Chapter 2 Kinematics in One Dimension

2.1 Displacement

There are two aspects to any motion. In a purely descriptive sense, there is the movement itself. Is it rapid or slow, for instance? Then, there is the issue of what causes the motion or what changes it, which requires that forces be considered. ***Kinematics*** deals with the concepts that are needed to describe motion. without any reference to forces. The present chapter discusses these concepts as they apply to motion in one dimension, and the next chapter treats two-dimensional motion. ***Dynamics*** deals with the effect that forces have on motion, a topic that is considered in Chapter 4. Together, kinematics and dynamics form the branch of physics known as ***mechanics.*** We turn now to the first of the kinematics concepts to be discussed, which is displacement.

To describe the motion of an object, we must be able to specify the location of the object at all times, and Figure 2.1 shows how to do this for one-dimensional motion. In this drawing, the initial position of a car is indicated by the vector labeled $\mathbf{x}_0$. The length of $\mathbf{x}_0$ is the distance of the car from an arbitrarily chosen origin. At a later time the car has moved to a new position, which is indicated by the vector $\mathbf{x}$. The ***displacement*** of the car $\Delta\mathbf{x}$ (read as "delta x" or "the change in x") is a vector drawn from the initial position to the final position. Displacement is a vector quantity in the sense discussed in Section 1.5, for it conveys both a magnitude (the distance between the initial and final positions) and a direction. The displacement can be related to $\mathbf{x}_0$ and $\mathbf{x}$ by noting from the drawing that

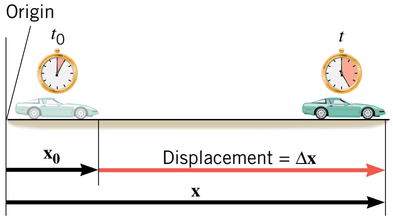


Figure 2.1 The displacement $\Delta\mathbf{x}$ is a vector that points from the initial position $\mathbf{x}_0$ to the final position $\mathbf{x}$.

$$\mathbf{x}_0 + \Delta\mathbf{x} = \mathbf{x} \quad \text{or} \quad \Delta\mathbf{x} = \mathbf{x} - \mathbf{x}_0$$

Thus, the displacement $\Delta\mathbf{x}$ is the difference between $\mathbf{x}$ and $\mathbf{x}_0$, and the Greek letter delta (Δ) is used to signify this difference. It is important to note that the change in any variable is always the final value minus the initial value.

DEFINITION OF DISPLACEMENT

The displacement is a vector that points from an object's initial position to its final position and has a magnitude that equals the shortest distance between the two positions.

SI Unit of Displacement: meter (m)

The SI unit for displacement is the meter (m), but there are other units as well, such as the centimeter and the inch. When converting between centimeters (cm) and inches (in.), remember that 2.54 cm = 1 in.

Often, we will deal with motion along a straight line. In such a case, a displacement in one direction along the line is assigned a positive value, and a displacement in the opposite direction is assigned a negative value. For instance, assume that a car is moving along an east/west direction and that a positive (+) sign is used to denote a direction due east. Then, $\Delta\mathbf{x} = +500$ m represents a displacement that points to the east and has a magnitude of 500 meters. Conversely, $\Delta\mathbf{x} = -500$ m is a displacement that has the same magnitude but points in the opposite direction, due west.

2.2 Speed and Velocity

AVERAGE SPEED

One of the most obvious features of an object in motion is how fast it is moving. If a car travels 200 meters in 10 seconds, we say its average speed is 20 meters per second, the

average speed being the distance traveled divided by the time required to cover the distance:

$$\text{Average speed} = \frac{\text{Distance}}{\text{Elapsed time}} \tag{2.1}$$

Equation 2.1 indicates that the unit for average speed is the unit for distance divided by the unit for time, or meters per second (m/s) in SI units. Example 1 illustrates how the idea of average speed is used.

Example 1 Distance Run by a Jogger

How far does a jogger run in 1.5 hours (5400 s) if his average speed is 2.22 m/s?

Reasoning The average speed of the jogger is the average distance per second that he travels. Thus, the distance covered by the jogger is equal to the average distance per second (his average speed) multiplied by the number of seconds (the elapsed time) that he runs.

Solution To find the distance run, we rewrite Equation 2.1 as

$$\text{Distance} = (\text{Average speed})(\text{Elapsed time}) = (2.22 \text{ m/s})(5400 \text{ s}) = \boxed{12\ 000 \text{ m}}$$

Speed is a useful idea, because it indicates how fast an object is moving. However, speed does not reveal anything about the direction of the motion. To describe both how fast an object moves and the direction of its motion, we need the vector concept of velocity.

AVERAGE VELOCITY

Suppose that the initial position of the car in Figure 2.1 is $\mathbf{x_0}$ when the time is t_0. A little later the car arrives at the final position $\mathbf{x}$ at the time t. The difference between these times is the time required for the car to travel between the two positions. We denote this difference by the shorthand notation Δt (read as "delta t"), where Δt represents the final time t minus the initial time t_0:

$$\Delta t = \underbrace{t - t_0}_{\text{Elapsed time}}$$

Note that Δt is defined in a manner analogous to $\Delta\mathbf{x}$, which is the final position minus the initial position ($\Delta\mathbf{x} = \mathbf{x} - \mathbf{x_0}$). Dividing the displacement $\Delta\mathbf{x}$ of the car by the elapsed time Δt gives the ***average velocity*** of the car. It is customary to denote the average value of a quantity by placing a horizontal bar above the symbol representing the quantity. The average velocity, then, is written as $\overline{\mathbf{v}}$, as specified in Equation 2.2:

Figure 2.2 In this time-lapse photo of traffic on the Los Angeles Freeway in California, the velocity of a car in the left lane (white headlights) is opposite to that of an adjacent car in the right lane (red taillights). (© Peter Essick/Aurora & Quanta Productions)

■ **DEFINITION OF AVERAGE VELOCITY**

$$\text{Average velocity} = \frac{\text{Displacement}}{\text{Elapsed time}}$$

$$\overline{\mathbf{v}} = \frac{\mathbf{x} - \mathbf{x_0}}{t - t_0} = \frac{\Delta\mathbf{x}}{\Delta t} \tag{2.2}$$

SI Unit of Average Velocity: meter per second (m/s)

Equation 2.2 indicates that the unit for average velocity is the unit for length divided by the unit for time, or meters per second (m/s) in SI units. Velocity can also be expressed in other units, such as kilometers per hour (km/h) or miles per hour (mi/h).

Average velocity is a vector that points in the same direction as the displacement in Equation 2.2. Figure 2.2 illustrates that the velocity of a car confined to move along a line can point either in one direction or in the opposite direction. As with displacgment, we will use plus and minus signs to indicate the two possible directions. If the displacement points in the positive direction, the average velocity is positive. Conversely, if the displacement points in the negative direction, the average velocity is negative. Example 2 illustrates these features of average velocity.

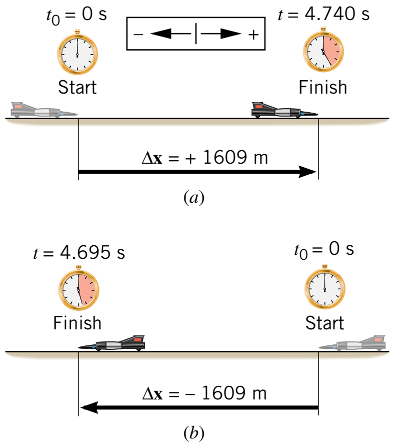


Figure 2.3 The arrows in the box at the top of the drawing indicate the positive and negative directions for the displacements of the car, as explained in Example 2.

