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The Riddle of the Sphinx

Let us consider that we are all partially insane.
It will explain us to each other; it will unriddle many
riddles; it will make clear and simple many things
which are involved in haunting and harassing
difficulties and obscurities now.

MARK TWAIN (1835–1910)

IF WE VISIT THE CITY OF GIZA in Egypt today, we cannot help but be overwhelmed by the massive sculpture known as the Great Sphinx, a creature with the head and the breasts of a woman, the body of a lion, the tail of a serpent, and the wings of a bird. Dating from before 2500 B.C., the Great Sphinx magnificently stretches 240 feet (73 meters) in length and rises about 66 feet (20 meters) above us. The width of its face measures an astounding 13 feet, 8 inches (4.17 meters).

Legend has it that a similarly enormous sphinx guarded the entrance to the ancient city of Thebes. The first recorded puzzle in human history comes out of that very legend. The Riddle of the Sphinx, as it came to be known, constitutes not only the point of departure for this book but the starting point for any study of the relationship between puzzles and mathematical ideas. As humankind's earliest puzzle, it is among the ten greatest of all time. Riddles are so common, we hardly ever reflect upon what they are. Their appeal is ageless and timeless. When children are posed a simple riddle, such as "Why did the chicken cross the road?" without any hesitation whatsoever, they seek an answer to it, as if impelled by some unconscious mythic instinct to do so.

Readers may wonder what a riddle shrouded in mythic lore has to do with mathematics. The answer to this will become obvious as they work their way through this chapter. Simply put, in its basic structure, the Riddle of the Sphinx is a model of how so-called insight thinking unfolds. And this form of thinking undergirds all mathematical discoveries.

The Puzzle

According to legend, when Oedipus approached the city of Thebes, he encountered a gigantic sphinx guarding the entrance to the city. The menacing beast confronted the mythic hero and posed the following riddle to him, warning that if he failed to answer it correctly, he would die instantly at the sphinx's hands:

> What has four feet in the morning, two at noon, and three at night?

THE OEDIPUS LEGEND

In Greek mythology, the oracle (prophet) at Delphi warned King Laius of Thebes that a son born to his wife, Queen Jocasta, would grow up to kill him. So, after Jocasta gave birth to a son, Laius ordered the baby taken to a mountain and left there to die. As fate would have it, a shepherd rescued the child and brought him to King Polybus of Corinth, who adopted the boy and named him Oedipus.

Oedipus learned about the ominous prophecy during his youth. Believing that Polybus was his real father, he fled to Thebes, of all places, to avoid the prophecy. On the road, he quarreled with a strange man and ended up killing him. At the entrance to Thebes, Oedipus was stopped by an enormous sphinx that vowed to kill him if he could not solve its riddle. Oedipus solved it. As a consequence, the sphinx took its own life. For ridding them of the monster, the Thebans asked Oedipus to be their king. He accepted and married Jocasta, the widowed queen.

Several years later, a plague struck Thebes. The oracle said that the plague would end when King Laius's murderer had been driven from Thebes. Oedipus investigated the murder, discovering that Laius was the man he had killed on his way to Thebes. To his horror, he learned that Laius was his real father and Jocasta his mother. In despair, Oedipus blinded himself. Jocasta hanged herself. Oedipus was then banished from Thebes. The prophecy pronounced at Delphi had come true.

The fearless Oedipus answered, "Humans, who crawl on all fours as babies, then walk on two legs as grown-ups, and finally need a cane in old age to get around." Upon hearing the correct answer, the astonished sphinx killed itself, and Oedipus entered Thebes as a hero for ridding the city of the terrible monster that had kept it captive for so long.

Various versions of the riddle exist. The previous one is adapted from the play *Oedipus Rex* by the Greek dramatist Sophocles (c. 496–406 B.C.). Following is another common statement of the riddle, also dating back to antiquity:

> What is it that has one voice and yet becomes four-footed, then two-footed, and finally three-footed?

