\theta = (67.2\text{ m})(\tan 50.0°) = (67.2\text{ m})(1.19) = \boxed{80.0\text{ m}}$$

The value of tan 50.0° is found by using a calculator.

Problem solving insight

The sine, cosine, or tangent may be used in calculations such as that in Example 3, depending on which side of the triangle has a known value and which side is asked for. However, ***the choice of which side of the triangle to label h_o (opposite) and which to label h_a (adjacent) can be made only after the angle θ is identified.***

Often the values for two sides of the right triangle in Figure 1.5 are available, and the value of the angle θ is unknown. The concept of ***inverse trigonometric functions*** plays an important role in such situations. Equations 1.4–1.6 give the inverse sine, inverse cosine, and inverse tangent in terms of the symbols used in the drawing. For instance, Equation 1.4 is read as "θ equals the angle whose sine is h_o/h."

$$\theta = \sin^{-1}\left(\frac{h_o}{h}\right) \tag{1.4}$$

$$\theta = \cos^{-1}\left(\frac{h_a}{h}\right) \tag{1.5}$$

$$\theta = \tan^{-1}\left(\frac{h_o}{h_a}\right) \tag{1.6}$$

The use of "-1" as an exponent in Equations 1.4–1.6 *does not mean* "take the reciprocal." For instance, $\tan^{-1}(h_o/h_a)$ does not equal $1/\tan(h_o/h_a)$. Another way to express the inverse trigonometric functions is to use arc sin, arc cos, and arc tan instead of $\sin^{-1}$, $\cos^{-1}$, and $\tan^{-1}$. Example 4 illustrates the use of an inverse trigonometric function.

Example 4 Using Inverse Trigonometric Functions

A lakefront drops off gradually at an angle θ, as Figure 1.7 indicates. For safety reasons, it is necessary to know how deep the lake is at various distances from the shore. To provide some information about the depth, a lifeguard rows straight out from the shore a distance of 14.0 m and drops a weighted fishing line. By measuring the length of the line, the lifeguard determines the depth to be 2.25 m. (a) What is the value of θ? (b) What would be the depth d of the lake at a distance of 22.0 m from the shore?

Reasoning Near the shore, the lengths of the opposite and adjacent sides of the right triangle in Figure 1.7 are $h_o = 2.25$ m and $h_a = 14.0$ m, relative to the angle θ. Having made this identification, we can use the inverse tangent to find the angle in part (a). For part (b) the opposite and adjacent sides farther from the shore become $h_o = d$ and $h_a = 22.0$ m. With the value for θ obtained in part (a), the tangent function can be used to find the unknown depth. Considering the way in which the lake bottom drops off in Figure 1.7, we expect the unknown depth to be greater than 2.25 m.

Solution

(a) Using the inverse tangent given in Equation 1.6, we find that

$$\theta = \tan^{-1}\left(\frac{h_o}{h_a}\right) = \tan^{-1}\left(\frac{2.25\text{ m}}{14.0\text{ m}}\right) = \boxed{9.13°}$$

(b) With $\theta = 9.13°$, the tangent function given in Equation 1.3 can be used to find the unknown depth farther from the shore, where $h_o = d$ and $h_a = 22.0$ m. Since $\tan\theta = h_o/h_a$, it follows that

$$h_o = h_a \tan\theta$$
$$d = (22.0\text{ m})(\tan 9.13°) = \boxed{3.54\text{ m}}$$

which is greater than 2.25 m, as expected.

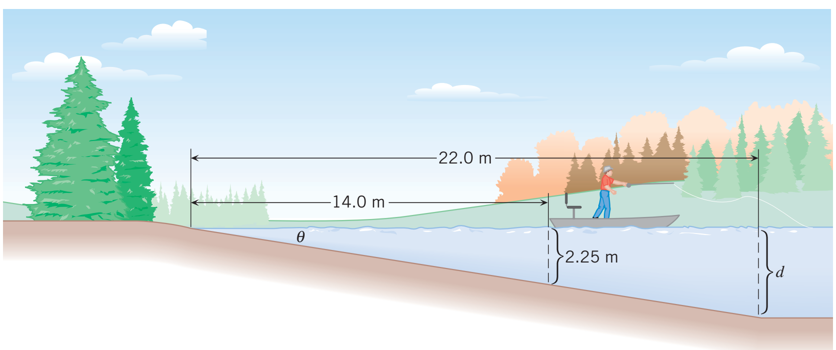

Figure 1.7 If the distance from the shore and the depth of the water at any one point are known, the angle θ can be found with the aid of trigonometry. Knowing the value of θ is useful, because then the depth d at another point can be determined.

The right triangle in Figure 1.5 provides the basis for defining the various trigonometric functions according to Equations 1.1–1.3. These functions always involve an angle and two sides of the triangle. There is also a relationship among the lengths of the three sides of a right triangle. This relationship is known as the ***Pythagorean theorem*** and is used often in this text.

■ PYTHAGOREAN THEOREM

The square of the length of the hypotenuse of a right triangle is equal to the sum of the squares of the lengths of the other two sides:

$$h^2 = h_o^2 + h_a^2 \qquad (1.7)$$

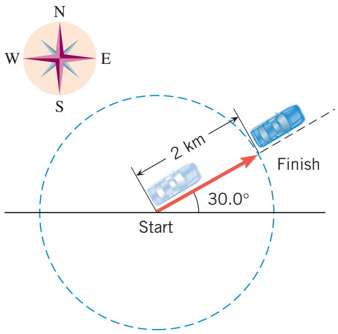

Figure 1.8 A vector quantity has a magnitude and a direction. The arrow in this drawing represents a displacement vector.

The velocity of this in-line skater is another example of a vector quantity, because the velocity has a magnitude (the speed of the skater) and a direction. (© Scott Markewitz/Taxi/Getty Images)

1.5 Scalars and Vectors

The volume of water in a swimming pool might be 50 cubic meters, or the winning time of a race could be 11.3 seconds. In cases like these, only the size of the numbers matters. In other words, *how much* volume or time is there? The "50" specifies the amount of water in units of cubic meters, while the "11.3" specifies the amount of time in seconds. Volume and time are examples of scalar quantities. A ***scalar quantity*** is one that can be described with a single number (including any units) giving its size or magnitude. Some other common scalars are temperature (e.g., 20 °C) and mass (e.g., 85 kg).

While many quantities in physics are scalars, there are also many that are not, and for these quantities the magnitude tells only part of the story. Consider Figure 1.8, which depicts a car that has moved 2 km along a straight line from start to finish. When describing the motion, it is incomplete to say that "the car moved a distance of 2 km." This statement would indicate only that the car ends up somewhere on a circle whose center is at the starting point and whose radius is 2 km. A complete description must include the direction along with the distance, as in the statement "the car moved a distance of 2 km in a direction 30° north of east." A quantity that deals inherently with both magnitude and direction is called a ***vector quantity.*** Because direction is an important characteristic of vectors, arrows are used to represent them; ***the direction of the arrow gives the direction of the vector.*** The colored arrow in Figure 1.8, for example, is called the displacement vector, because it shows how the car is displaced from its starting point. Chapter 2 discusses this particular vector.

The length of the arrow in Figure 1.8 represents the magnitude of the displacement vector. If the car had moved 4 km instead of 2 km from the starting point, the arrow would have been drawn twice as long. ***By convention, the length of a vector arrow is proportional to the magnitude of the vector.***

In physics there are many important kinds of vectors, and the practice of using the length of an arrow to represent the magnitude of a vector applies to each of them. All forces, for instance, are vectors. In common usage a force is a push or a pull, and the direction in which a force acts is just as important as the strength or magnitude of the force. The magnitude of a force is measured in SI units called newtons (N). An arrow representing a force of 20 newtons is drawn twice as long as one representing a force of 10 newtons.

The fundamental distinction between scalars and vectors is the characteristic of direction. Vectors have it, and scalars do not. Conceptual Example 5 helps to clarify this distinction and explains what is meant by the "direction" of a vector.

Conceptual Example 5
Vectors, Scalars, and the Role of Plus and Minus Signs

There are places where the temperature is +20 °C at one time of the year and −20 °C at another time. Do the plus and minus signs that signify positive and negative temperatures imply that temperature is a vector quantity?

Reasoning and Solution A vector has a physical direction associated with it, due east or due west, for example. The question, then, is whether such a direction is associated with temperature. In particular, do the plus and minus signs that go along with temperature imply this kind of direction? On a thermometer, the algebraic signs simply mean that the temperature is a number less than or greater than zero on the scale and have nothing to do with east, west, or any other physical direction. Temperature, then, is not a vector. It is a scalar, and scalars can sometimes be negative. ***The fact that a quantity is positive or negative does not necessarily mean that the quantity is a scalar or a vector.***

Often, for the sake of convenience, quantities such as volume, time, displacement, and force are represented by symbols. This text follows the usual practice of writing vectors in boldface symbols* **(this is boldface)** and writing scalars in italic symbols (*this is italic*). Thus, a displacement vector is written as "**A** = 750 m, due east," where the **A** is a

* A vector quantity can also be represented without boldface symbols, by including an arrow above the symbol—e.g., $\vec{A}$.

boldface symbol. By itself, however, separated from the direction, the magnitude of this vector is a scalar quantity. Therefore, the magnitude is written as "$A = 750$ m," where the A is an italic symbol.

Check Your Understanding 1

Which of the following statements, if any, involves a vector? (a) I walked 2 miles along the beach. (b) I walked 2 miles due north along the beach. (c) I jumped off a cliff and hit the water traveling at 17 miles per hour. (d) I jumped off a cliff and hit the water traveling straight down at 17 miles per hour. (e) My bank account shows a negative balance of −25 dollars. *(The answers are given at the end of the book.)*

Background: These questions deal with the concepts of vectors and scalars, and the difference between them.

For similar questions (including calculational counterparts), consult Self-Assessment Test 1.1. This test is described at the end of Section 1.6.