Example 2 The World's Fastest Jet-Engine Car

Andy Green in the car *ThrustSSC* set a world record of 341.1 m/s (763 mi/h) in 1997. The car was powered by two jet engines, and it was the first one officially to exceed the speed of sound. To establish such a record, the driver makes two runs through the course, one in each direction, to nullify wind effects. Figure 2.3*a* shows that the car first travels from left to right and covers a distance of 1609 m (1 mile) in a time of 4.740 s. Figure 2.3*b* shows that in the reverse direction, the car covers the same distance in 4.695 s. From these data, determine the average velocity for each run.

Reasoning Average velocity is defined as the displacement divided by the elapsed time. In using this definition we recognize that the displacement is not the same as the distance traveled. Displacement takes the direction of the motion into account, and distance does not. During both runs, the car covers the same distance of 1609 m. However, for the first run the displacement is $\Delta\mathbf{x} = +1609$ m, while for the second it is $\Delta\mathbf{x} = -1609$ m. The plus and minus signs are essential, because the first run is to the right, which is the positive direction, and the second run is in the opposite or negative direction.

Solution According to Equation 2.2, the average velocities are

Run 1 $$\overline{\mathbf{v}} = \frac{\Delta\mathbf{x}}{\Delta t} = \frac{+1609\text{ m}}{4.740\text{ s}} = \boxed{+339.5\text{ m/s}}$$

Run 2 $$\overline{\mathbf{v}} = \frac{\Delta\mathbf{x}}{\Delta t} = \frac{-1609\text{ m}}{4.695\text{ s}} = \boxed{-342.7\text{ m/s}}$$

In these answers the algebraic signs convey the directions of the velocity vectors. In particular, for Run 2 the minus sign indicates that the average velocity, like the displacement, points to the left in Figure 2.3*b*. The magnitudes of the velocities are 339.5 and 342.7 m/s. The average of these numbers is 341.1 m/s and is recorded in the record book.

INSTANTANEOUS VELOCITY

Suppose the magnitude of your average velocity for a long trip was 20 m/s. This value, being an average, does not convey any information about how fast you were moving at any instant during the trip. Surely there were times when your car traveled faster than 20 m/s and times when it traveled more slowly. The ***instantaneous velocity*** $\mathbf{v}$ of the car indicates how fast the car moves and the direction of the motion at each instant of time. The magnitude of the instantaneous velocity is called the ***instantaneous speed,*** and it is the number (with units) indicated by the speedometer.

The instantaneous velocity at any point during a trip can be obtained by measuring the time interval Δt for the car to travel a *very small* displacement $\Delta\mathbf{x}$. We can then compute the average velocity over this interval. If the time Δt is small enough, the instantaneous velocity does not change much during the measurement. Then, the instantaneous velocity $\mathbf{v}$ at the point of interest is approximately equal to ($\approx$) the average velocity $\overline{\mathbf{v}}$ computed over the interval, or $\mathbf{v} \approx \overline{\mathbf{v}} = \Delta\mathbf{x}/\Delta t$ (for sufficiently small Δt). In fact, in the limit that Δt becomes infinitesimally small, the instantaneous velocity and the average velocity become equal, so that

$$\mathbf{v} = \lim_{\Delta t \to 0} \frac{\Delta\mathbf{x}}{\Delta t} \tag{2.3}$$

The notation $\lim_{\Delta t \to 0} (\Delta\mathbf{x}/\Delta t)$ means that the ratio $\Delta\mathbf{x}/\Delta t$ is defined by a limiting process in which smaller and smaller values of Δt are used, so small that they approach zero. As smaller values of Δt are used, $\Delta\mathbf{x}$ also becomes smaller. However, the ratio $\Delta\mathbf{x}/\Delta t$ does *not* become zero but, rather, approaches the value of the instantaneous velocity. For brevity, we will use the word *velocity* to mean "instantaneous velocity" and *speed* to mean "instantaneous speed."

In a wide range of motions, the velocity changes from moment to moment. To describe the manner in which it changes, the concept of acceleration is needed.

2.3 Acceleration

The velocity of a moving object may change in a number of ways. For example, it may increase, as it does when the driver of a car steps on the gas pedal to pass the car ahead. Or it may decrease, as it does when the driver applies the brakes to stop at a red light. In either case, the change in velocity may occur over a short or a long time interval.

To describe how the velocity of an object changes during a given time interval, we now introduce the new idea of acceleration; this idea depends on two concepts that we have previously encountered, velocity and time. Specifically, the notion of acceleration emerges when the *change* in the velocity is combined with the time during which the change occurs.

The meaning of ***average acceleration*** can be illustrated by considering a plane during takeoff. Figure 2.4 focuses attention on how the plane's velocity changes along the runway. During an elapsed time interval $\Delta t = t - t_0$, the velocity changes from an initial value of $\mathbf{v_0}$ to a final value of $\mathbf{v}$. The change $\Delta\mathbf{v}$ in the plane's velocity is its final velocity minus its initial velocity, so that $\Delta\mathbf{v} = \mathbf{v} - \mathbf{v_0}$. The average acceleration $\bar{\mathbf{a}}$ is defined in the following manner, to provide a measure of how much the velocity changes per unit of elapsed time.

DEFINITION OF AVERAGE ACCELERATION

$$\text{Average acceleration} = \frac{\text{Change in velocity}}{\text{Elapsed time}}$$

$$\bar{\mathbf{a}} = \frac{\mathbf{v} - \mathbf{v_0}}{t - t_0} = \frac{\Delta\mathbf{v}}{\Delta t} \tag{2.4}$$

SI Unit of Average Acceleration: meter per second squared (m/s^2)

The average acceleration $\bar{\mathbf{a}}$ is a vector that points in the same direction as $\Delta\mathbf{v}$, the change in the velocity. Following the usual custom, plus and minus signs indicate the two possible directions for the acceleration vector when the motion is along a straight line.

We are often interested in an object's acceleration at a particular instant of time. The ***instantaneous acceleration* a** can be defined by analogy with the procedure used in Section 2.2 for instantaneous velocity:

$$\mathbf{a} = \lim_{\Delta t \to 0} \frac{\Delta\mathbf{v}}{\Delta t} \tag{2.5}$$

Equation 2.5 indicates that the instantaneous acceleration is a limiting case of the average acceleration. When the time interval Δt for measuring the acceleration becomes extremely small (approaching zero in the limit), the average acceleration and the instantaneous acceleration become equal. Moreover, in many situations the acceleration is constant, so the acceleration has the same value at any instant of time. In the future, we will use the word *acceleration* to mean "instantaneous acceleration." Example 3 deals with the acceleration of a plane during takeoff.

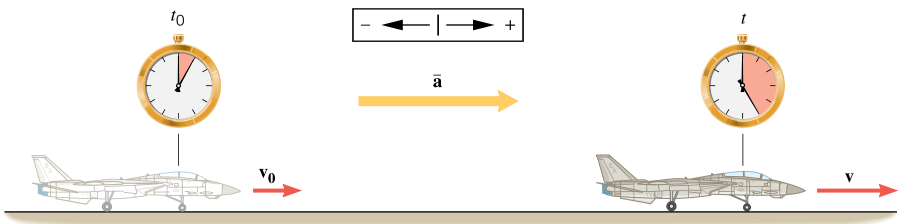


Figure 2.4 During takeoff, the plane accelerates from an initial velocity $\mathbf{v_0}$ to a final velocity $\mathbf{v}$ during the time interval $\Delta t = t - t_0$.

Example 3 Acceleration and Increasing Velocity

Suppose the plane in Figure 2.4 starts from rest ($\mathbf{v}_0 = 0$ m/s) when $t_0 = 0$ s. The plane accelerates down the runway and at $t = 29$ s attains a velocity of $\mathbf{v} = +260$ km/h, where the plus sign indicates that the velocity points to the right. Determine the average acceleration of the plane.