Whatever its version, the Riddle of the Sphinx is the prototype for all riddles (and puzzles, for that matter). It is intentionally constructed to harbor a nonobvious answer—namely, that life's three phases of infancy, adulthood, and old age are comparable, respectively, to the three phases of a day (morning, noon, and night). Its function in the Oedipus story, moreover, suggests that puzzles may have originated as tests of intelligence and thus as probes of human mentality. The biblical story of Samson is further proof of this. At his wedding feast, Samson, obviously wanting to impress the relatives of his wife-to-be, posed the following riddle to his Philistine guests (Judges 14:14):

> Out of the eater came forth meat and out of the strong came forth sweetness.

He gave the Philistines seven days to come up with the answer, convinced that they were incapable of solving it. Samson contrived his riddle to describe something that he once witnessed—a swarm of bees that made honey in a lion's carcass. Hence, the wording of the riddle: the "eater" = "swarm of bees"; "the strong" = the "lion"; and "came forth sweetness" = "made honey." The deceitful guests, however, took advantage of the seven days to coerce the answer from Samson's wife. When they gave Samson the correct response, the mighty biblical hero became enraged and declared war against all Philistines. The ensuing conflict eventually led to his own destruction.

The ancients saw riddles as tests of intelligence and thus as a means through which they could gain knowledge. This explains why the Greek priests and priestesses (called oracles) expressed their prophecies in the form of riddles. The implicit idea was, evidently, that only people who could penetrate the language of the message would unravel its concealed prophecy.

However, not all riddles were devised to test the acumen of mythic heroes. The biblical kings Solomon and Hiram, for example, organized riddle contests simply for the pleasure of outwitting each other. The ancient Romans made riddling a recreational activity of the Saturnalia, a religious event that they celebrated from December 17 to 23. By the fourth century A.D., riddles had, in fact, become so popular for their "recreational value" that the memory of their mythic origin started to fade. In the tenth century, Arabic scholars used riddles for pedagogical reasons—namely, to train students of the law to detect linguistic ambiguities. This coincided with the establishment of the first law schools in Europe.

Shortly after the invention of the printing press in the fifteenth century, some of the first books ever printed for popular entertainment were collections of riddles. One of these, titled *The Merry Book of Riddles*, was published in 1575. Here is a riddle from that work:

> He went to the wood and caught it,
> He sate him downe and sought it;
> Because he could not finde it,
> Home with him he brought it.
>
> (answer: a thorn caught on a foot)

By the eighteenth century, riddles were regularly included in many newspapers and periodicals. Writers and scholars often composed riddles. The American inventor Benjamin Franklin (1706–1790), for instance, devised riddles under the pen name of Richard Saunders. He included them in his *Poor Richard's Almanack*, first published in 1732. The almanac became an unexpected success, due in large part to the popularity of its riddle section. In France, no less a literary figure than the great satirist Voltaire (1694–1778) penned brain-teasing riddles, such as the following one:

> What of all things in the world is the longest, the shortest, the swiftest, the slowest, the most divisible and most extended, most regretted, most neglected, without which nothing can be done, and with which many do nothing, which destroys all that is little and ennobles all that is great?
>
> (answer: time)

The ever-increasing popularity of riddles in the nineteenth century brought about a demand for more variety. This led to the invention of a new riddle genre, known as the *charade*. Charades are solved one syllable or line at a time, by unraveling the meanings suggested by separate syllables, words, or lines. In the nineteenth century, this led to the *mime charade*, which became, and continues to be, a highly popular game at social gatherings. It is played by members of separate teams who act out the meanings of various syllables of a word, an entire word, or a phrase in pantomime. If the answer to the charade is, for example, "baseball," the syllables *base* and *ball* of that word are the ones normally pantomimed. By the end of the century, riddles were firmly embedded in European and American recreational culture and remain so to this very day.

Mathematical Annotations

The question that the legendary sphinx asked Oedipus seems to defy an answer at first. What bizarre creature could possibly have four, then two, and finally three legs, in that order? Wresting an answer from the riddle requires us to think imaginatively, not linearly. This very type of imaginative thinking undergirds all true mathematical inquiry.

Problem-Solving

PROBLEM-SOLVING METHODS AND STRATEGIES

Deduction: This involves applying previous knowledge to the problem.