1.6 Vector Addition and Subtraction

ADDITION

Often it is necessary to add one vector to another, and the process of addition must take into account both the magnitude and the direction of the vectors. The simplest situation occurs when the vectors point along the same direction—that is, when they are colinear, as in Figure 1.9. Here, a car first moves along a straight line, with a displacement vector **A** of 275 m, due east. Then, the car moves again in the same direction, with a displacement vector **B** of 125 m, due east. These two vectors add to give the total displacement vector **R**, which would apply if the car had moved from start to finish in one step. The symbol **R** is used because the total vector is often called the ***resultant vector.*** With the tail of the second arrow located at the head of the first arrow, the two lengths simply add to give the length of the total displacement. This kind of vector addition is identical to the familiar addition of two scalar numbers ($2 + 3 = 5$) *and can be carried out here only because the vectors point along the same direction.* In such cases we add the individual magnitudes to get the magnitude of the total, knowing in advance what the direction must be. Formally, the addition is written as follows:

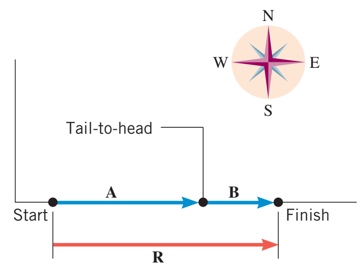

Figure 1.9 Two colinear displacement vectors **A** and **B** add to give the resultant displacement vector **R**.

$$\mathbf{R} = \mathbf{A} + \mathbf{B}$$
$$\mathbf{R} = 275 \text{ m, due east} + 125 \text{ m, due east} = 400 \text{ m, due east}$$

Perpendicular vectors are frequently encountered, and Figure 1.10 indicates how they can be added. This figure applies to a car that first travels with a displacement vector **A** of 275 m, due east, and then with a displacement vector **B** of 125 m, due north. The two vectors add to give a resultant displacement vector **R**. Once again, the vectors to be added are arranged in a tail-to-head fashion, and the resultant vector points from the tail of the first to the head of the last vector added. The resultant displacement is given by the vector equation

$$\mathbf{R} = \mathbf{A} + \mathbf{B}$$

The addition in this equation cannot be carried out by writing $R = 275 \text{ m} + 125 \text{ m}$, because the vectors have different directions. Instead, we take advantage of the fact that the triangle in Figure 1.10 is a right triangle and use the Pythagorean theorem (Equation 1.7). According to this theorem, the magnitude of **R** is

$$R = \sqrt{(275 \text{ m})^2 + (125 \text{ m})^2} = 302 \text{ m}$$

The angle θ in Figure 1.10 gives the direction of the resultant vector. Since the lengths of all three sides of the right triangle are now known, either $\sin\theta$, $\cos\theta$, or $\tan\theta$ can be used to determine θ. Noting that $\tan\theta = B/A$ and using the inverse trigonometric function, we find that:

$$\theta = \tan^{-1}\left(\frac{B}{A}\right) = \tan^{-1}\left(\frac{125 \text{ m}}{275 \text{ m}}\right) = 24.4°$$

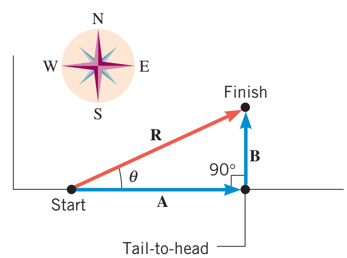

Figure 1.10 The addition of two perpendicular displacement vectors **A** and **B** gives the resultant vector **R**.

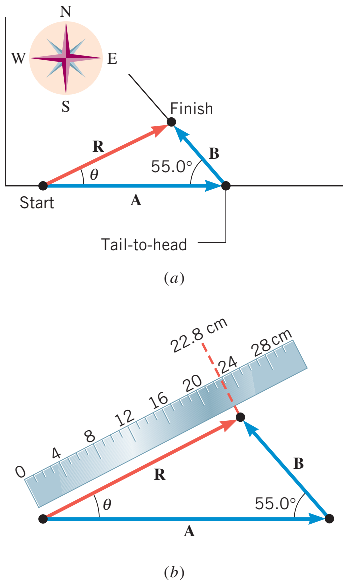

Figure 1.11 (*a*) The two displacement vectors **A** and **B** are neither colinear nor perpendicular but even so they add to give the resultant vector **R**. (*b*) In one method for adding them together, a graphical technique is used.

Thus, the resultant displacement of the car has a magnitude of 302 m and points north of east at an angle of 24.4°. This displacement would bring the car from the start to the finish in Figure 1.10 in a single straight-line step.

When two vectors to be added are not perpendicular, the tail-to-head arrangement does not lead to a right triangle, and the Pythagorean theorem cannot be used. Figure 1.11*a* illustrates such a case for a car that moves with a displacement **A** of 275 m, due east, and then with a displacement **B** of 125 m in a direction 55.0° north of west. As usual, the resultant displacement vector **R** is directed from the tail of the first to the head of the last vector added. The vector addition is still given according to

$$\mathbf{R} = \mathbf{A} + \mathbf{B}$$

However, the magnitude of **R** is not $R = \sqrt{A^2 + B^2}$, because the vectors **A** and **B** are not perpendicular and the Pythagorean theorem does not apply. Some other means must be used to find the magnitude and direction of the resultant vector.

One approach uses a graphical technique. In this method, a diagram is constructed in which the arrows are drawn tail to head. The lengths of the vector arrows are drawn to scale, and the angles are drawn accurately (with a protractor, perhaps). Then, the length of the arrow representing the resultant vector is measured with a ruler. This length is converted to the magnitude of the resultant vector by using the scale factor with which the drawing is constructed. In Figure 1.11*b*, for example, a scale of one centimeter of arrow length for each 10.0 m of displacement is used, and it can be seen that the length of the arrow representing **R** is 22.8 cm. Since each centimeter corresponds to 10.0 m of displacement, the magnitude of **R** is 228 m. The angle θ, which gives the direction of **R**, can be measured with a protractor to be $\theta = 26.7°$ north of east.

Check Your Understanding 2

Two vectors, **A** and **B**, are added by means of vector addition to give a resultant vector **R**: **R** = **A** + **B**. The magnitudes of **A** and **B** are 3 and 8 m, but they can have any orientation. What is (a) the maximum possible value and (b) the minimum possible value for the magnitude of **R**? *(The answers are given at the end of the book.)*

Background: These questions deal with adding two vectors by means of the tail-to-head method. They illustrate how the two vectors must be oriented relative to each other to produce a resultant vector that has the greatest possible magnitude and one that has the least possible magnitude.

For similar questions (including conceptual counterparts), consult Self-Assessment Test 1.1. The test is described at the end of this section.

SUBTRACTION

The subtraction of one vector from another is carried out in a way that depends on the following fact. *When a vector is multiplied by −1, the magnitude of the vector remains the same, but the direction of the vector is reversed.* Conceptual Example 6 illustrates the meaning of this statement.

Conceptual Example 6 Multiplying a Vector by −1

Consider the two vectors described as follows:

1. A woman climbs 1.2 m up a ladder, so that her displacement vector **D** is 1.2 m, upward along the ladder, as in Figure 1.12*a*.
2. A man is pushing with 450 N of force on his stalled car, trying to move it eastward. The force vector **F** that he applies to the car is 450 N, due east, as in Figure 1.13*a*.

What are the physical meanings of the vectors −**D** and −**F**?

Reasoning and Solution A displacement vector of −**D** is (−1)**D** and has the same magnitude as the vector **D**, but is opposite in direction. Thus, −**D** would represent the displacement of a woman climbing 1.2 m down the ladder, as in Figure 1.12*b*. Similarly, a force vector of −**F** has the same magnitude as the vector **F** but has the opposite direction. As a result, −**F**

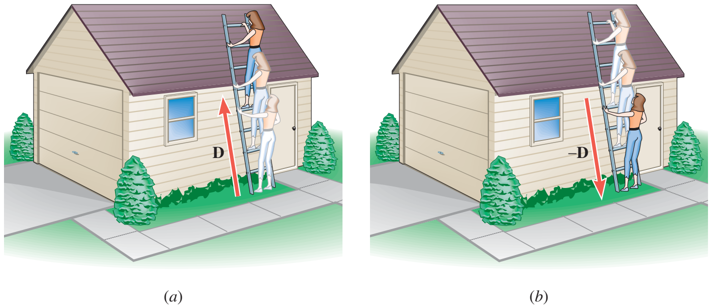

Figure 1.12 (*a*) The displacement vector for a woman climbing 1.2 m up a ladder is **D**. (*b*) The displacement vector for a woman climbing 1.2 m down a ladder is **−D**.

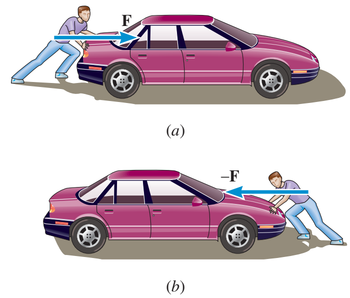

Figure 1.13 (*a*) The force vector for a man pushing on a car with 450 N of force in a direction due east is **F**. (*b*) The force vector for a man pushing on a car with 450 N of force in a direction due west is **−F**.

would represent a force of 450 N applied to the car in a direction of due west, as in Figure 1.13*b*.

Related Homework: *Conceptual Question 15, Problem 62*

In practice, vector subtraction is carried out exactly like vector addition, except that one of the vectors added is multiplied by a scalar factor of −1. To see why, look at the two vectors **A** and **B** in Figure 1.14*a*. These vectors add together to give a third vector **C**, according to **C = A + B**. Therefore, we can calculate vector **A** as **A = C − B**, which is an example of vector subtraction. However, we can also write this result as **A = C + (−B)** and treat it as vector addition. Figure 1.14*b* shows how to calculate vector **A** by adding the vectors **C** and **−B**. Notice that vectors **C** and **−B** are arranged tail to head and that any suitable method of vector addition can be employed to determine **A**.

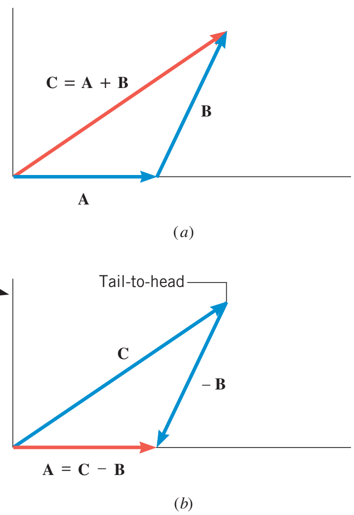

Figure 1.14 (*a*) Vector addition according to **C = A + B**. (*b*) Vector subtraction according to **A = C − B = C + (−B)**.

Self-Assessment Test 1.1

Test your understanding of the concepts discussed in Sections 1.5 and 1.6:

- Adding and Subtracting Vectors by the Tail-to-Head Method
- Using the Pythagorean Theorem and Trigonometry to Find the Magnitude and Direction of the Resultant Vector

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1.7 *The Components of a Vector*

VECTOR COMPONENTS

Suppose a car moves along a straight line from start to finish in Figure 1.15, the corresponding displacement vector being **r**. The magnitude and direction of the vector **r** give the distance and direction traveled along the straight line. However, the car could also arrive at the finish point by first moving due east, turning through 90°, and then moving due north. This alternative path is shown in the drawing and is associated with the two displacement vectors **x** and **y**. The vectors **x** and **y** are called the *x* vector component and the *y* vector component of **r**.

Vector components are very important in physics and have two basic features that are apparent in Figure 1.15. One is that the components add together to equal the original vector:

$$\mathbf{r} = \mathbf{x} + \mathbf{y}$$

The components **x** and **y**, when added vectorially, convey exactly the same meaning as does the original vector **r**: they indicate how the finish point is displaced relative to the starting point. In general, ***the components of any vector can be used in place of the vector itself in any calculation where it is convenient to do so.*** The other feature of vector components that is apparent in Figure 1.15 is that **x** and **y** are not just any two vectors that add together to give the original vector **r**: they are perpendicular vectors. This perpendicularity is a valuable characteristic, as we will soon see.