Reasoning The average acceleration of the plane is defined as the change in its velocity divided by the elapsed time. The change in the plane's velocity is its final velocity $\mathbf{v}$ minus its initial velocity $\mathbf{v}_0$, or $\mathbf{v} - \mathbf{v}_0$. The elapsed time is the final time t minus the initial time t_0, or $t - t_0$.

Problem solving insight
The change in any variable is the final value minus the initial value: for example, the change in velocity is $\Delta\mathbf{v} = \mathbf{v} - \mathbf{v}_0$, and the change in time is $\Delta t = t - t_0$.

Solution The average acceleration is expressed by Equation 2.4 as

$$\bar{\mathbf{a}} = \frac{\mathbf{v} - \mathbf{v}_0}{t - t_0} = \frac{260 \text{ km/h} - 0 \text{ km/h}}{29 \text{ s} - 0 \text{ s}} = \boxed{+9.0 \frac{\text{km/h}}{\text{s}}}$$

The average acceleration calculated in Example 3 is read as "nine kilometers per hour per second." Assuming the acceleration of the plane is constant, a value of $+9.0 \dfrac{\text{km/h}}{\text{s}}$ means the velocity changes by $+9.0$ km/h during each second of the motion. During the first second, the velocity increases from 0 to 9.0 km/h; during the next second, the velocity increases by another 9.0 km/h to 18 km/h, and so on. Figure 2.5 illustrates how the velocity changes during the first two seconds. By the end of the 29th second, the velocity is 260 km/h.

It is customary to express the units for acceleration solely in terms of SI units. One way to obtain SI units for the acceleration in Example 3 is to convert the velocity units from km/h to m/s:

$$\left(260 \frac{\cancel{\text{km}}}{\cancel{\text{h}}}\right)\left(\frac{1000 \text{ m}}{1 \cancel{\text{km}}}\right)\left(\frac{1 \cancel{\text{h}}}{3600 \text{ s}}\right) = 72 \frac{\text{m}}{\text{s}}$$

The average acceleration then becomes

$$\bar{\mathbf{a}} = \frac{72 \text{ m/s} - 0 \text{ m/s}}{29 \text{ s} - 0 \text{ s}} = +2.5 \text{ m/s}^2$$

where we have used $2.5 \dfrac{\text{m/s}}{\text{s}} = 2.5 \dfrac{\text{m}}{\text{s}\cdot\text{s}} = 2.5 \dfrac{\text{m}}{\text{s}^2}$. An acceleration of $2.5 \dfrac{\text{m}}{\text{s}^2}$ is read as "2.5 meters per second per second" (or "2.5 meters per second squared") and means that the velocity changes by 2.5 m/s during each second of the motion.

Example 4 deals with a case where the motion becomes slower as time passes.

Example 4 Acceleration and Decreasing Velocity

A drag racer crosses the finish line, and the driver deploys a parachute and applies the brakes to slow down, as Figure 2.6 illustrates. The driver begins slowing down when $t_0 = 9.0$ s and

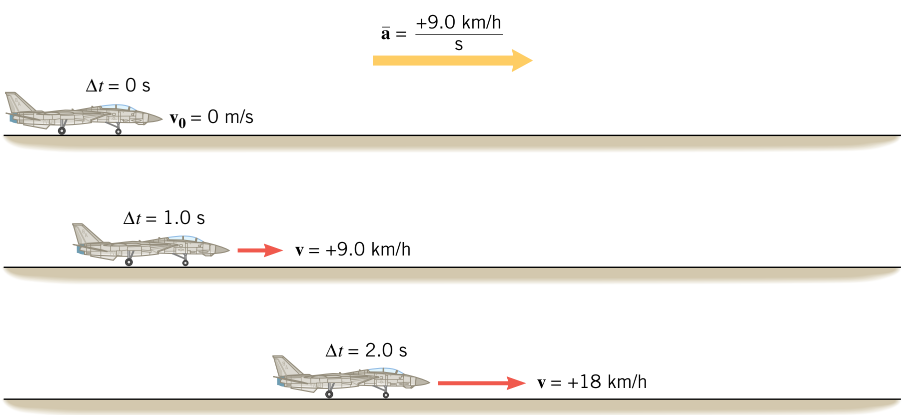


Figure 2.5 An acceleration of $+9.0 \dfrac{\text{km/h}}{\text{s}}$ means that the velocity of the plane changes by $+9.0$ km/h during each second of the motion. The "+" direction for **a** and **v** is to the right.

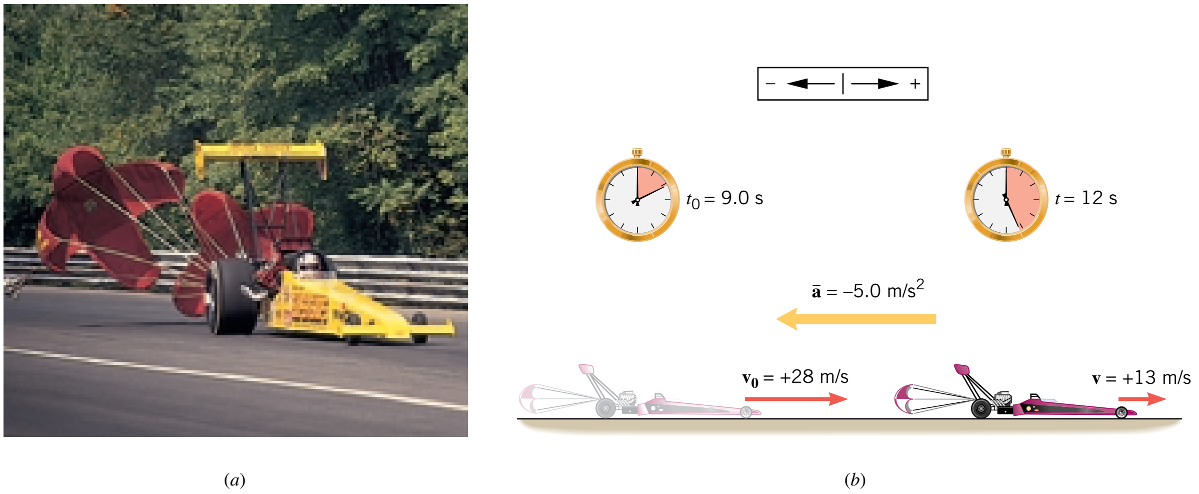


Figure 2.6 (*a*) To slow down, a drag racer deploys a parachute and applies the brakes. (*b*) The velocity of the car is decreasing, giving rise to an average acceleration $\bar{\mathbf{a}}$ that points opposite to the velocity. (© Geoff Stunkard)

the car's velocity is $\mathbf{v_0} = +28$ m/s. When $t = 12.0$ s, the velocity has been reduced to $\mathbf{v} = +13$ m/s. What is the average acceleration of the dragster?

Reasoning The average acceleration of an object is always specified as its change in velocity, $\mathbf{v} - \mathbf{v_0}$, divided by the elapsed time, $t - t_0$. This is true whether the final velocity is less than the initial velocity or greater than the initial velocity.