Induction: This involves reasoning from particular facts given in the problem, to reach a general conclusion.

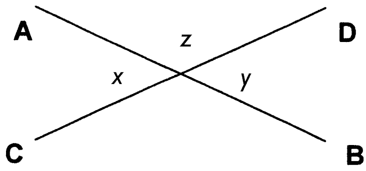
Insight thinking: This involves making guesses or following up on hunches that come from trial-and-error approaches to the problem.

Riddles highlight how puzzles differ in general from typical mathematical problems, such as those found in school textbooks. The latter are designed to help students do something systematically (for example, add large numbers, solve equations, prove theorems, etc.). To grasp the difference, consider two typical textbook problems. Here's the first one:

> Prove that the vertically opposite angles formed when two straight lines intersect are equal.

The method used to solve this type of problem is called **deduction**. It involves applying previous knowledge to the problem at hand.

Start by drawing a diagram that shows all the relevant features of the problem. The two straight lines can be labeled **AB** and **CD**, and two of the four vertically opposite angles formed by their intersection can be labeled *x* and *y*. One of the angles between *x* and *y* can be labeled *z*, as shown:



The problem asks us, in effect, to prove that *x* and *y* (being vertically opposite angles) are equal. There are, of course, two other vertically opposite angles formed by the intersection of the two lines, but they need not be considered here because the method of proof and the end result are the same. The proof hinges on previous knowledge—specifically, that a straight line is an angle of 180 degrees. Consider **CD** first. As a straight line, it is (as mentioned) an angle of 180 degrees. Now, notice that **CD** is composed of two smaller angles on the diagram, *x* and *z*. So, logically, these two must add up to 180 degrees—a statement that can be represented with the equation $x + z = 180°$. The equation reads as follows: "Angle *x* and angle *z* when added together equal 180°."

Now, consider **AB**. Notice that it, too, is composed of two smaller angles on the diagram, *y* and *z*. These two angles must also add up to 180 degrees—a fact that can be similarly represented with an equation: $y + z = 180°$. The two equations just discussed are listed as follows:

1. $x + z = 180°$
2. $y + z = 180°$

They can be rewritten as follows:

3. $x = 180° - z$
4. $y = 180° - z$

If you have forgotten your high school algebra, the reason we can do this is that whatever is done to one side of an equation must also be done to the other. Think of the two sides of an equation as the two pans on a balancing scale, with equal weights in each pan. The weights are analogous to the expressions on either side of an equation. If we want to maintain balance, any weight we take from one of the pans (such as the left one) we must also take from the other (the right one). In like fashion, if we subtract *z* from the left side of equation 1, we must also subtract it from its right side. The result is equation 3, which shows that *z* has been subtracted from both sides. Note that when z is subtracted from itself on the left side ($z - z$), it leaves 0—a result that is not normally indicated. Subtracting z from both sides of equation 2 yields equation 4.

Now, since two things that are equal to the same thing are equal to each other (for example, if Alex is six feet tall and Sarah is six feet tall, then the two people are equal in height), we can deduce that $x = y$, since equation 3 shows that x is equal to ($180° - z$), and equation 4 shows that y is equal to the same expression ($180° - z$). It is not necessary to figure out what the value of the expression is. Whatever it is, the fact remains that both x and y will be equal to it. We can now conclude that "any two vertically opposite angles produced by the intersection of two straight lines are equal," because we did not assign a specific value to either angle. When a proof is generalizable in this way, it is called a **theorem**.

POLYGONS

A polygon is a closed plane (two-dimensional) figure. Examples of polygons are triangles, quadrilaterals such as rectangles and squares, pentagons (five-sided figures), and hexagons (six-sided figures).

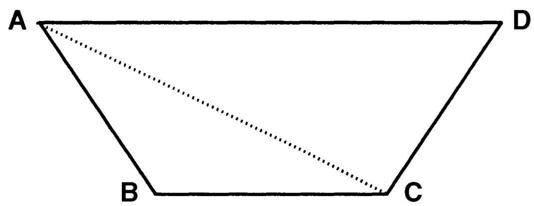
The sum of the three angles in any triangle is 180°, no matter what type of triangle it is (see chapter 5).