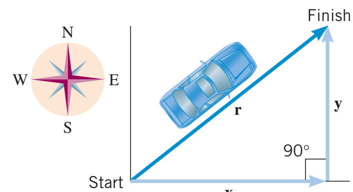


Figure 1.15 The displacement vector **r** and its vector components **x** and **y**.

Any type of vector may be expressed in terms of its components, in a way similar to that illustrated for the displacement vector in Figure 1.15. Figure 1.16 shows an arbitrary vector **A** and its vector components $\mathbf{A}_x$ and $\mathbf{A}_y$. The components are drawn parallel to convenient x and y axes and are perpendicular. They add vectorially to equal the original vector **A**:

$$\mathbf{A} = \mathbf{A}_x + \mathbf{A}_y$$

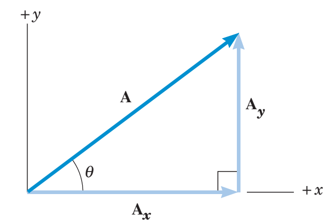


Figure 1.16 An arbitrary vector **A** and its vector components $\mathbf{A}_x$ and $\mathbf{A}_y$.

There are times when a drawing such as Figure 1.16 is not the most convenient way to represent vector components, and Figure 1.17 presents an alternative method. The disadvantage of this alternative is that the tail-to-head arrangement of $\mathbf{A}_x$ and $\mathbf{A}_y$ is missing, an arrangement that is a nice reminder that $\mathbf{A}_x$ and $\mathbf{A}_y$ add together to equal **A**.

The definition that follows summarizes the meaning of vector components:

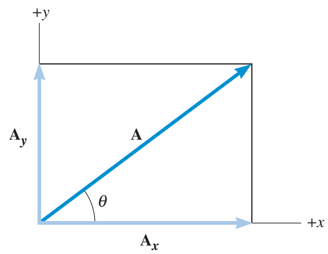


Figure 1.17 This alternative way of drawing the vector **A** and its vector components is completely equivalent to that shown in Figure 1.16.

DEFINITION OF VECTOR COMPONENTS

In two dimensions, the vector components of a vector **A** are two perpendicular vectors $\mathbf{A}_x$ and $\mathbf{A}_y$ that are parallel to the x and y axes, respectively, and add together vectorially so that $\mathbf{A} = \mathbf{A}_x + \mathbf{A}_y$.

The values calculated for vector components depend on the orientation of the vector relative to the axes used as a reference. Figure 1.18 illustrates this fact for a vector **A** by showing two sets of axes, one set being rotated clockwise relative to the other. With respect to the black axes, vector **A** has perpendicular vector components $\mathbf{A}_x$ and $\mathbf{A}_y$; with respect to the colored rotated axes, vector **A** has different vector components $\mathbf{A}_x'$ and $\mathbf{A}_y'$. The choice of which set of components to use is purely a matter of convenience.

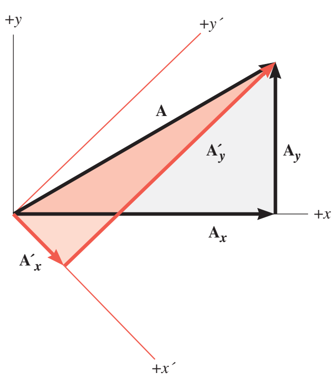


Figure 1.18 The vector components of the vector depend on the orientation of the axes used as a reference.

SCALAR COMPONENTS

It is often easier to work with the ***scalar components,*** A_x and A_y (note the italic symbols), rather than the vector components $\mathbf{A}_x$ and $\mathbf{A}_y$. Scalar components are positive or negative numbers (with units) that are defined as follows. The component A_x has a magnitude that is equal to that of $\mathbf{A}_x$ and is given a positive sign if $\mathbf{A}_x$ points along the $+x$ axis and a negative sign if $\mathbf{A}_x$ points along the $-x$ axis. The component A_y is defined in a similar manner. The following table shows an example of vector and scalar components:

Vector Components	Scalar Components	Unit Vectors
$\mathbf{A}_x$ = 8 meters, directed along the $+x$ axis	$A_x = +8$ meters	$\mathbf{A}_x = (+8 \text{ meters})\,\hat{\mathbf{x}}$
$\mathbf{A}_y$ = 10 meters, directed along the $-y$ axis	$A_y = -10$ meters	$\mathbf{A}_y = (-10 \text{ meters})\,\hat{\mathbf{y}}$

In this text, when we use the term "component," we will be referring to a scalar component, unless otherwise indicated.

Another method of expressing vector components is to use unit vectors. A ***unit vector*** is a vector that has a magnitude of 1, but no dimensions. We will use a caret (^) to distinguish it from other vectors. Thus,

$\hat{\mathbf{x}}$ is a dimensionless unit vector of length 1 that points in the positive x direction, and

$\hat{\mathbf{y}}$ is a dimensionless unit vector of length 1 that points in the positive y direction.

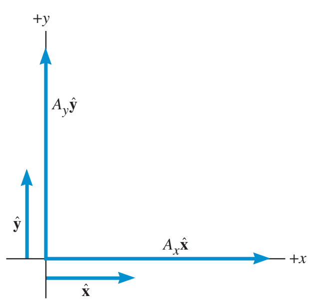
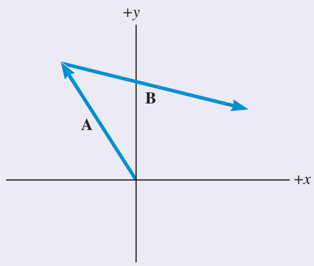


Figure 1.19 The dimensionless unit vectors $\hat{\mathbf{x}}$ and $\hat{\mathbf{y}}$ have magnitudes equal to 1, and they point in the $+x$ and $+y$ directions, respectively. Expressed in terms of unit vectors, the vector components of the vector **A** are $A_x\,\hat{\mathbf{x}}$ and $A_y\,\hat{\mathbf{y}}$.

These unit vectors are illustrated in Figure 1.19. With the aid of unit vectors, the vector components of an arbitrary vector **A** can be written as $\mathbf{A}_x = A_x\,\hat{\mathbf{x}}$ and $\mathbf{A}_y = A_y\,\hat{\mathbf{y}}$, where A_x and A_y are its scalar components (see the drawing and third column of the table above). The vector **A** is then written as $\mathbf{A} = A_x\,\hat{\mathbf{x}} + A_y\,\hat{\mathbf{y}}$.

Check Your Understanding 3

Two vectors, **A** and **B**, are shown in the drawing that follows. (a) What are the signs (+ or −) of the scalar components A_x and A_y of vector **A**? (b) What are the signs of the scalar components B_x and B_y of vector **B**? (c) What are the signs of the scalar components R_x and R_y of the resultant vector **R**, where **R** = **A** + **B**? (*The answers are given at the end of the book.*)



Background: Two concepts play a role in this question: the scalar components of a vector, and how two vectors are added by means of the tail-to-head method to produce a resultant vector.

For similar questions (including calculational counterparts), consult Self-Assessment Test 1.2. This test is described at the end of Section 1.8.

RESOLVING A VECTOR INTO ITS COMPONENTS

If the magnitude and direction of a vector are known, it is possible to find the components of the vector. The process of finding the components is called "resolving the vector into its components." As Example 7 illustrates, this process can be carried out with the aid of trigonometry, because the two perpendicular vector components and the original vector form a right triangle.

Example 7 Finding the Components of a Vector

A displacement vector **r** has a magnitude of $r = 175$ m and points at an angle of 50.0° relative to the x axis in Figure 1.20. Find the x and y components of this vector.

Reasoning We will base our solution on the fact that the triangle formed in Figure 1.20 by the vector **r** and its components **x** and **y** is a right triangle. This fact enables us to use the trigonometric sine and cosine functions, as defined in Equations 1.1 and 1.2.

Solution 1 The y component can be obtained using the 50.0° angle and Equation 1.1, $\sin\theta = y/r$:

$$y = r\sin\theta = (175\text{ m})(\sin 50.0°) = \boxed{134\text{ m}}$$

In a similar fashion, the x component can be obtained using the 50.0° angle and Equation 1.2, $\cos\theta = x/r$:

$$x = r\cos\theta = (175\text{ m})(\cos 50.0°) = \boxed{112\text{ m}}$$

Solution 2 The angle α in Figure 1.20 can also be used to find the components. Since $\alpha + 50.0° = 90.0°$, it follows that $\alpha = 40.0°$. The solution using α yields the same answers as in Solution 1:

$$\cos\alpha = \frac{y}{r}$$

$$y = r\cos\alpha = (175\text{ m})(\cos 40.0°) = \boxed{134\text{ m}}$$

$$\sin\alpha = \frac{x}{r}$$

$$x = r\sin\alpha = (175\text{ m})(\sin 40.0°) = \boxed{112\text{ m}}$$

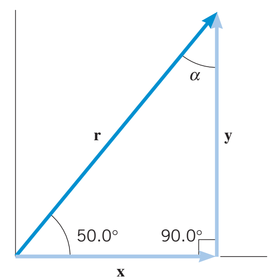


Figure 1.20 The x and y components of the displacement vector **r** can be found using trigonometry.

Problem solving insight
Either acute angle of a right triangle can be used to determine the components of a vector. The choice of angle is a matter of convenience.

Since the vector components and the original vector form a right triangle, the Pythagorean theorem can be applied to check the validity of calculations such as those in Example 7. Thus, with the components obtained in Example 7, the theorem can be used to verify that the magnitude of the original vector is indeed 175 m, as given initially:

$$r = \sqrt{(112\text{ m})^2 + (134\text{ m})^2} = 175\text{ m}$$

Problem solving insight
You can check to see whether the components of a vector are correct by substituting them into the Pythagorean theorem and verifying that the result is the magnitude of the original vector.

Problem solving insight

It is possible for one of the components of a vector to be zero. This does not mean that the vector itself is zero, however. ***For a vector to be zero, every vector component must individually be zero.*** Thus, in two dimensions, saying that $\mathbf{A} = 0$ is equivalent to saying that $\mathbf{A}_x = 0$ and $\mathbf{A}_y = 0$. Or, stated in terms of scalar components, if $\mathbf{A} = 0$, then $A_x = 0$ and $A_y = 0$.

Problem solving insight

Two vectors are equal if, and only if, they have the same magnitude and direction. Thus, if one displacement vector points east and another points north, they are *not* equal, even if each has the same magnitude of 480 m. In terms of vector components, two vectors, $\mathbf{A}$ and $\mathbf{B}$, are equal if, and only if, each vector component of one is equal to the corresponding vector component of the other. In two dimensions, if $\mathbf{A} = \mathbf{B}$, then $\mathbf{A}_x = \mathbf{B}_x$ and $\mathbf{A}_y = \mathbf{B}_y$. Alternatively, using scalar components, we write that $A_x = B_x$ and $A_y = B_y$.