Solution The average acceleration is, according to Equation 2.4,

$$\bar{\mathbf{a}} = \frac{\mathbf{v} - \mathbf{v_0}}{t - t_0} = \frac{13\text{ m/s} - 28\text{ m/s}}{12.0\text{ s} - 9.0\text{ s}} = \boxed{-5.0\text{ m/s}^2}$$

Figure 2.7 shows how the velocity of the dragster changes during the braking, assuming that the acceleration is constant throughout the motion. The acceleration calculated in Example 4 is negative, indicating that the acceleration points to the left in the drawing. As a result, the acceleration and the velocity point in *opposite* directions. ***Whenever the acceleration and velocity vectors have opposite directions, the object slows down and is said to be "decelerating."*** In contrast, the acceleration and velocity vectors in Figure 2.5 point in the *same* direction, and the object speeds up.

2.4 Equations of Kinematics for Constant Acceleration

In discussing the equations of kinematics, it will be convenient to assume that the object is located at the origin $\mathbf{x_0} = 0$ m when $t_0 = 0$ s. With this assumption, the displace-

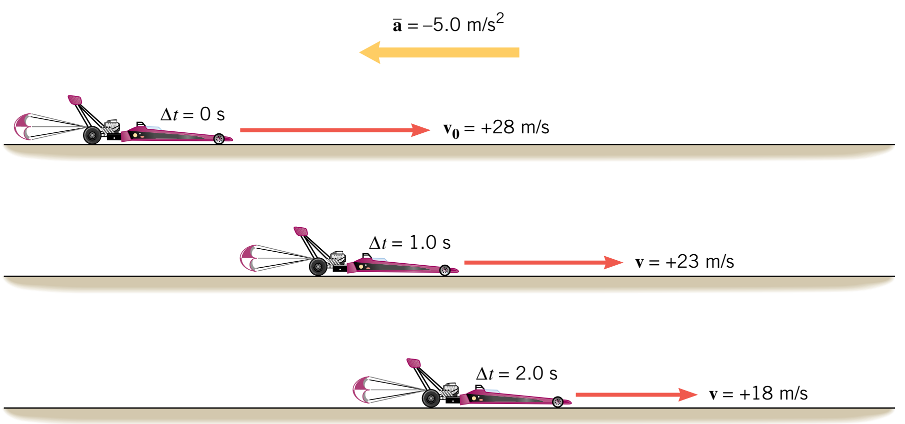


Figure 2.7 Here, an acceleration of -5.0 m/s^2 means the velocity decreases by 5.0 m/s during each second of elapsed time.

ment $\Delta \mathbf{x} = \mathbf{x} - \mathbf{x}_0$ becomes $\Delta \mathbf{x} = \mathbf{x}$. Furthermore, it is customary to dispense with the use of boldface symbols for the displacement, velocity, and acceleration vectors in the equations that follow. We will, however, continue to convey the directions of these vectors with plus or minus signs.

Consider an object that has an initial velocity of v_0 at time $t_0 = 0$ s and moves for a time t with a constant acceleration a. For a complete description of the motion, it is also necessary to know the final velocity and displacement at time t. The final velocity v can be obtained directly from Equation 2.4:

$$\bar{a} = a = \frac{v - v_0}{t} \quad \text{or} \quad v = v_0 + at \qquad \text{(constant acceleration)} \tag{2.4}$$

The displacement x at time t can be obtained from Equation 2.2, if a value for the average velocity $\bar{v}$ can be obtained. Considering the assumption that $\mathbf{x}_0 = 0$ m at $t_0 = 0$ s, we have

$$\bar{v} = \frac{x - x_0}{t - t_0} = \frac{x}{t} \quad \text{or} \quad x = \bar{v}t \tag{2.2}$$

Because the acceleration is constant, the velocity increases at a constant rate. Thus, the average velocity $\bar{v}$ is midway between the initial and final velocities:

$$\bar{v} = \tfrac{1}{2}(v_0 + v) \qquad \text{(constant acceleration)} \tag{2.6}$$

Equation 2.6, like Equation 2.4, applies only if the acceleration is constant and cannot be used when the acceleration is changing. The displacement at time t can now be determined as

$$x = \bar{v}t = \tfrac{1}{2}(v_0 + v)t \qquad \text{(constant acceleration)} \tag{2.7}$$

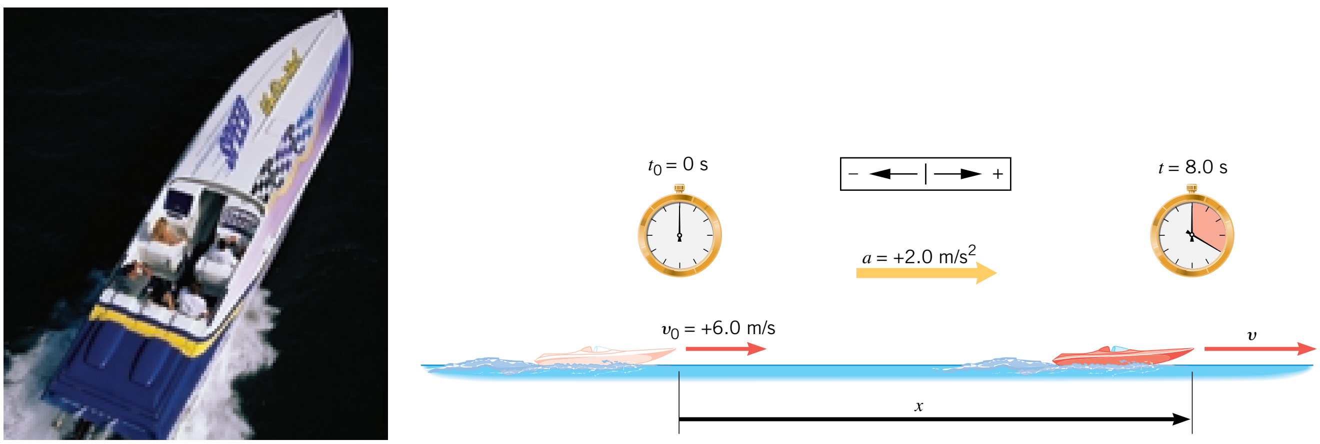
Notice in Equations 2.4 ($v = v_0 + at$) and 2.7 [$x = \tfrac{1}{2}(v_0 + v)t$] that there are five kinematic variables:

1. x = displacement
2. $a = \bar{a}$ = acceleration (constant)
3. v = final velocity at time t
4. v_0 = initial velocity at time $t_0 = 0$ s
5. t = time elapsed since $t_0 = 0$ s

Each of the two equations contains four of these variables, so if three of them are known, the fourth variable can always be found. Example 5 illustrates how Equations 2.4 and 2.7 are used to describe the motion of an object.

Example 5 The Displacement of a Speedboat

The speedboat in Figure 2.8 has a constant acceleration of $+2.0\ \text{m/s}^2$. If the initial velocity of the boat is $+6.0$ m/s, find its displacement after 8.0 seconds.



(*a*) (*b*)

Figure 2.18 (*a*) An accelerating speedboat. (*b*) The boat's displacement x can be determined if the boat's acceleration, initial velocity, and time of travel are known. (© Onne van der Wal/Corbis Images)

Reasoning Numerical values for the three known variables are listed in the data table below. We wish to determine the displacement x of the speedboat, so it is an unknown variable. Therefore, we have placed a question mark in the displacement column of the data table.

Speedboat Data				
x	a	v	v_0	t
?	$+2.0\ \text{m/s}^2$		$+6.0\ \text{m/s}$	8.0 s

We can use $x = \frac{1}{2}(v_0 + v)t$ to find the displacement of the boat if a value for the final velocity v can be found. To find the final velocity, it is necessary to use the value given for the acceleration, because it tells us how the velocity changes, according to $v = v_0 + at$.