Here's our second textbook problem:

Develop a formula for the number of degrees in any polygon.

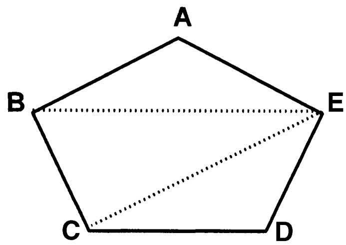
Solving this problem entails a different kind of strategy, known as **induction**. This involves extracting a generalization on the basis of observed facts. Consider a triangle first—the **polygon** with the least number of sides. The sum of the angles in a triangle is 180 degrees (see chapter 5 for the relevant proof).

Next, consider any quadrilateral (a four-sided figure). **ABCD** is one such figure:



Notice that this figure can be divided into two triangles as shown (triangle **ABC** and triangle **ADC**). By doing this, we have discovered that the sum of the angles in the quadrilateral is equivalent to the sum of the angles in two triangles, namely, 180° + 180° = 360°.

Next, consider the case of a pentagon (a five-sided figure). **ABCDE**, as follows, is one such figure:



Since the pentagon can be divided into three triangles, as shown (triangle **ABE**, triangle **BEC**, and triangle **ECD**), we have again discovered a simple fact—namely, that the sum of its angles is equivalent to the sum of the angles in three triangles: 180° + 180° + 180° = 540°.

Continuing in this way, we can show just as easily that the number of angles in a hexagon (a six-sided figure) is equal to the sum of the angles in four triangles; in a heptagon (a seven-sided figure), to the sum of the angles in five triangles; and so on. Let's now attempt to generalize what we have apparently discovered. The letter *n* can be used to represent any number of sides, and the term *n-gon* can be used to refer to any polygon—that is, to a polygon with an unspecified number of sides. The previous observations suggest that the number of triangles that can be drawn in any polygon is "two less" than the number of sides that make up the polygon. For example, in a quadrilateral, we can draw two triangles, which is "two less" than the number of its sides (4), or (4 – 2); in a pentagon, we can draw three triangles, which is, again, "two less" than the number of its sides (5), or (5 – 2); and so on. In the case of a triangle, this rule also applies, since we can draw in it one and only one triangle (itself). This also is "two less" than the number of its sides (3), or (3 – 2). In an *n-gon*, therefore, we can draw (*n* – 2) triangles. To summarize:

TABLE 1-1: CALCULATING THE TRIANGLES IN A POLYGON

Number of Sides in the Polygon	Number of Triangles That Can Be Drawn in the Polygon
3 (= triangle)	(3 – 2) = 1 triangle
4 (= quadrilateral)	(4 – 2) = 2 triangles

Number of Sides in the Polygon	Number of Triangles That Can Be Drawn in the Polygon
5 (= pentagon)	(5 – 2) = 3 triangles
6 (= hexagon)	(6 – 2) = 4 triangles
7 (= heptagon)	(7 – 2) = 5 triangles
. . .	. . .
n (= *n*-gon)	(*n* – 2) triangles

Since we know that there are 180 degrees in a triangle, then there will be (4 – 2) 180° in a quadrilateral, (5 – 2) 180° in a pentagon, and so on. Thus, in an *n-gon*, there will be (*n* – 2) 180°:

TABLE 1-2: DETERMINING THE DEGREES IN A POLYGON

Number of Sides in the Polygon	Number of Triangles That Can Be Drawn in the Polygon	Sum of Degrees of the Angles in the Polygon
3	(3 – 2) = 1	180° × 1 = 180°
4	(4 – 2) = 2	180° × 2 = 360°
5	(5 – 2) = 3	180° × 3 = 540°
6	(6 – 2) = 4	180° × 4 = 720°
7	(7 – 2) = 5	180° × 5 = 900°
. . .	. . .	. . .
n	(*n* – 2)	180° × (*n* – 2) = 180° (*n* – 2)