1.8 Addition of Vectors by Means of Components

The components of a vector provide the most convenient and accurate way of adding (or subtracting) any number of vectors. For example, suppose that vector $\mathbf{A}$ is added to vector $\mathbf{B}$. The resultant vector is $\mathbf{C}$, where $\mathbf{C} = \mathbf{A} + \mathbf{B}$. Figure 1.21*a* illustrates this vector addition, along with the *x* and *y* vector components of $\mathbf{A}$ and $\mathbf{B}$. In part *b* of the drawing, the vectors $\mathbf{A}$ and $\mathbf{B}$ have been removed, because we can use the vector components of these vectors in place of them. The vector component $\mathbf{B}_x$ has been shifted downward and arranged tail to head with the vector component $\mathbf{A}_x$. Similarly, the vector component $\mathbf{A}_y$ has been shifted to the right and arranged tail to head with the vector component $\mathbf{B}_y$. The *x* components are colinear and add together to give the *x* component of the resultant vector $\mathbf{C}$. In like fashion, the *y* components are colinear and add together to give the *y* component of $\mathbf{C}$. In terms of scalar components, we can write

$$C_x = A_x + B_x \quad \text{and} \quad C_y = A_y + B_y$$

The vector components $\mathbf{C}_x$ and $\mathbf{C}_y$ of the resultant vector form the sides of the right triangle shown in Figure 1.21*c*. Thus, we can find the magnitude of $\mathbf{C}$ by using the Pythagorean theorem:

$$C = \sqrt{C_x^{\,2} + C_y^{\,2}}$$

The angle θ that $\mathbf{C}$ makes with the *x* axis is given by $\theta = \tan^{-1}(C_y/C_x)$. Example 8 illustrates how to add several vectors using the component method.

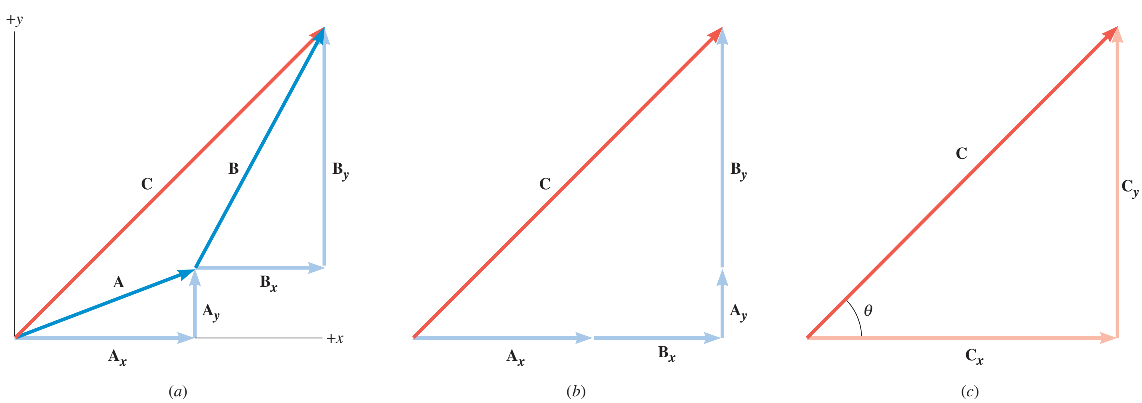


Figure 1.21 (*a*) The vectors $\mathbf{A}$ and $\mathbf{B}$ add together to give the resultant vector $\mathbf{C}$. The *x* and *y* vector components of $\mathbf{A}$ and $\mathbf{B}$ are also shown. (*b*) The drawing illustrates that $\mathbf{C}_x = \mathbf{A}_x + \mathbf{B}_x$ and $\mathbf{C}_y = \mathbf{A}_y + \mathbf{B}_y$. (*c*) Vector $\mathbf{C}$ and its components form a right triangle.

Example 8 The Component Method of Vector Addition

A jogger runs 145 m in a direction 20.0° east of north (displacement vector **A**) and then 105 m in a direction 35.0° south of east (displacement vector **B**). Determine the magnitude and direction of the resultant vector **C** for these two displacements.

Reasoning Figure 1.22*a* shows the vectors **A** and **B**, assuming that the y axis corresponds to the direction due north. Since the vectors are not given in component form, we will begin by using the given magnitudes and directions to find the components. Then, the components of **A** and **B** can be used to find the components of the resultant **C**. Finally, with the aid of the Pythagorean theorem and trigonometry, the components of **C** can be used to find its magnitude and direction.

Solution The first two rows of the following table give the x and y components of the vectors **A** and **B**. Note that the component B_y is negative, because $\mathbf{B}_y$ points downward, in the negative y direction in the drawing.

Vector	x Component	y Component
A	$A_x = (145\text{ m})\sin 20.0° = 49.6\text{ m}$	$A_y = (145\text{ m})\cos 20.0° = 136\text{ m}$
B	$B_x = (105\text{ m})\cos 35.0° = 86.0\text{ m}$	$B_y = -(105\text{ m})\sin 35.0° = -60.2\text{ m}$
C	$C_x = A_x + B_x = 135.6\text{ m}$	$C_y = A_y + B_y = 76\text{ m}$

The third row in the table gives the x and y components of the resultant vector **C**: $C_x = A_x + B_x$ and $C_y = A_y + B_y$. Part *b* of the drawing shows **C** and its vector components. The magnitude of **C** is given by the Pythagorean theorem as

$$C = \sqrt{C_x^2 + C_y^2} = \sqrt{(135.6\text{ m})^2 + (76\text{ m})^2} = \boxed{155\text{ m}}$$

The angle θ that **C** makes with the x axis is

$$\theta = \tan^{-1}\left(\frac{C_y}{C_x}\right) = \tan^{-1}\left(\frac{76\text{ m}}{135.6\text{ m}}\right) = \boxed{29°}$$

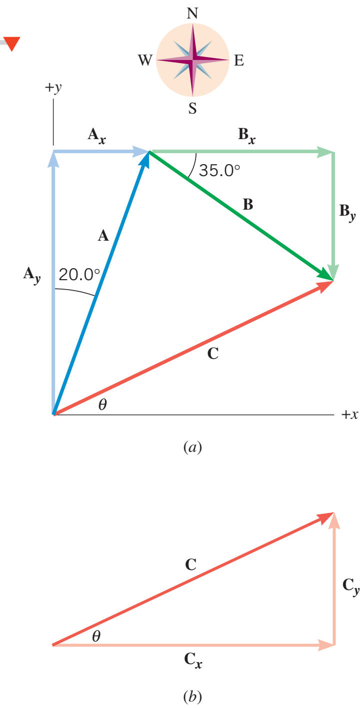
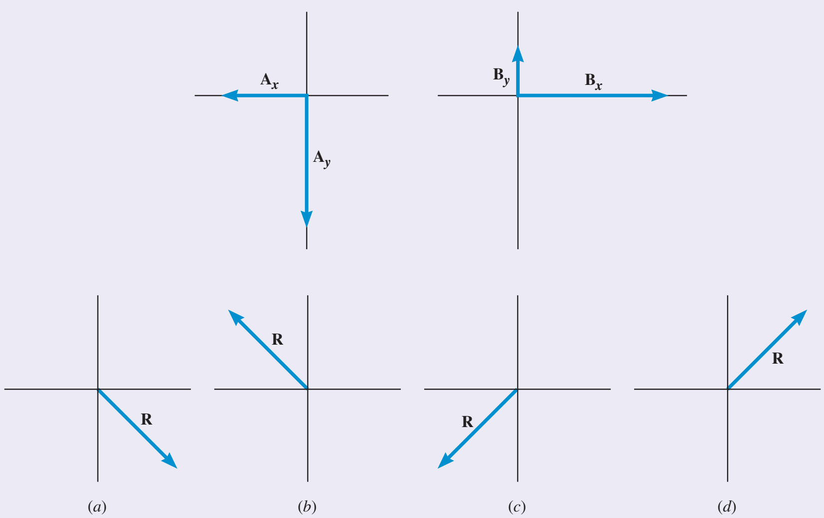


Figure 1.22 (*a*) The vectors **A** and **B** add together to give the resultant vector **C**. The vector components of **A** and **B** are also shown. (*b*) The resultant vector **C** can be obtained once its components have been found.

Check Your Understanding 4

Two vectors, **A** and **B**, have vector components that are shown (to the same scale) in the first row of drawings. Which vector **R** in the second row of drawings is the vector sum of **A** and **B**? *(The answer is given at the end of the book.)*



Background: The concept of adding vectors by means of vector components is featured in this question. Note that the x components of **A** and **B** point in opposite directions, as do the y components.

For similar questions (including calculational counterparts), consult Self-Assessment Test 1.2. The test is described at the end of this section.

Concept Simulation 1.1

This simulation illustrates how two vectors can be added by the tail-to-head method and by the component method. The user can change the magnitude and direction of each vector, and the simulation shows how their components, as well as those of the resultant vector, change.

Related Homework: *Problems 40, 45, 55*

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In later chapters we will often use the component method for vector addition. For future reference, the main features of the reasoning strategy used in this technique are summarized below.

Reasoning Strategy

The Component Method of Vector Addition

1. For each vector to be added, determine the x and y components relative to a conveniently chosen x, y coordinate system. Be sure to take into account the directions of the components by using plus and minus signs to denote whether the components point along the positive or negative axes.
2. Find the algebraic sum of the x components, which is the x component of the resultant vector. Similarly, find the algebraic sum of the y components, which is the y component of the resultant vector.
3. Use the x and y components of the resultant vector and the Pythagorean theorem to determine the magnitude of the resultant vector.
4. Use either the inverse sine, inverse cosine, or inverse tangent function to find the angle that specifies the direction of the resultant vector.

Self-Assessment Test 1.2

Test your understanding of the concepts discussed in Sections 1.7 and 1.8:

- The Vector and Scalar Components of a Vector Component Method
- Adding Vectors by the

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1.9 *Concepts & Calculations*

This chapter has presented an introduction to the mathematics of trigonometry and vectors, which will be used throughout this text. Therefore, in this last section we consider several examples in order to review some of the important features of this mathematics. The three-part format of these examples stresses the role of conceptual understanding in problem solving. First, the problem statement is given. Then, there is a concept question-and-answer section, which is followed by the solution section. The purpose of the concept question-and-answer section is to provide help in understanding the solution and to illustrate how a review of the concepts can help in anticipating some of the characteristics of the numerical answers.

Concepts & Calculations Example 9 Equal Vectors

Figure 1.23 shows two displacement vectors **A** and **B**. Vector **A** points at an angle of 22.0° above the x axis but has an unknown magnitude. Vector **B** has an x component of B_x = 35.0 m but has an unknown y component B_y. These two vectors are equal. Find the magnitude of **A** and the value of B_y.