Solution The final velocity is

$$v = v_0 + at = 6.0\ \text{m/s} + (2.0\ \text{m/s}^2)(8.0\ \text{s}) = +22\ \text{m/s} \qquad (2.4)$$

The displacement of the boat can now be obtained:

$$x = \tfrac{1}{2}(v_0 + v)t = \tfrac{1}{2}(6.0\ \text{m/s} + 22\ \text{m/s})(8.0\ \text{s}) = \boxed{+110\ \text{m}} \qquad (2.7)$$

A calculator would give the answer as 112 m, but this number must be rounded to 110 m, since the data are accurate to only two significant figures.

The solution to Example 5 involved two steps: finding the final velocity v and then calculating the displacement x. It would be helpful if we could find an equation that allows us to determine the displacement in a single step. Using Example 5 as a guide, we can obtain such an equation by substituting the final velocity v from Equation 2.4 ($v = v_0 + at$) into Equation 2.7 [$x = \frac{1}{2}(v_0 + v)t$]:

$$x = \tfrac{1}{2}(v_0 + v)t = \tfrac{1}{2}(v_0 + \boxed{v_0 + at})t = \tfrac{1}{2}(2v_0 t + at^2)$$

$$x = v_0 t + \tfrac{1}{2}at^2 \qquad \text{(constant acceleration)} \qquad (2.8)$$

You can verify that Equation 2.8 gives the displacement of the speedboat directly without the intermediate step of determining the final velocity. The first term ($v_0 t$) on the right side of this equation represents the displacement that would result if the acceleration were zero and the velocity remained constant at its initial value of v_0. The second term ($\frac{1}{2}at^2$) gives the additional displacement that arises because the velocity changes (a is not zero) to values that are different from its initial value. We now turn to another example of accelerated motion.

The physics of **catapulting a jet from an aircraft carrier.**

Example 6 Catapulting a Jet

A jet is taking off from the deck of an aircraft carrier, as Figure 2.9 shows. Starting from rest, the jet is catapulted with a constant acceleration of $+31\ \text{m/s}^2$ along a straight line and reaches a velocity of $+62\ \text{m/s}$. Find the displacement of the jet.

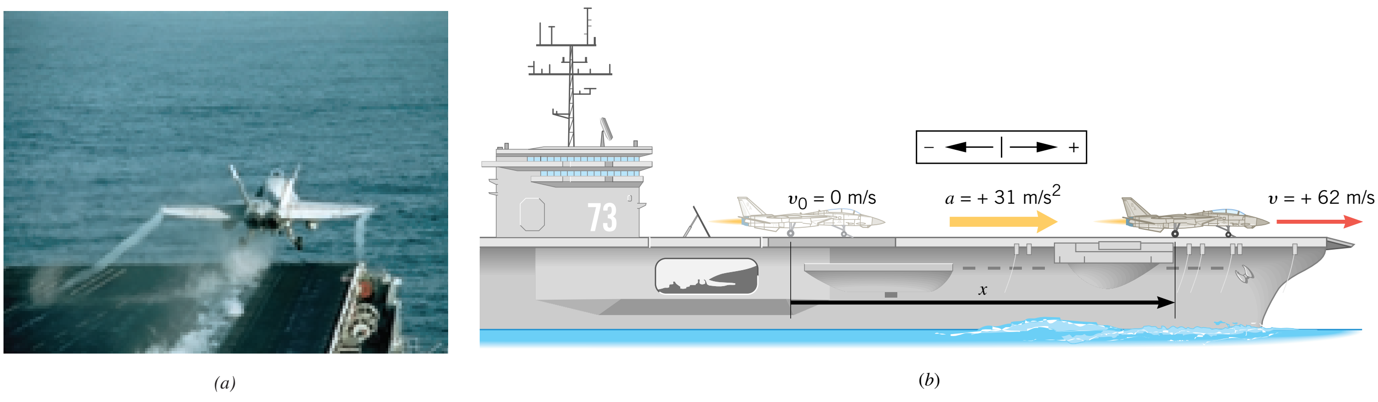


Figure 2.9 (*a*) A plane is being launched from an aircraft carrier. (*b*) During the launch, a catapult accelerates the jet down the flight deck. (© George Hall/Corbis Images)

Reasoning The data are as follows:

Jet Data				
x	a	v	v_0	t
?	$+31\ \text{m/s}^2$	$+62\ \text{m/s}$	$0\ \text{m/s}$	

Problem solving insight
"Implied data" are important. For instance, in Example 6 the phrase "starting from rest" means that the initial velocity is zero ($v_0 = 0$ m/s).

The initial velocity v_0 is zero, since the jet starts from rest. The displacement x of the aircraft can be obtained from $x = \frac{1}{2}(v_0 + v)t$, if we can determine the time t during which the plane is being accelerated. But t is controlled by the value of the acceleration. With larger accelerations, the jet reaches its final velocity in shorter times, as can be seen by solving Equation 2.4 ($v = v_0 + at$) for t.

Solution Solving Equation 2.4 for t, we find

$$t = \frac{v - v_0}{a} = \frac{62\ \text{m/s} - 0\ \text{m/s}}{31\ \text{m/s}^2} = 2.0\ \text{s}$$

Since the time is now known, the displacement can be found by using Equation 2.7:

$$x = \tfrac{1}{2}(v_0 + v)t = \tfrac{1}{2}(0\ \text{m/s} + 62\ \text{m/s})(2.0\ \text{s}) = \boxed{+62\ \text{m}} \quad (2.7)$$

When a, v, and v_0 are known, but the time t is not known, as in Example 6, it is possible to calculate the displacement x in a single step. Solving Equation 2.4 for the time [$t = (v - v_0)/a$] and then substituting into Equation 2.7 [$x = \frac{1}{2}(v_0 + v)t$] reveals that

$$x = \tfrac{1}{2}(v_0 + v)t = \tfrac{1}{2}(v_0 + v)\boxed{\frac{v - v_0}{a}} = \frac{v^2 - v_0^2}{2a}$$

Solving for v^2 shows that

$$v^2 = v_0^2 + 2ax \qquad \text{(constant acceleration)} \quad (2.9)$$

It is a straightforward exercise to verify that Equation 2.9 can be used to find the displacement of the jet in Example 6 without having to solve first for the time.

Table 2.1 presents a summary of the equations that we have been considering. These equations are called the ***equations of kinematics.*** Each equation contains four variables, as indicated by the check marks (✓) in the table. The next section shows how to apply the equations of kinematics.

2.5 Applications of the Equations of Kinematics

The equations of kinematics can be used for any moving object, as long as the acceleration of the object is constant. However, to avoid errors when using these equations, it helps to follow a few sensible guidelines and to be alert for a few situations that can arise during your calculations.

Table 2.1 *Equations of Kinematics for Constant Acceleration*

Equation Number	Equation	Variables				
		x	a	v	v_0	t
(2.4)	$v = v_0 + at$	—	✓	✓	✓	✓
(2.7)	$x = \frac{1}{2}(v_0 + v)t$	✓	—	✓	✓	✓
(2.8)	$x = v_0t + \frac{1}{2}at^2$	✓	✓	—	✓	✓
(2.9)	$v^2 = v_0^2 + 2ax$	✓	✓	✓	✓	—

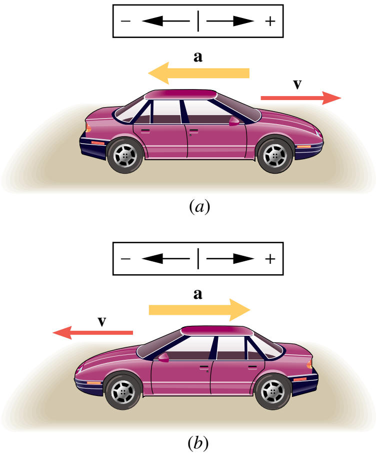


Figure 2.10 When a car decelerates along a straight road, the acceleration vector points opposite to the velocity vector, as Conceptual Example 7 discusses.