The formula can be written as

$$(n - 2)\ 180°$$

or as

$$180°\ (n - 2).$$

Now we can determine the number of degrees in any polygon in a straightforward fashion. For example, in the case of an octagon, $n = 8$. Plugging this value into our formula will yield the number of degrees in an octagon:

$$(n - 2)\ 180° = (8 - 2)\ 180° = 6 \times 180° = 1{,}080°.$$

The thing to note about this problem's solution is that it involves generalizing from particular instances. That is the sum and the substance of

COMMUTATIVITY

Changing the order of the factors (numbers) in a multiplication does not change the result (the product). This property of multiplication is known as **commutativity**. Examples include:

$2 \times 3 = 3 \times 2 = 6$

$4 \times 9 = 9 \times 4 = 36$

In general (n = any number, m = any other number),

$$n \times m = m \times n.$$

It can also be written as

$$nm = mn.$$

So, applying the principle of commutativity to our case, we get

$$180° (n - 2) = (n - 2)\ 180°.$$

This same property, incidentally, holds for addition. Examples are:

$2 + 3 = 3 + 2 = 5$

$4 + 9 = 9 + 4 = 13$

In general,

$$n + m = m + n.$$

Commutativity does not hold for either subtraction or division, as you can see for yourself (≠ stands for "does not equal"). Some examples are:

$7 - 4 \neq 4 - 7$

$9 \div 3 \neq 3 \div 9$

inductive reasoning. However, a caveat is in order with respect to such reasoning. Consider the following arithmetical computations—multiplications are on the left and additions are on the right:

Multiplication	**=**	**Addition?**
$2 \times 2 = 4$		$2 + 2 = 4$
$\frac{3}{2} \times 3 = 4\frac{1}{2}$		$\frac{3}{2} + 3 = 4\frac{1}{2}$
$\frac{4}{3} \times 4 = 5\frac{1}{3}$		$\frac{4}{3} + 4 = 5\frac{1}{3}$
$\frac{5}{4} \times 5 = 6\frac{1}{4}$		$\frac{5}{4} + 5 = 6\frac{1}{4}$

From these examples, we might conclude that multiplying numbers always produces the same result as adding them. But, of course, that is not true. Therefore, certain conditions apply when using the method of induction to solve problems. We will return to this topic in chapter 5.

Insight Thinking

What distinguishes the Riddle of the Sphinx from problems such as those we just solved is that the solution strategy is not as predictable. Solving riddles requires **insight thinking**. This can be characterized, essentially, as the act or the outcome of intuitively grasping the inward or hidden nature of a problem. Humanity's first puzzle is a model of how insight thinking unfolds.

The relevant insight required to solve the Riddle of the Sphinx is not to interpret its words literally but to do so metaphorically. Most riddles are based on the various meanings of a word. Consider the following example:

> What has four wheels and flies?
>
> (answer: a garbage truck)

The answer makes sense only when we realize that the word *flies* has two meanings—as a verb ("to move through the air") and as a noun ("an insect with two wings"). A garbage truck is indeed something that has "four wheels" and "flies" that surround it, given that flies are attracted to garbage.

It might be instructive to turn the tables around and create a riddle ourselves. Take, for example, the word *smile*. In English, a smile is said to be something that, like clothing, can be worn. This is why we speak of "wearing a smile," "taking a smile off one's face," and so on. Now, we propitiously can use this very linguistic convention to phrase our riddle:

> I am neither clothes nor shoes, yet I can be worn and taken off when not needed any longer. What am I?

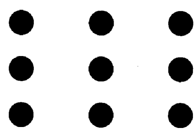
Parenthetically, riddles can also be composed to provide humor. Take, for example, the classic children's riddle "Why did the chicken cross the road?" The number of replies to this question is infinite. Here are three possible answers:

1. To get to the other side.
2. Because it was taken across by a farmer.
3. Because a fox was chasing it.

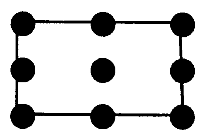
All three answers tend to evoke moderate laughter, similar to the kind that the punch line of a joke would elicit. Riddles of this kind abound, revealing that they have a lot in common with humor.