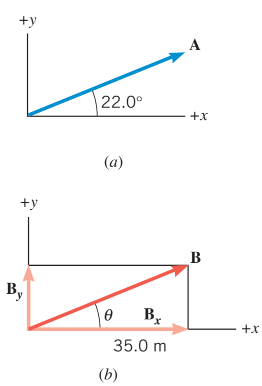


Figure 1.23 The two displacement vectors **A** and **B** are equal. Example 9 discusses what this equality means.

Concept Questions and Answers What does the fact that vector **A** equals vector **B** imply about the magnitudes and directions of the vectors?

Answer When two vectors are equal, each has the same magnitude and each has the same direction.

What does the fact that vector **A** equals vector **B** imply about the x and y components of the vectors?

Answer When two vectors are equal, the x component of vector **A** equals the x component of vector **B** ($A_x = B_x$) and the y component of vector **A** equals the y component of vector **B** ($A_y = B_y$).

Solution We focus on the fact that the x components of the vectors are the same and the y components of the vectors are the same. This allows us to write that

$$\underbrace{A \cos 22.0°}_{\substack{\text{Component } A_x \\ \text{of vector } \mathbf{A}}} = \underbrace{35.0 \text{ m}}_{\substack{\text{Component } B_x \\ \text{of vector } \mathbf{B}}} \quad (1.8)$$

$$\underbrace{A \sin 22.0°}_{\substack{\text{Component } A_y \\ \text{of vector } \mathbf{A}}} = \underbrace{B_y}_{\substack{\text{Component } B_y \\ \text{of vector } \mathbf{B}}} \quad (1.9)$$

Dividing Equation 1.9 by Equation 1.8 shows that

$$\frac{A \sin 22.0°}{A \cos 22.0°} = \frac{B_y}{35.0 \text{ m}}$$

$$B_y = (35.0 \text{ m}) \frac{\sin 22.0°}{\cos 22.0°} = (35.0 \text{ m}) \tan 22.0° = \boxed{14.1 \text{ m}}$$

Solving Equation 1.8 directly for A gives

$$A = \frac{35.0 \text{ m}}{\cos 22.0°} = \boxed{37.7 \text{ m}}$$

Concepts & Calculations Example 10 Adding Vectors

Figure 1.24*a* shows two displacement vectors **A** and **B**, which add together to give a resultant displacement **C**. Find the magnitude and direction of **C**.

Concept Questions and Answers Does the Pythagorean theorem apply directly to this problem, so that the magnitude of vector **C** is given by $C = \sqrt{A^2 + B^2}$?

Answer The magnitude of the vector **C** is not given by the Pythagorean theorem in the form $C = \sqrt{A^2 + B^2}$, because the vectors **A** and **B** are not perpendicular (see Figure 1.24*a*). It is only when the two vectors are perpendicular that this equation for the magnitude applies.

Does the component method for vector addition apply to this problem, even though the vectors are not perpendicular?

Answer Yes. The component method applies whether or not the vectors are perpendicular, and it applies to any number of vectors. We will use it in our solution.

Solution The components of the vectors **A** and **B** can be determined from the data in Figure 1.24*a* and are listed in the first two rows of the following table. Note that the vector **A** points along the x axis and, therefore, has no y component.

Vector	x Component	y Component
A	$A_x = 47.0$ m	$A_y = 0$ m
B	$B_x = (62.0 \text{ m}) \cos 35.0° = 50.8$ m	$B_y = (62.0 \text{ m}) \sin 35.0° = 35.6$ m
C	$C_x = A_x + B_x = 97.8$ m	$C_y = A_y + B_y = 35.6$ m

Figure 1.24*b* shows the components of the resultant vector, and the third row in the table gives their values. The components C_x and C_y are perpendicular, so that we can use the Pythagorean theorem to find the magnitude of **C**:

$$C = \sqrt{C_x^2 + C_y^2} = \sqrt{(97.8 \text{ m})^2 + (35.6 \text{ m})^2} = \boxed{104 \text{ m}}$$

The value of the directional angle θ is

$$\theta = \tan^{-1}\left(\frac{C_y}{C_x}\right) = \tan^{-1}\left(\frac{35.6 \text{ m}}{97.8 \text{ m}}\right) = \boxed{20.0°}$$

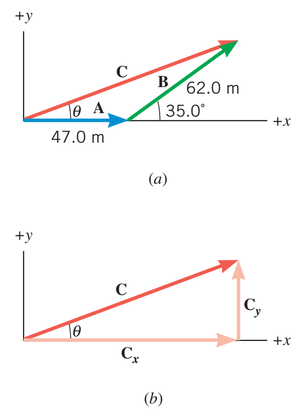


Figure 1.24 (*a*) Two displacement vectors **A** and **B** add together to give a resultant vector **C**. (*b*) The components of **C** are $\mathbf{C_x}$ and $\mathbf{C_y}$.

At the end of the problem set for this chapter, you will find homework problems that contain both conceptual and quantitative parts. These problems are grouped under the heading *Concepts & Calculations, Group Learning Problems.* They are designed for use by students working alone or in small learning groups. The conceptual part of each problem provides a convenient focus for group discussions.

Concept Summary

This summary presents an abridged version of the chapter, including the important equations and all available learning aids. For convenient reference, the learning aids (including the text's examples) are placed next to or immediately after the relevant equation or discussion. The following learning aids may be found on-line at **www.wiley.com/college/cutnell:**

Interactive LearningWare examples are solved according to a five-step interactive format that is designed to help you develop problem-solving skills.	**Concept Simulations** are animated versions of text figures or animations that illustrate important concepts. You can control parameters that affect the display, and we encourage you to experiment.
Interactive Solutions offer specific models for certain types of problems in the chapter homework. The calculations are carried out interactively.	**Self-Assessment Tests** include both qualitative and quantitative questions. Extensive feedback is provided for both incorrect and correct answers, to help you evaluate your understanding of the material.

Topic	***Discussion***	***Learning Aids***
	1.2 Units	
	The SI system of units includes the meter (m), the kilogram (kg), and the second (s) as the base units for length, mass, and time, respectively.	
Meter	One meter is the distance that light travels in a vacuum in a time of 1/299 792 458 second.	
Kilogram	One kilogram is the mass of a standard cylinder of platinum–iridium alloy kept at the International Bureau of Weights and Measures.	
Second	One second is the time for a certain type of electromagnetic wave emitted by cesium-133 atoms to undergo 9 192 631 770 wave cycles.	
	1.3 The Role of Units in Problem Solving	
Conversion of units	To convert a number from one unit to another, multiply the number by the ratio of the two units. For instance, to convert 979 meters to feet, multiply 979 meters by the factor (3.281 foot/1 meter).	**Examples 1, 2**
Dimension	The dimension of a quantity represents its physical nature and the type of unit used to specify it. Three such dimensions are length [L], mass [M], time [T].	
Dimensional analysis	A method for checking mathematical relations for the consistency of their dimensions.	
	1.4 Trigonometry	
Sine, cosine, and tangent of an angle θ	The sine, cosine, and tangent functions of an angle θ are defined in terms of a right triangle that contains θ: $$\sin\theta = \frac{h_o}{h} \quad (1.1) \qquad \cos\theta = \frac{h_a}{h} \quad (1.2) \qquad \tan\theta = \frac{h_o}{h_a} \quad (1.3)$$ where h_o and h_a are, respectively, the lengths of the sides opposite and adjacent to the angle θ, and h is the length of the hypotenuse.	**Example 3** **Interactive Solution 1.17**
Inverse trigonometric functions	The inverse sine, inverse cosine, and inverse tangent functions are $$\theta = \sin^{-1}\left(\frac{h_o}{h}\right) \quad (1.4) \qquad \theta = \cos^{-1}\left(\frac{h_a}{h}\right) \quad (1.5)$$ $$\theta = \tan^{-1}\left(\frac{h_o}{h_a}\right) \quad (1.6)$$	**Example 4**
Pythagorean theorem	The Pythagorean theorem states that the square of the length of the hypotenuse of a right triangle is equal to the sum of the squares of the lengths of the other two sides: $$h^2 = h_o^2 + h_a^2 \qquad (1.7)$$	
	1.5 Scalars and Vectors	
Scalars and vectors	A scalar quantity is described completely by its size, which is also called its magnitude. A vector quantity has both a magnitude and a direction. Vectors are often represented by arrows, the length of the arrow being proportional to the magnitude of the vector and the direction of the arrow indicating the direction of the vector.	**Example 5**

Topic	Discussion	Learning Aids
	1.6 Vector Addition and Subtraction	
Graphical method of vector addition and subtraction	One procedure for adding vectors utilizes a graphical technique, in which the vectors to be added are arranged in a tail-to-head fashion. The resultant vector is drawn from the tail of the first vector to the head of the last vector.	
	The subtraction of a vector is treated as the addition of a vector that has been multiplied by a scalar factor of −1. Multiplying a vector by −1 reverses the direction of the vector.	**Example 6** Interactive Solution 1.29

Use Self-Assessment Test 1.1 to evaluate your understanding of Sections 1.5 and 1.6.

Topic	Discussion	Learning Aids
	1.7 The Components of a Vector	
Vector components	In two dimensions, the vector components of a vector **A** are two perpendicular vectors $\mathbf{A}_x$ and $\mathbf{A}_y$ that are parallel to the x and y axes, respectively, and that add together vectorially so that $\mathbf{A} = \mathbf{A}_x + \mathbf{A}_y$.	
Scalar components	The scalar component A_x has a magnitude that is equal to that of $\mathbf{A}_x$ and is given a positive sign if $\mathbf{A}_x$ points along the $+x$ axis and a negative sign if $\mathbf{A}_x$ points along the $-x$ axis. The scalar component A_y is defined in a similar manner.	Interactive Solution 1.37 **Example 7**
Condition for a vector to be zero	A vector is zero if, and only if, each of its vector components is zero.	
Condition for two vectors to be equal	Two vectors are equal if, and only if, they have the same magnitude and direction. Alternatively, two vectors are equal in two dimensions if the x vector components of each are equal and the y vector components of each are equal.	
	1.8 Addition of Vectors by Means of Components	
	If two vectors **A** and **B** are added to give a resultant vector **C** such that $\mathbf{C} = \mathbf{A} + \mathbf{B}$, then $C_x = A_x + B_x$ and $C_y = A_y + B_y$ where C_x, A_x, and B_x are the scalar components of the vectors along the x direction, and C_y, A_y, and B_y are the scalar components of the vectors along the y direction.	**Example 8** Concept Simulation 1.1 Interactive Solution 1.49

Use Self-Assessment Test 1.2 to evaluate your understanding of Sections 1.7 and 1.8.

Conceptual Questions

1. The following table lists four variables along with their units:

Variable	Units
x	meters (m)
v	meters per second (m/s)
t	seconds (s)
a	meters per second squared (m/s^2)

These variables appear in the following equations, along with a few numbers that have no units. In which of the equations are the units on the left side of the equals sign consistent with the units on the right side?