Decide at the start which directions are to be called positive (+) and negative (−) relative to a conveniently chosen coordinate origin. This decision is arbitrary, but important because displacement, velocity, and acceleration are vectors, and their directions must always be taken into account. In the examples that follow, the positive and negative directions will be shown in the drawings that accompany the problems. It does not matter which direction is chosen to be positive. However, once the choice is made, it should not be changed during the course of the calculation.

As you reason through a problem before attempting to solve it, be sure to interpret the terms "decelerating" or "deceleration" correctly, should they occur in the problem statement. These terms are the source of frequent confusion, and Conceptual Example 7 offers help in understanding them.

Conceptual Example 7 Deceleration Versus Negative Acceleration

A car is traveling along a straight road and is decelerating. Does the car's acceleration a necessarily have a negative value?

Reasoning and Solution We begin with the meaning of the term "decelerating," which has nothing to do with whether the acceleration a is positive or negative. The term means only that the acceleration vector points opposite to the velocity vector and indicates that the moving object is slowing down. When a moving object slows down, its instantaneous speed (the magnitude of the instantaneous velocity) decreases. One possibility is that the velocity vector of the car points to the right, in the positive direction, as Figure 2.10*a* shows. The term "decelerating" implies that the acceleration vector points opposite, or to the left, which is the negative direction. Here, the value of the acceleration a would indeed be negative. However, there is another possibility. The car could be traveling to the left, as in Figure 2.10*b*. Now, since the velocity vector points to the left, the acceleration vector would point opposite or to the right, according to the meaning of the term "decelerating." But right is the positive direction, so the acceleration a would have a positive value in Figure 2.10*b*. We see, then, that ***a decelerating object does not necessarily have a negative acceleration.***

Related Homework: *Problems 20, 38*

Sometimes there are two possible answers to a kinematics problem, each answer corresponding to a different situation. Example 8 discusses one such case.

Example 8 An Accelerating Spacecraft

The physics of
the acceleration caused by a retrorocket.

The spacecraft shown in Figure 2.11*a* is traveling with a velocity of +3250 m/s. Suddenly the retrorockets are fired, and the spacecraft begins to slow down with an acceleration whose magnitude is 10.0 m/s^2. What is the velocity of the spacecraft when the displacement of the craft is +215 km, relative to the point where the retrorockets began firing?

Reasoning Since the spacecraft is slowing down, the acceleration must be opposite to the velocity. The velocity points to the right in the drawing, so the acceleration points to the left, in the negative direction; thus, $a = -10.0$ m/s^2. The three known variables are listed as follows:

Spacecraft Data

x	a	v	v_0	t
+215 000 m	−10.0 m/s^2	?	+3250 m/s	

The final velocity v of the spacecraft can be calculated using Equation 2.9, since it contains the four pertinent variables.

Solution From Equation 2.9 ($v^2 = v_0^2 + 2ax$), we find that

$$v = \pm\sqrt{v_0^2 + 2ax} = \pm\sqrt{(3250 \text{ m/s})^2 + 2(-10.0 \text{ m/s}^2)(215\,000 \text{ m})}$$
$$= \boxed{+2500 \text{ m/s}} \quad \text{and} \quad \boxed{-2500 \text{ m/s}}$$

Both of these answers correspond to the *same* displacement ($x = +215$ km), but each arises in a different part of the motion. The answer $v = +2500$ m/s corresponds to the situation in Fig-

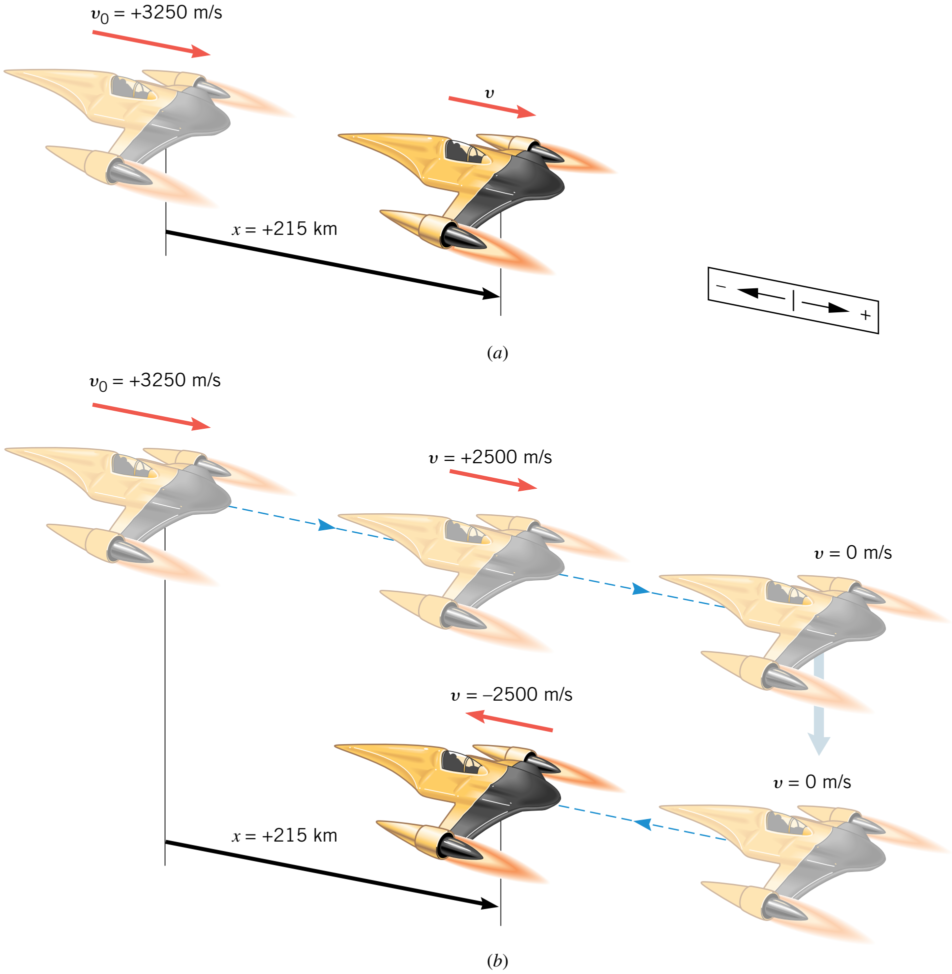


Figure 2.11 (*a*) Because of an acceleration of $-10.0\ \text{m/s}^2$, the spacecraft changes its velocity from v_0 to v. (*b*) Continued firing of the retrorockets changes the direction of the craft's motion.

ure 2.11*a*, where the spacecraft has slowed to a speed of 2500 m/s, but is still traveling to the right. The answer $v = -2500$ m/s arises because the retrorockets eventually bring the spacecraft to a momentary halt and cause it to reverse its direction. Then it moves to the left, and its speed increases due to the continually firing rockets. After a time, the velocity of the craft becomes $v = -2500$ m/s, giving rise to the situation in Figure 2.11*b*. In both parts of the drawing the spacecraft has the same displacement, but a greater travel time is required in part *b* compared to part *a*.

The motion of two objects may be interrelated, so they share a common variable. The fact that the motions are interrelated is an important piece of information. In such cases, data for only two variables need be specified for each object.

Often the motion of an object is divided into segments, each with a different acceleration. When solving such problems, it is important to realize that the final velocity for one segment is the initial velocity for the next segment, as Example 9 illustrates.