Insight thinking is the defining characteristic of how most (if not all) puzzles are solved. As an example, consider the following classic puzzle:

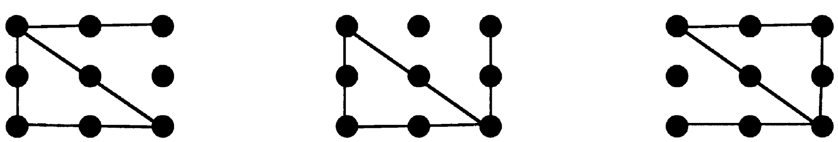
> Without letting your pencil leave the paper, can you draw four straight lines through the following nine dots?



At first, people tend to approach this puzzle by joining up the dots as if they were located on the perimeter (boundary) of an imaginary square or a flattened box:

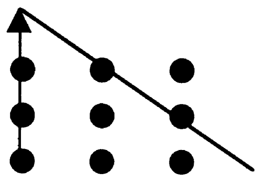


But this reading of the puzzle does not yield a solution, no matter how many times one tries to draw four straight lines without lifting the pencil. A dot is always "left over," as the following three attempts show:

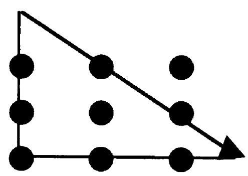


At this point, intuition comes into play: "What would happen if I extend one or more of the four lines beyond the box?" That hunch turns out, in fact, to be the relevant insight.

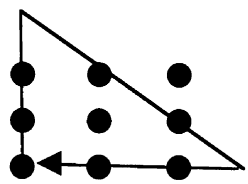
Start by putting the pencil on, say, the bottom left dot, tracing a straight line upward through the two dots above it and stopping at a point "outside the box," when you can see that it is in line diagonally with the two dots below it. You could start with any of the four corner dots and produce a solution (as you may wish to confirm yourself):



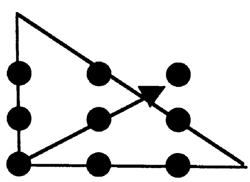
Second, trace a straight line diagonally downward through the two dots. Stop when you see that your second line is in horizontal alignment with the three bottom dots:



Draw your third line through the bottom dots:



Finally, draw your fourth line through the remaining dots:



Incidentally, this puzzle is the probable source of the common expression "thinking outside the box." The reason for this is self-explanatory.

Solving puzzles may, at times, involve the use of other forms of thinking. But it is the intuitive trial-and-error form that dominates. The word *puzzle*, incidentally, comes from the Middle English word *poselen*, "to bewilder, confuse." It is an apt term because, unlike the typical problems found in mathematics textbooks, puzzles at first generate bewilderment and confusion, at the same time that they challenge our wits. As Helene

Hovanec has stated in her delectable book *The Puzzler's Paradise* (see Further Reading), the lure of puzzles lies in the fact that they "simultaneously conceal the answers yet cry out to be solved," piquing solvers to pit "their own ingenuity against that of the constructors."

Consider one more classic puzzle, devised by the French Jesuit poet and scholar Claude-Gaspar Bachet de Mézirac (1581–1638)—a puzzle that he included in his 1612 collection titled *Problèmes plaisans et délectables qui se font par les nombres* ("Amusing and Delightful Number Problems"):

> What is the least number of weights that can be used on a scale to weigh any whole number of pounds of sugar from 1 to 40 inclusive, if the weights can be placed on either of the scale pans?

We might, at first, be tempted to conclude that six weights of 1, 2, 4, 8, 16, and 32 pounds would do the trick. The reasoning would go somewhat as follows. We could weigh 1 pound of sugar by putting the 1-pound weight on the left pan, pouring sugar into the right pan until both pans balance. We could weigh 2 pounds of sugar by putting the 2-pound weight on the left pan, pouring sugar on the right pan until the pans balance. We could weigh 3 pounds of sugar by putting the 1-pound and the 2-pound weights on the left pan, pouring sugar on the right pan until the pans balance. And so on, and so forth. In this way, we could weigh any number of integral (whole-number) pounds of sugar from 1 pound to 40 pounds.