(a) $x = vt$

(b) $x = vt + \frac{1}{2}at^2$

(c) $v = at$

(d) $v = at + \frac{1}{2}at^3$

(e) $v^3 = 2ax^2$

(f) $t = \sqrt{\dfrac{2x}{a}}$

2. The variables x and v have the units shown in the table that accompanies question 1. Is it possible for x and v to be related to an angle θ according to $\tan\theta = x/v$? Account for your answer.

3. You can always add two numbers that have the same units. However, you cannot always add two numbers that have the same dimensions. Explain why not, and include an example in your explanation.

4. (a) Is it possible for two quantities to have the same dimensions but different units? (b) Is it possible for two quantities to have the same units but different dimensions? In each case, support your answer with an example and an explanation.

5. In the equation $y = c^n at^2$ you wish to determine the integer value (1, 2, etc.) of the exponent n. The dimensions of y, a, and t are known. It is also known that c has no dimensions. Can dimensional analysis be used to determine n? Account for your answer.

6. Using your calculator, verify that $\sin\theta$ divided by $\cos\theta$ is equal to $\tan\theta$, for an angle θ. Try 30°, for example. Prove that this result is true in general by using the definitions for $\sin\theta$, $\cos\theta$, and $\tan\theta$ given in Equations 1.1–1.3.

7. Sin θ and cos θ are called sinusoidal functions of the angle θ. The way in which these functions change as θ changes leads to a characteristic pattern when they are graphed. This pattern arises many times in physics. (a) To familiarize yourself with the sinusoidal pattern, use a calculator and construct a graph, with $\sin\theta$ plotted on the vertical axis and θ on the horizontal axis. Use 15° increments for θ between 0° and 720°. (b) Repeat for cos θ.

8. Which of the following quantities (if any) can be considered a vector: (a) the number of people attending a football game, (b) the number of days in a month, and (c) the number of pages in a book? Explain your reasoning.

9. Which of the following displacement vectors (if any) are equal? Explain your reasoning.

Vector	Magnitude	Direction
A	100 m	30° north of east
B	100 m	30° south of west
C	50 m	30° south of west
D	100 m	60° east of north

10. Are two vectors with the same magnitude necessarily equal? Give your reasoning.

11. A cube has six faces and twelve edges. You start at one corner, are allowed to move only along the edges, and may not retrace your path along any edge. Consistent with these rules, there are a number of ways to arrive back at your starting point. For instance, you could move around the four edges that make up one of the square faces. The four corresponding displacement vectors would add to zero. How many ways are there to arrive back at your starting point that involve *eight* displacement vectors that add to zero? Describe each possibility, using drawings for clarity.

12. (a) Is it possible for one component of a vector to be zero, while the vector itself is not zero? (b) Is it possible for a vector to be zero, while one component of the vector is not zero? Explain.

13. Can two nonzero perpendicular vectors be added together so their sum is zero? Explain.

14. Can three or more vectors with unequal magnitudes be added together so their sum is zero? If so, show by means of a tail-to-head arrangement of the vectors how this could occur.

15. In preparation for this question, review Conceptual Example 6. Vectors **A** and **B** satisfy the vector equation $\mathbf{A} + \mathbf{B} = \mathbf{0}$. (a) How does the magnitude of **B** compare with the magnitude of **A**? (b) How does the direction of **B** compare with the direction of **A**? Give your reasoning.

16. Vectors **A**, **B**, and **C** satisfy the vector equation $\mathbf{A} + \mathbf{B} = \mathbf{C}$, and their magnitudes are related by the scalar equation $A^2 + B^2 = C^2$. How is vector **A** oriented with respect to vector **B**? Account for your answer.

17. Vectors **A**, **B**, and **C** satisfy the vector equation $\mathbf{A} + \mathbf{B} = \mathbf{C}$, and their magnitudes are related by the scalar equation $A + B = C$. How is vector **A** oriented with respect to vector **B**? Explain your reasoning.

18. The magnitude of a vector has doubled, its direction remaining the same. Can you conclude that the magnitude of each component of the vector has doubled? Explain your answer.

19. The tail of a vector is fixed to the origin of an x, y axis system. Originally the vector points along the $+x$ axis. As time passes, the vector rotates counterclockwise. Describe how the sizes of the x and y components of the vector compare to the size of the original vector for rotational angles of (a) 90°, (b) 180°, (c) 270°, and (d) 360°.

20. A vector has a component of zero along the x axis of a certain axis system. Does this vector necessarily have a component of zero along the x axis of another (rotated) axis system? Use a drawing to justify your answer.

Problems

Problems that are not marked with a star are considered the easiest to solve. Problems that are marked with a single star (*) are more difficult, while those marked with a double star (**) are the most difficult.

ssm Solution is in the Student Solutions Manual. **www Solution is available on the World Wide Web at www.wiley.com/college/cutnell**
This icon represents a biomedical application.

Section 1.2 Units, Section 1.3 The Role of Units in Problem Solving

1. ssm The mass of the parasitic wasp *Caraphractus cintus* can be as small as 5×10^{-6} kg. What is this mass in (a) grams (g), (b) milligrams (mg), and (c) micrograms (μg)?

2. Vesna Vulovic survived the longest fall on record without a parachute when her plane exploded and she fell 6 miles, 551 yards. What is this distance in meters?

3. How many seconds are there in (a) one hour and thirty-five minutes and (b) one day?

4. Bicyclists in the Tour de France reach speeds of 34.0 miles per hour (mi/h) on flat sections of the road. What is this speed in (a) kilometers per hour (km/h) and (b) meters per second (m/s)?

5. ssm The largest diamond ever found had a size of 3106 carats. One carat is equivalent to a mass of 0.200 g. Use the fact that 1 kg (1000 g) has a weight of 2.205 lb under certain conditions, and determine the weight of this diamond in pounds.

6. A bottle of wine known as a magnum contains a volume of 1.5 liters. A bottle known as a jeroboam contains 0.792 U.S. gallons. How many magnums are there in one jeroboam?

7. The following are dimensions of various physical parameters that will be discussed later on in the text. Here [L], [T], and [M] denote, respectively, dimensions of length, time, and mass.

	Dimension		Dimension
Distance (x)	[L]	Acceleration (a)	$[L]/[T]^2$
Time (t)	[T]	Force (F)	$[M][L]/[T]^2$
Mass (m)	[M]	Energy (E)	$[M][L]^2/[T]^2$
Speed (v)	[L]/[T]		

Which of the following equations are dimensionally correct?

(a) $F = ma$ **(d)** $E = max$

(b) $x = \frac{1}{2}at^3$ **(e)** $v = \sqrt{Fx/m}$

(c) $E = \frac{1}{2}mv$

8. The variables x, v, and a have the dimensions of [L], [L]/[T], and $[L]/[T]^2$, respectively. These variables are related by an equation that has the form $v^n = 2ax$, where n is an integer constant (1, 2, 3, etc.) without dimensions. What must be the value of n, so that both sides of the equation have the same dimensions? Explain your reasoning.

* **9.** **ssm** The depth of the ocean is sometimes measured in fathoms (1 fathom = 6 feet). Distance on the surface of the ocean is sometimes measured in nautical miles (1 nautical mile = 6076 feet). The water beneath a surface rectangle 1.20 nautical miles by 2.60 nautical miles has a depth of 16.0 fathoms. Find the volume of water (in cubic meters) beneath this rectangle.

* **10.** A spring is hanging down from the ceiling, and an object of mass m is attached to the free end. The object is pulled down, thereby stretching the spring, and then released. The object oscillates up and down, and the time T required for one complete up-and-down oscillation is given by the equation $T = 2\pi\sqrt{m/k}$, where k is known as the spring constant. What must be the dimension of k for this equation to be dimensionally correct?

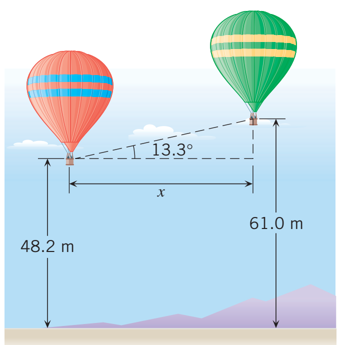
Section 1.4 Trigonometry

11. You are driving into St. Louis, Missouri, and in the distance you see the famous Gateway-to-the-West arch. This monument rises to a height of 192 m. You estimate your line of sight with the top of the arch to be 2.0° above the horizontal. Approximately how far (in kilometers) are you from the base of the arch?

12. An observer, whose eyes are 1.83 m above the ground, is standing 32.0 m away from a tree. The ground is level, and the tree is growing perpendicular to it. The observer's line of sight with the treetop makes an angle of 20.0° above the horizontal. How tall is the tree?

13. **ssm** **www** A highway is to be built between two towns, one of which lies 35.0 km south and 72.0 km west of the other. What is the shortest length of highway that can be built between the two towns, and at what angle would this highway be directed with respect to due west?

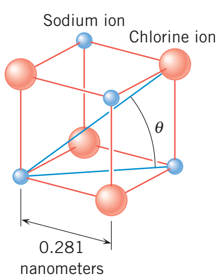
14. The two hot-air balloons in the drawing are 48.2 and 61.0 m above the ground. A person in the left balloon observes that the right balloon is 13.3° above the horizontal. What is the horizontal distance x between the two balloons?



Problem 14

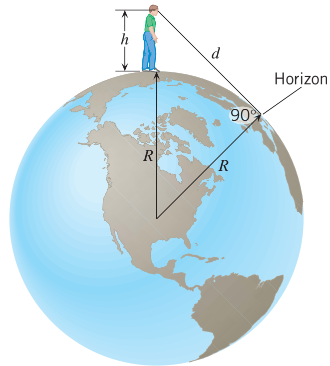
15. The silhouette of a Christmas tree is an isosceles triangle. The angle at the top of the triangle is 30.0°, and the base measures 2.00 m across. How tall is the tree?

* **16.** The drawing shows sodium and chlorine ions positioned at the corners of a cube that is part of the crystal structure of sodium chloride (common table salt). The edge of the cube is 0.281 nm (1 nm = 1 nanometer = 10^{-9} m) in length. Find the distance (in nanometers) between the sodium ion located at one corner of the cube and the chlorine ion located on the diagonal at the opposite corner.



* **17.** **Interactive Solution 1.17** at **www.wiley.com/college/cutnell** presents a method for modeling this problem. What is the value of the angle θ in the drawing that accompanies problem 16?

* **18.** A person is standing at the edge of the water and looking out at the ocean (see the drawing). The height of the person's eyes above the water is h = 1.6 m, and the radius of the earth is $R = 6.38 \times 10^6$ m. (a) How far is it to the horizon? In other words, what is the distance d from the person's eyes to the horizon? *(Note: At the horizon the angle between the line of sight and the radius of the earth is 90°.)* (b) Express this distance in miles.