Example 9 A Motorcycle Ride

A motorcycle, starting from rest, has an acceleration of $+2.6\ \text{m/s}^2$. After the motorcycle has traveled a distance of 120 m, it slows down with an acceleration of $-1.5\ \text{m/s}^2$ until its velocity is $+12$ m/s (see Figure 2.12). What is the total displacement of the motorcycle?

Reasoning The total displacement is the sum of the displacements for the first ("speeding up") and second ("slowing down") segments. The displacement for the first segment is +120 m. The displacement for the second segment can be found if the initial velocity for this segment can be determined, since values for two other variables are already known ($a = -1.5\ \text{m/s}^2$ and $v = +12$ m/s). The initial velocity for the second segment can be determined, since it is the final velocity of the first segment.

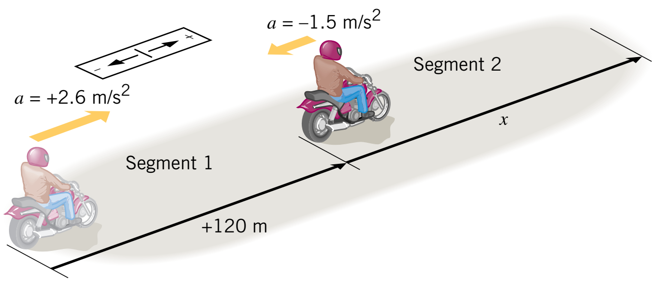


Figure 2.12 This motorcycle ride consists of two segments, each with a different acceleration.

Solution Recognizing that the motorcycle starts from rest ($v_0 = 0$ m/s), we can determine the final velocity v of the first segment from the given data:

Problem solving insight
It is only when values are available for at least three of the five kinematic variables (x, a, v, v_0, and t) that the equations in Table 2.1 can be used to determine the fourth and fifth variables.

Segment 1 Data				
x	a	v	v_0	t
+120 m	$+2.6\ \text{m/s}^2$	?	0 m/s	

From Equation 2.9 ($v^2 = v_0^2 + 2ax$), it follows that

$$v = +\sqrt{v_0^2 + 2ax} = +\sqrt{(0\ \text{m/s})^2 + 2(2.6\ \text{m/s}^2)(120\ \text{m})} = +25\ \text{m/s}$$

Now we can use +25 m/s as the initial velocity for the second segment, along with the remaining data listed below:

Segment 2 Data				
x	a	v	v_0	t
?	$-1.5\ \text{m/s}^2$	+12 m/s	+25 m/s	

The displacement for segment 2 can be obtained by solving $v^2 = v_0^2 + 2ax$ for x:

$$x = \frac{v^2 - v_0^2}{2a} = \frac{(12\ \text{m/s})^2 - (25\ \text{m/s})^2}{2(-1.5\ \text{m/s}^2)} = +160\ \text{m}$$

The total displacement of the motorcycle is 120 m + 160 m = $\boxed{280\ \text{m}}$.

2.6 Freely Falling Bodies

Everyone has observed the effect of gravity as it causes objects to fall downward. In the absence of air resistance, it is found that all bodies at the same location above the earth fall vertically with the same acceleration. Furthermore, if the distance of the fall is small compared to the radius of the earth, the acceleration remains essentially constant throughout the descent. This idealized motion, in which air resistance is neglected and the acceleration is nearly constant, is known as ***free-fall.*** Since the acceleration is constant in free-fall, the equations of kinematics can be used.

The acceleration of a freely falling body is called the ***acceleration due to gravity,*** and its magnitude (without any algebraic sign) is denoted by the symbol g. The acceleration due to gravity is directed downward, toward the center of the earth. Near the earth's surface, g is approximately

$$g = 9.80\ \text{m/s}^2 \quad \text{or} \quad 32.2\ \text{ft/s}^2$$

Unless circumstances warrant otherwise, we will use either of these values for g in subsequent calculations. In reality, however, g decreases with increasing altitude and varies slightly with latitude.

Figure 2.13*a* shows the well-known phenomenon of a rock falling faster than a sheet of paper. The effect of air resistance is responsible for the slower fall of the paper, for when air is removed from the tube, as in Figure 2.13*b*, the rock and the paper have exactly

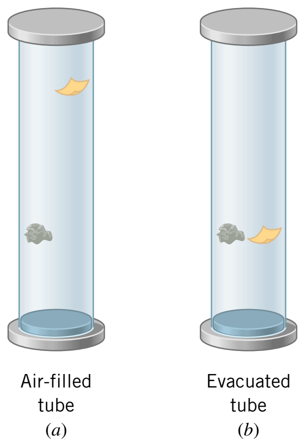


Figure 2.13 (*a*) In the presence of air resistance, the acceleration of the rock is greater than that of the paper. (*b*) In the absence of air resistance, both the rock and the paper have the same acceleration.

the same acceleration due to gravity. In the absence of air, the rock and the paper both exhibit free-fall motion. Free-fall is closely approximated for objects falling near the surface of the moon, where there is no air to retard the motion. A nice demonstration of lunar free-fall was performed by astronaut David Scott, who dropped a hammer and a feather simultaneously from the same height. Both experienced the same acceleration due to lunar gravity and consequently hit the ground at the same time. The acceleration due to gravity near the surface of the moon is approximately one-sixth as large as that on the earth.

When the equations of kinematics are applied to free-fall motion, it is natural to use the symbol y for the displacement, since the motion occurs in the vertical or y direction. Thus, when using the equations in Table 2.1 for free-fall motion, we will simply replace x with y. There is no significance to this change. The equations have the same algebraic form for either the horizontal or vertical direction, provided that the acceleration remains constant during the motion. We now turn our attention to several examples that illustrate how the equations of kinematics are applied to freely falling bodies.

Example 10 A Falling Stone

A stone is dropped from rest from the top of a tall building, as Figure 2.14 indicates. After 3.00 s of free-fall, what is the displacement y of the stone?

Reasoning The upward direction is chosen as the positive direction. The three known variables are shown in the box below. The initial velocity v_0 of the stone is zero, because the stone is dropped from rest. The acceleration due to gravity is negative, since it points downward in the negative direction.

Stone Data

y	a	v	v_0	t
?	$-9.80\ \mathrm{m/s^2}$		$0\ \mathrm{m/s}$	$3.00\ \mathrm{s}$

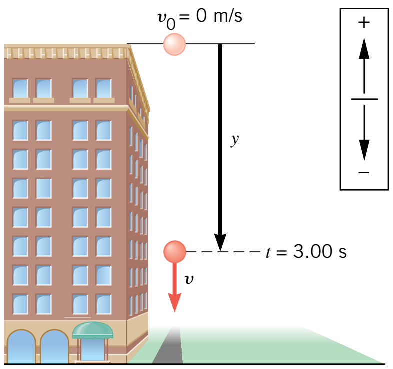


Figure 2.14 The stone, starting with zero velocity at the top of the building, is accelerated downward by gravity.

Equation 2.8 contains the appropriate variables and offers a direct solution to the problem. Since the stone moves downward, and upward is the positive direction, we expect the displacement y to have a negative value.

Solution Using Equation 2.8, we find that

$$y = v_0 t + \tfrac{1}{2}at^2 = (0\ \mathrm{m/s})(3.00\ \mathrm{s}) + \tfrac{1}{2}(-9.80\ \mathrm{m/s^2})(3.00\ \mathrm{s})^2 = \boxed{-44.1\ \mathrm{m}}$$

The answer for y is negative, as expected.

Example 11 The Velocity of a Falling Stone

After 3.00 s of free-fall, what is the velocity v of the stone in Figure 2.14?

Reasoning Because of the acceleration due to gravity, the magnitude of the stone's downward velocity increases by 9.80 m/s during each second of free-fall. The data for the stone are the same as in Example 10, and Equation 2.4 offers a direct solution for the final velocity. Since the stone is moving downward in the negative direction, the value determined for v should be negative.