EXPONENTS

An *exponent* (also called a *power*) is a superscript digit or letter attached to the right of a number, indicating how many times the number is to be multiplied by itself. For example, in 3^4 the superscript digit 4 indicates that the number 3 is to be multiplied by itself four times:

$$3^4 = 3 \times 3 \times 3 \times 3.$$

The term 3^4 is read: "3 to the power of four" or "3 to the fourth power."

Exponential representation is shorthand form for repeated multiplication. Examples include:

$$2^1 = 2$$
$$3^2 = 3 \times 3$$
$$5^3 = 5 \times 5 \times 5$$
$$\ldots$$
$$n^4 = n \times n \times n \times n$$

(continued)

Any number to the zero power is always 1, no matter what the number is (see chapter 6). Examples include:

$3^0 = 1$
$9^0 = 1$
. . .
$n^0 = 1$

However, since the puzzle allows us to put the weights on both pans of the scale, the weighing can be done—Aha!—with only four weights of 1, 3, 9, and 27 pounds. The reason for this is remarkably simple—placing a weight on the right pan, along with the sugar, is equivalent to taking its weight away from the total weight on the left pan. Think about this for a moment. For example, if 2 pounds of sugar are to be weighed, we would put the 3-pound weight on the left pan and the 1-pound weight on the right pan. The result is that there are 2 pounds less on the right pan. We will therefore get a balance when we pour the missing 2 pounds of sugar on the right pan.

The four weights are, upon closer scrutiny, powers of 3:

$1 = 3^0$

$3 = 3^1$

$9 = 3^2$

$27 = 3^3$

The choice of these weights works because each of the whole numbers from 1 to 40 (= the required weights) turns out to be either a multiple or a power of 3, or else one more or less than a multiple or a power of 3. Thus, each of the first forty integers can be expressed with the first four powers of 3:

$1 = 3^0$	$(= 1)$
$2 = 3^1 - 3^0$	$(= 3 - 1)$
$3 = 3^1$	$(= 3)$
$4 = 3^1 + 3^0$	$(= 3 + 1)$
$5 = 3^2 - 3^1 - 3^0 = 3^2 - (3^1 + 3^0)$	$(= 9 - 3 - 1 = 6 - 1)$
. . .	. . .
$40 = 3^3 + 3^2 + 3^1 + 3^0$	$(= 27 + 9 + 3 + 1 = 39 + 1)$

Since the four powers of 3 represent our weights, all we have to do is "translate" addition in the previous layout as the action of putting weights on the left pan and subtraction as the action of putting weights on the right pan (along with the sugar). The following chart gives an indication of how this can be done. Readers may wish to complete it on their own:

TABLE 1-3: MÉZIRAC'S WEIGHT PUZZLE

Amount of Sugar to Be Weighed	Weight to Be Placed on the Left Pan	Weight Added to the Right Pan along with the Sugar
1	3^0 (= 1)	None
2	3^1 (= 3)	3^0 (= 1)
3	3^1 (= 3)	None
4	$3^1 + 3^0$ (= 3 + 1)	None
5	3^2 (= 9)	$3^1 + 3^0$ (= 4)
. . .	. . .	. . .
40	$3^3 + 3^2 + 3^1 + 3^0$ (= 27 + 9 + 3 + 1)	None

Reflections

The Riddle of the Sphinx is the first example in history of a true puzzle. Its origin in myth resonates to this day in stories composed for children. The heroes in such stories typically face challenges that are designed to test not only their physical mettle but also their mental ability to solve riddles. As such narrative traditions suggest, we perceive riddles as "miniature revelations" of truth. What are philosophy and science, after all, if not attempts to answer the riddles that life poses?

Mathematical inquiry, too, seems to be guided by an inborn need to model perplexing ideas in the form of puzzles. This is perhaps why some of the greatest questions of mathematical history were originally framed as puzzles. Solving them required a large dose of insight thinking. In many cases, the insight took centuries and even millennia to come to fruition. But, eventually, it did, leading to significant progress in mathematics. It would seem that in order to enter the "Thebes" of mathematical knowledge, we must first solve challenging riddles, not unlike the Riddle of the Sphinx.