Problem 18

* **19.** **ssm** What is the value of each of the angles of a triangle whose sides are 95, 150, and 190 cm in length? *(Hint: Consider using the law of cosines given in Appendix E.)*

** **20.** A regular tetrahedron is a three-dimensional object that has four faces, each of which is an equilateral triangle. Each of the edges of such an object has a length L. The height H of a regular tetrahedron is the perpendicular distance from one corner to the center of the opposite triangular face. Show that the ratio between H and L is $H/L = \sqrt{2/3}$.

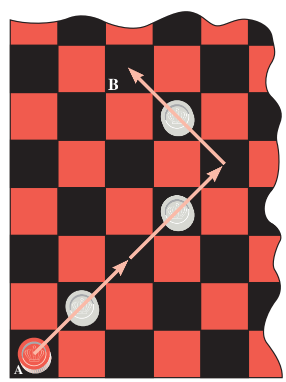
Section 1.6 Vector Addition and Subtraction

21. A force vector $\mathbf{F}_1$ points due east and has a magnitude of 200 newtons. A second force $\mathbf{F}_2$ is added to $\mathbf{F}_1$. The resultant of the two vectors has a magnitude of 400 newtons and points along the east/west line. Find the magnitude and direction of $\mathbf{F}_2$. Note that there are two answers.

22. (a) Two workers are trying to move a heavy crate. One pushes on the crate with a force **A**, which has a magnitude of 445 newtons and is directed due west. The other pushes with a force **B**, which has a magnitude of 325 newtons and is directed due north. What are the magnitude and direction of the resultant force **A** + **B** applied to the crate? (b) Suppose that the second worker applies a force −**B** instead of **B**. What then are the magnitude and direction of the resultant force **A** − **B** applied to the crate? In both cases express the direction relative to due west.

23. **ssm** Displacement vector **A** points due east and has a magnitude of 2.00 km. Displacement vector **B** points due north and has a magnitude of 3.75 km. Displacement vector **C** points due west and has a magnitude of 2.50 km. Displacement vector **D** points due south and has a magnitude of 3.00 km. Find the magnitude and direction (relative to due west) of the resultant vector **A** + **B** + **C** + **D**.

24. The drawing shows a triple jump on a checkerboard, starting at the center of square A and ending on the center of square B. Each side of a square measures 4.0 cm. What is the magnitude of the displacement of the colored checker during the triple jump?

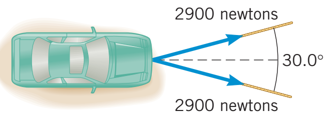


Problem 24

25. **ssm** **www** One displacement vector **A** has a magnitude of 2.43 km and points due north. A second displacement vector **B** has a magnitude of 7.74 km and also points due north. (a) Find the magnitude and direction of **A** − **B**. (b) Find the magnitude and direction of **B** − **A**.

26. Two bicyclists, starting at the same place, are riding toward the same campground by two different routes. One cyclist rides 1080 m due east and then turns due north and travels another 1430 m before reaching the campground. The second cyclist starts out by heading due north for 1950 m and then turns and heads directly toward the campground. (a) At the turning point, how far is the second cyclist from the campground? (b) What direction (measured relative to due east) must the second cyclist head during the last part of the trip?

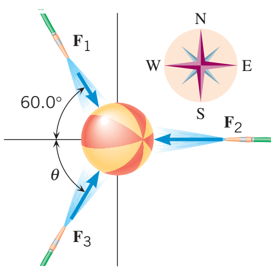
* **27.** ssm www A car is being pulled out of the mud by two forces that are applied by the two ropes shown in the drawing. The dashed line in the drawing bisects the 30.0° angle. The magnitude of the force applied by each rope is 2900 newtons. Arrange the force vectors tail to head and use the graphical technique to answer the following questions. (a) How much force would a single rope need to apply to accomplish the same effect as the two forces added together? (b) How would the single rope be directed relative to the dashed line?



* **28.** In wandering, a grizzly bear makes a displacement of 1563 m due west, followed by a displacement of 3348 m in a direction 32.0° north of west. What are (a) the magnitude and (b) the direction of the displacement needed for the bear to *return to its starting point?* Specify the direction relative to due east.

* **29.** Before starting this problem, review **Interactive Solution 1.29** at **www.wiley.com/college/cutnell.** Vector **A** has a magnitude of 12.3 units and points due west. Vector **B** points due north. (a) What is the magnitude of **B** if **A** + **B** has a magnitude of 15.0 units? (b) What is the direction of **A** + **B** relative to due west? (c) What is the magnitude of **B** if **A** − **B** has a magnitude of 15.0 units? (d) What is the direction of **A** − **B** relative to due west?

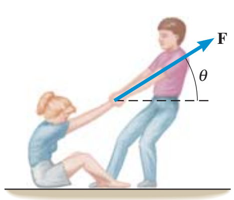
* **30.** At a picnic, there is a contest in which hoses are used to shoot water at a beach ball from three directions. As a result, three forces act on the ball, $\mathbf{F}_1$, $\mathbf{F}_2$, and $\mathbf{F}_3$ (see the drawing). The magnitudes of $\mathbf{F}_1$ and $\mathbf{F}_2$ are $F_1 = 50.0$ newtons and $F_2 = 90.0$ newtons. Using a scale drawing and the graphical technique, determine (a) the magnitude of $\mathbf{F}_3$ and (b) the angle θ such that the resultant force acting on the ball is zero.



Section 1.7 The Components of a Vector

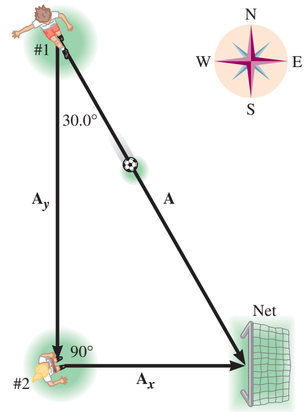
31. ssm The speed of an object and the direction in which it moves constitute a vector quantity known as the velocity. An ostrich is running at a speed of 17.0 m/s in a direction of 68.0° north of west. What is the magnitude of the ostrich's velocity component that is directed (a) due north and (b) due west?

32. Your friend has slipped and fallen. To help her up, you pull with a force **F**, as the drawing shows. The vertical component of this force is 130 newtons, and the horizontal component is 150 newtons. Find (a) the magnitude of **F** and (b) the angle θ.



33. An ocean liner leaves New York City and travels 18.0° north of east for 155 km. How far east and how far north has it gone? In other words, what are the magnitudes of the components of the ship's displacement vector in the directions (a) due east and (b) due north?

34. Soccer player #1 is 8.6 m from the goal, as the drawing shows. If she kicks the ball directly into the net, the ball has a displacement labeled **A**. If, on the other hand, she first kicks it to player #2, who then kicks it into the net, the ball undergoes two successive displacements, $\mathbf{A}_y$ and $\mathbf{A}_x$. What are the magnitude and direction of $\mathbf{A}_x$ and $\mathbf{A}_y$?



Problem 34

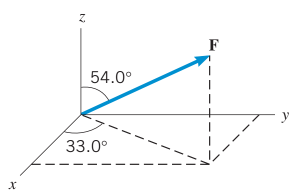
35. ssm Two ropes are attached to a heavy box to pull it along the floor. One rope applies a force of 475 newtons in a direction due west; the other applies a force of 315 newtons in a direction due south. As we will see later in the text, force is a vector quantity. (a) How much force should be applied by a single rope, and (b) in what direction (relative to due west), if it is to accomplish the same effect as the two forces added together?

36. On takeoff, an airplane climbs with a speed of 180 m/s at an angle of 34° above the horizontal. The speed and direction of the airplane constitute a vector quantity known as the velocity. The sun is shining directly overhead. How fast is the shadow of the plane moving along the ground? (That is, what is the magnitude of the horizontal component of the plane's velocity?)

* **37.** To review the solution to a similar problem, consult **Interactive Solution 1.37** at **www.wiley.com/college/cutnell.** The magnitude of the force vector **F** is 82.3 newtons. The x component of this vector is directed along the $+x$ axis and has a magnitude of 74.6 newtons. The y component points along the $+y$ axis. (a) Find the direction of **F** relative to the $+x$ axis. (b) Find the component of **F** along the $+y$ axis.

* **38.** A force vector points at an angle of 52° above the $+x$ axis. It has a y component of +290 newtons. Find (a) the magnitude and (b) the x component of the force vector.

** **39.** ssm www The drawing shows a force vector that has a magnitude of 475 newtons. Find the (a) x, (b) y, and (c) z components of the vector.



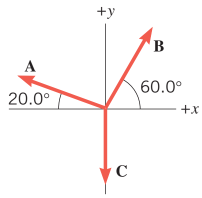
Section 1.8 Addition of Vectors by Means of Components

40. As an aid in working this problem, consult **Concept Simulation 1.1** at **www.wiley.com/college/cutnell.** Two forces are applied to a tree stump to pull it out of the ground. Force $\mathbf{F}_A$ has a magnitude of 2240 newtons and points 34.0° south of east, while force $\mathbf{F}_B$ has a magnitude of 3160 newtons and points due south. Using the component method, find the magnitude and direction of the resultant force $\mathbf{F}_A + \mathbf{F}_B$ that is applied to the stump. Specify the direction with respect to due east.

41. ssm A golfer, putting on a green, requires three strokes to "hole the ball." During the first putt, the ball rolls 5.0 m due east. For the second putt, the ball travels 2.1 m at an angle of 20.0° north of east. The third putt is 0.50 m due north. What displacement (magnitude and direction relative to due east) would have been needed to "hole the ball" on the very first putt?

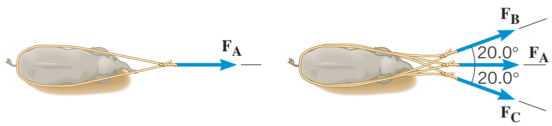
42. You are on a treasure hunt and your map says "Walk due west for 52 paces, then walk 30.0° north of west for 42 paces, and finally walk due north for 25 paces." What is the magnitude of the component of your displacement in the direction (a) due north and (b) due west?

43. Find the resultant of the three displacement vectors in the drawing by means of the component method. The magnitudes of the vectors are $A = 5.00$ m, $B = 5.00$ m, and $C = 4.00$ m.



Problem 43

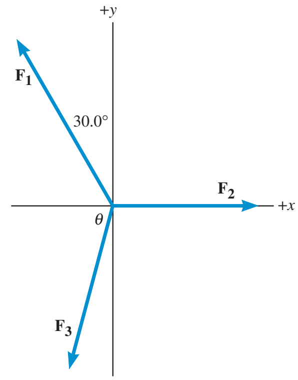
44. A baby elephant is stuck in a mud hole. To help pull it out, game keepers use a rope to apply force $\mathbf{F}_A$, as part *a* of the drawing shows. By itself, however, force $\mathbf{F}_A$ is insufficient. Therefore, two additional forces $\mathbf{F}_B$ and $\mathbf{F}_C$ are applied, as in part *b* of the drawing. Each of these additional forces has the same magnitude *F*. The magnitude of the resultant force acting on the elephant in part *b* of the drawing is twice that in part *a*. Find the ratio F/F_A.