Explorations

Riddles

1. What can be thrown away when it is caught but must be kept when it is not caught?

2. What possible creature is unlike its mother and does not resemble its father? You should also know that it is of mingled race and incapable of producing its own progeny.

3. I scare away my master's foes by bearing weapons in my jaws, yet I flee before the lashings of a little child. What am I?

4. It is something red, blue, purple, and green. Everyone can easily see it, yet no one can touch it or even reach it. What is it?

5. Before my birth I had a name, but it changed the instant I was born. And when I am no more, I will be called by my father's name. In sum, I change my name three days in a row, yet I live but one day. Who or what am I?

6. What belongs to you, which others use more than you do?

7. Create riddles based on the following words:

A. justice

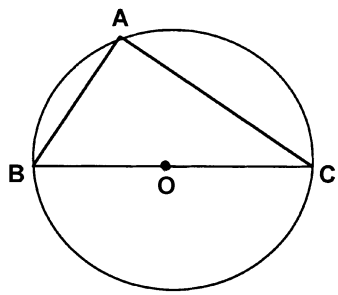
B. friendship

C. love

D. time

Deductive Reasoning

8. A triangle, **ABC**, is inscribed in a semicircle ("half circle"), with its base, **BC**, resting on the diameter. Prove that the angle opposite the base, ∠**BAC**, is equal to 90 degrees. The sign ∠ stands for "angle":



You may want to use these facts to develop your proof:

- The sum of the three angles in a triangle is 180 degrees.
- The diameter is a straight line made up of two radii (**OC** and **OB**).
- The radii of a circle are all equal.
- An isosceles triangle is a triangle with two equal sides.
- The angles in an isosceles triangle opposite the equal sides are equal.

Inductive Reasoning

9. Multiply several numbers by 9. Add up the digits of each product. If the result of the addition is a number that is more than one digit, add up the digits. Keep doing this until you get a one-digit number. For example:

$9 \times 50 = 450$

Add the digits of the product: $4 + 5 + 0 = 9$

$9 \times 43 = 387$

Add the digits of the product: $3 + 8 + 7 = 18$ (two digits)

Add the digits of the sum: $1 + 8 = 9$

$9 \times 693 = 6{,}237$

Add the digits of the product: $6 + 2 + 3 + 7 = 18$ (two digits)

Add the digits of the sum: $1 + 8 = 9$

Do you detect an emerging pattern here? If so, what is it?

10. Now, use the pattern discovered in the previous problem to determine which of the following numbers is a multiple of 9:

A. 477

B. 648

C. 8,765

D. 738

E. 9,878

11. Consider the squares of the numbers from 1 to 20:

$1^2 = 1 \times 1 = 1$

$2^2 = 2 \times 2 = 4$

$3^2 = 3 \times 3 = 9$

$4^2 = 4 \times 4 = 16$

$5^2 = 5 \times 5 = 25$

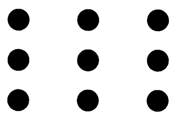
. . .

$20^2 = 20 \times 20 = 400$

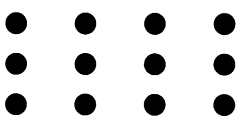
Do you detect a pattern here? If so, what can you predict about the square of 22 and the square of 23?

Insight Thinking

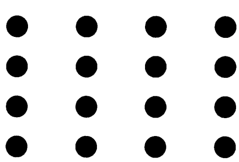
12. Recall the previous Nine-Dot Puzzle. It was solved with four lines. Can it be solved with only three straight lines? That is, can you connect the nine dots without lifting your pencil, using only three straight lines?



13. In the following version of the puzzle, there are twelve dots. Connect them without lifting your pencil. What is the least number of straight lines required to do so?



14. Finally, connect sixteen dots without lifting your pencil. What is the least number of straight lines required this time around?



Further Reading

The following list contains collections of puzzles and complementary treatments of the role of puzzles in the development of mathematics.

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