(*a*) (*b*)

45. As preparation for this problem, consult **Concept Simulation 1.1** at **www.wiley.com/college/cutnell.** On a safari, a team of naturalists sets out toward a research station located 4.8 km away in a direction 42° north of east. After traveling in a straight line for 2.4 km, they stop and discover that they have been traveling 22° north of east, because their guide misread his compass. What are (a) the magnitude and (b) the direction (relative to due east) of the displacement vector now required to bring the team to the research station?

* **46.** Three forces are applied to an object, as indicated in the drawing. Force $\mathbf{F}_1$ has a magnitude of 21.0 newtons (21.0 N) and is directed 30.0° to the left of the $+y$ axis. Force $\mathbf{F}_2$ has a magnitude of 15.0 N and points along the $+x$ axis. What must be the magnitude and direction (specified by the angle θ in the drawing) of the third force $\mathbf{F}_3$ such that the vector sum of the three forces is 0 N?



Problem 46

* **47.** **ssm** Vector **A** has a magnitude of 6.00 units and points due east. Vector **B** points due north. (a) What is the magnitude of **B**, if the vector **A + B** points 60.0° north of east? (b) Find the magnitude of **A + B**.

* **48.** The route followed by a hiker consists of three displacement vectors **A**, **B**, and **C**. Vector **A** is along a measured trail and is 1550 m in a direction 25.0° north of east. Vector **B** is not along a measured trail, but the hiker uses a compass and knows that the direction is 41.0° east of south. Similarly, the direction of vector **C** is 35.0° north of west. The hiker ends up back where she started, so the resultant displacement is zero, or **A + B + C = 0.** Find the magnitudes of (a) vector **B** and (b) vector **C**.

* **49.** **Interactive Solution 1.49** at **www.wiley.com/college/cutnell** presents the solution to a problem that is similar to this one. Vector **A** has a magnitude of 145 units and points 35.0° north of west. Vector **B** points 65.0° east of north. Vector **C** points 15.0° west of south. These three vectors add to give a resultant vector that is zero. Using components, find the magnitudes of (a) vector **B** and (b) vector **C**.

* **50.** A grasshopper makes four jumps. The displacement vectors are (1) 27.0 cm, due west; (2) 23.0 cm, 35.0° south of west; (3) 28.0 cm, 55.0° south of east; and (4) 35.0 cm, 63.0° north of east. Find the magnitude and direction of the resultant displacement. Express the direction with respect to due west.

Additional Problems

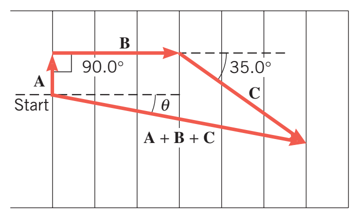
51. A chimpanzee sitting against his favorite tree gets up and walks 51 m due east and 39 m due south to reach a termite mound, where he eats lunch. (a) What is the shortest distance between the tree and the termite mound? (b) What angle does the shortest distance make with respect to due east?

52. The gondola ski lift at Keystone, Colorado, is 2830 m long. On average, the ski lift rises 14.6° above the horizontal. How high is the top of the ski lift relative to the base?

53. **ssm** Vector **A** points along the $+y$ axis and has a magnitude of 100.0 units. Vector **B** points at an angle of 60.0° above the $+x$ axis and has a magnitude of 200.0 units. Vector **C** points along the $+x$ axis and has a magnitude of 150.0 units. Which vector has (a) the largest *x* component and (b) the largest *y* component?

54. Consider the equation $v = \frac{1}{3}zxt^2$. The dimensions of the variables *x*, *v*, and *t* are [L], [L]/[T], and [T], respectively. What must be the dimensions of the variable *z*, such that both sides of the equation have the same dimensions? Show how you determined your answer.

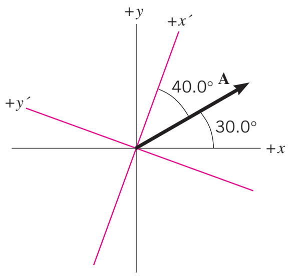
55. As an aid in visualizing the concepts in this problem, consult **Concept Simulation 1.1** at **www.wiley.com/college/cutnell.** A football player runs the pattern given in the drawing by the three displacement vectors **A**, **B**, and **C**. The magnitudes of these vectors are $A = 5.00$ m, $B = 15.0$ m, and $C = 18.0$ m. Using the component method, find the magnitude and direction θ of the resultant vector **A + B + C**.



56. A circus performer begins his act by walking out along a nearly horizontal high wire. He slips and falls to the safety net, 25.0 ft below. The magnitude of his displacement from the beginning of the walk to the net is 26.7 ft. (a) How far out along the high wire did he walk? (b) Find the angle that his displacement vector makes below the horizontal.

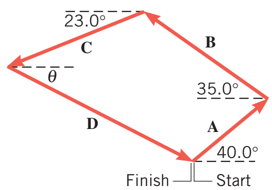
57. **ssm** The *x* vector component of a displacement vector **r** has a magnitude of 125 m and points along the negative *x* axis. The *y* vector component has a magnitude of 184 m and points along the negative *y* axis. Find the magnitude and direction of **r**. Specify the direction with respect to the negative *x* axis.

* **58.** The vector **A** in the drawing has a magnitude of 750 units. Determine the magnitude and direction of the x and y components of the vector **A**, relative to (a) the black axes and (b) the colored axes.



Problem 58

* **59.** **ssm** A sailboat race course consists of four legs, defined by the displacement vectors **A**, **B**, **C**, and **D**, as the drawing indicates. The magnitudes of the first three vectors are $A = 3.20$ km, $B = 5.10$ km, and $C = 4.80$ km. The finish line of the course coincides with the starting line. Using the data in the drawing, find the distance of the fourth leg and the angle θ.



Problem 59

* **60.** A jogger travels a route that has two parts. The first is a displacement **A** of 2.50 km due south, and the second involves a displacement **B** that points due east. (a) The resultant displacement **A + B** has a magnitude of 3.75 km. What is the magnitude of **B**, and what is the direction of **A + B** relative to due south? (b) Suppose that **A − B** had a magnitude of 3.75 km. What then would be the magnitude of **B**, and what is the direction of **A − B** relative to due south?

* **61.** Three deer, A, B, and C, are grazing in a field. Deer B is located 62 m from deer A at an angle of 51° north of west. Deer C is located 77° north of east relative to deer A. The distance between deer B and C is 95 m. What is the distance between deer A and C? *(Hint: Consider the law of cosines given in Appendix E.)*

* **62.** Before starting this problem, review Conceptual Example 6. The force vector $\mathbf{F_A}$ has a magnitude of 90.0 newtons and points due east. The force vector $\mathbf{F_B}$ has a magnitude of 135 newtons and points 75° north of east. Use the graphical method and find the magnitude and direction of (a) $\mathbf{F_A} - \mathbf{F_B}$ (give the direction with respect to due east) and (b) $\mathbf{F_B} - \mathbf{F_A}$ (give the direction with respect to due west).

** **63.** What are the x and y components of the vector that must be added to the following three vectors, so that the sum of the four vectors is zero? Due east is the $+x$ direction, and due north is the $+y$ direction.

A = 113 units, 60.0° south of west
B = 222 units, 35.0° south of east
C = 177 units, 23.0° north of east

Concepts & Calculations Group Learning Problems

Note: Each of these problems consists of Concept Questions followed by a related quantitative Problem. They are designed for use by students working alone or in small learning groups. The Concept Questions involve little or no mathematics and are intended to stimulate group discussions. They focus on the concepts with which the problems deal. Recognizing the concepts is the essential initial step in any problem-solving technique.

64. **Concept Questions** (a) Considering the fact that 3.28 ft = 1 m, which is the larger unit for measuring area, 1 ft^2 or 1 m^2? (b) Consider a 1330-ft^2 apartment. With your answer to part (a) in mind and without doing any calculations, decide whether this apartment has an area that is greater than or less than 1330 m^2.

Problem In a 1330-ft^2 apartment, how many square meters of area are there? Be sure that your answer is consistent with your answers to the Concept Questions.

65. **Concept Question** The corners of a square lie on a circle of diameter D. Each side of the square has a length L. Is L smaller or larger than D? Explain your reasoning using the Pythagorean theorem.

Problem The diameter D of the circle is 0.35 m. Each side of the square has a length L. Find L. Be sure that your answer is consistent with your answer to the Concept Question.

66. **Concept Question** Can the x or y component of a vector ever have a greater magnitude than the vector itself has? Give your reasoning.

Problem A force vector has a magnitude of 575 newtons and points at an angle of 36.0° below the positive x axis. What are (a) the x scalar component and (b) the y scalar component of the vector? Verify that your answers are consistent with your answer to the Concept Question.

67. **Concept Questions** The components of vector **A** are A_x and A_y, and the angle that it makes with respect to the positive x axis is θ. (a) Does increasing the component A_x (while holding A_y constant) increase or decrease the angle θ? (b) Does increasing the component A_y (while holding A_x constant) increase or decrease the angle θ? Account for your answers.

Problem (a) The components of displacement vector **A** are $A_x = 12$ m and $A_y = 12$ m. Find θ. (b) The components of displacement vector **A** are $A_x = 17$ m and $A_y = 12$ m. Find θ. (c) The components of displacement vector **A** are $A_x = 12$ m and $A_y = 17$ m. Find θ. Be sure that your answers are consistent with your answers to the Concept Questions.

68. **Concept Questions** Vector **A** points due west, while vector **B** points due south. (a) Does the direction **A + B** point north or south of due west? (b) Does the direction of **A − B** point north or south of due west? Give your reasoning in each case.

Problem Vector **A** has a magnitude of 63 units and points due west, while vector **B** has the same magnitude and points due south. Find the magnitude and direction of (a) **A + B** and (b) **A − B**. Specify the directions relative to due west. Verify that your answers agree with your answers to the Concept Questions.

69. **Concept Questions** A pilot flies her route in two straight-line segments. The displacement vector **A** for the first segment has a magnitude of 244 km and a direction 30.0° north of east. The displacement vector **B** for the second segment has a magnitude of 175 km and a direction due west. The resultant displacement vector is **R** = **A + B** and makes an angle θ with the direction due east. Make a drawing to scale showing the vectors **A** and **B** placed tail to head and the resultant vector **R**. Without doing any calculations decide whether (a) the magnitude of **R** is greater or smaller than the magnitude of **A**, (b) the magnitude of **R** is greater or smaller than the magnitude of **B**, and (c) the angle θ is greater than, smaller than, or equal to 30.0°.

Problem Using the component method, find the magnitude of **R** and the directional angle θ. Check to see that your answers are consistent with your answers to the Concept Questions.