Solution Using Equation 2.4, we obtain

$$v = v_0 + at = 0\ \mathrm{m/s} + (-9.80\ \mathrm{m/s^2})(3.00\ \mathrm{s}) = \boxed{-29.4\ \mathrm{m/s}}$$

The velocity is negative, as expected.

The acceleration due to gravity is always a downward-pointing vector. It describes how the speed increases for an object that is falling freely downward. This same acceleration also describes how the speed decreases for an object moving upward under the influence of gravity alone, in which case the object eventually comes to a momentary halt and then falls back to earth. Examples 12 and 13 show how the equations of kinematics are applied to an object that is moving upward under the influence of gravity.

Example 12 How High Does It Go?

Problem solving insight
"Implied data" are important. In Example 12, for instance, the phrase "how high does the coin go" refers to the maximum height, which occurs when the final velocity v in the vertical direction is $v = 0$ m/s.

A football game customarily begins with a coin toss to determine who kicks off. The referee tosses the coin up with an initial speed of 5.00 m/s. In the absence of air resistance, how high does the coin go above its point of release?

Reasoning The coin is given an upward initial velocity, as in Figure 2.15. But the acceleration due to gravity points downward. Since the velocity and acceleration point in opposite directions, the coin slows down as it moves upward. Eventually, the velocity of the coin becomes $v = 0$ m/s at the highest point. Assuming that the upward direction is positive, the data can be summarized as shown below:

Coin Data				
y	a	v	v_0	t
?	$-9.80\ \text{m/s}^2$	0 m/s	+5.00 m/s	

With these data, we can use Equation 2.9 ($v^2 = v_0^2 + 2ay$) to find the maximum height y.

Solution Rearranging Equation 2.9, we find that the maximum height of the coin above its release point is

$$y = \frac{v^2 - v_0^2}{2a} = \frac{(0\ \text{m/s})^2 - (5.00\ \text{m/s})^2}{2(-9.80\ \text{m/s}^2)} = \boxed{1.28\ \text{m}}$$

Example 13 How Long Is It in the Air?

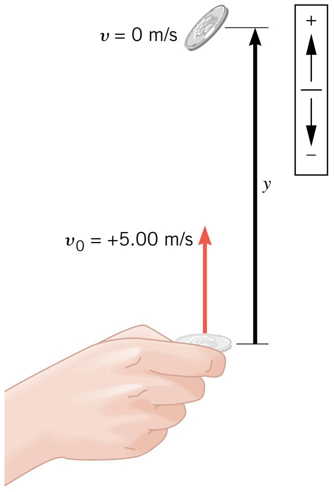


Figure 2.15 At the start of a football game, a referee tosses a coin upward with an initial velocity of $v_0 = +5.00$ m/s. The velocity of the coin is momentarily zero when the coin reaches its maximum height.

In Figure 2.15, what is the total time the coin is in the air before returning to its release point?

Reasoning During the time the coin travels upward, gravity causes its speed to decrease to zero. On the way down, however, gravity causes the coin to regain the lost speed. Thus, the time for the coin to go up is equal to the time for it to come down. In other words, the total travel time is twice the time for the upward motion. The data for the coin during the upward trip are the same as in Example 12. With these data, we can use Equation 2.4 ($v = v_0 + at$) to find the upward travel time.

Solution Rearranging Equation 2.4, we find that

$$t = \frac{v - v_0}{a} = \frac{0\ \text{m/s} - 5.00\ \text{m/s}}{-9.80\ \text{m/s}^2} = 0.510\ \text{s}$$

The total up-and-down time is twice this value, or $\boxed{1.02\ \text{s}}$.

It is possible to determine the total time by another method. When the coin is tossed upward and returns to its release point, the displacement for the *entire trip* is $y = 0$ m. With this value for the displacement, Equation 2.8 ($y = v_0t + \frac{1}{2}at^2$) can be used to find the time for the entire trip directly.

Examples 12 and 13 illustrate that the expression "freely falling" does not necessarily mean an object is falling down. A freely falling object is any object moving either upward or downward under the influence of gravity alone. In either case, the object always experiences the same *downward acceleration* due to gravity, a fact that is the focus of the next example.

Conceptual Example 14 Acceleration Versus Velocity

There are three parts to the motion of the coin in Figure 2.15. On the way up, the coin has a velocity vector that is directed upward and has a decreasing magnitude. At the top of its path, the coin momentarily has a zero velocity. On the way down, the coin has a downward-pointing velocity vector with an increasing magnitude. In the absence of air resistance, does the acceleration of the coin, like the velocity, change from one part of the motion to another?

Reasoning and Solution Since air resistance is absent, the coin is in free-fall motion. Therefore, the acceleration vector is that due to gravity and has the same magnitude and the same direction at all times. It has a magnitude of 9.80 m/s^2 and points downward during both the upward and downward portions of the motion. Furthermore, just because the coin's instantaneous velocity is zero at the top of the motional path, don't think that the acceleration vector is also zero there. Acceleration is the rate at which velocity changes, and the velocity at the top is changing, even though at one instant it is zero. In fact, the acceleration at the top has the same magnitude of 9.80 m/s^2 and the same downward direction as during the rest of the motion. Thus, ***the coin's velocity vector changes from moment to moment, but its acceleration vector does not change.***

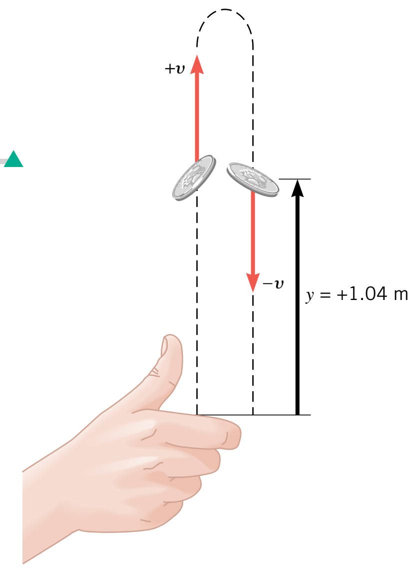


Figure 2.16 For a given displacement along the motional path, the upward speed of the coin is equal to its downward speed, but the two velocities point in opposite directions.

The motion of an object that is thrown upward and eventually returns to earth contains a symmetry that is useful to keep in mind from the point of view of problem solving. The calculations just completed indicate that a time symmetry exists in free-fall motion, in the sense that the time required for the object to reach maximum height equals the time for it to return to its starting point.

A type of symmetry involving the speed also exists. Figure 2.16 shows the coin considered in Examples 12 and 13. At any displacement y above the point of release, the coin's speed during the upward trip equals the speed at the same point during the downward trip. For instance, when $y = +1.04$ m, Equation 2.9 gives two possible values for the final velocity v, assuming that the initial velocity is $v_0 = +5.00$ m/s:

$$v^2 = v_0{}^2 + 2ay = (5.00 \text{ m/s})^2 + 2(-9.80 \text{ m/s}^2)(1.04 \text{ m}) = 4.62 \text{ m}^2/\text{s}^2$$
$$v = \pm 2.15 \text{ m/s}$$

The value $v = +2.15$ m/s is the velocity of the coin on the upward trip, and $v = -2.15$ m/s is the velocity on the downward trip. The speed in both cases is identical and equals 2.15 m/s. Likewise, the speed just as the coin returns to its point of release is 5.00 m/s, which equals the initial speed. This symmetry involving the speed arises because the coin loses 9.80 m/s in speed each second on the way up and gains back the same amount each second on the way down. In Conceptual Example 15, we use just this kind of symmetry to guide our reasoning as we analyze the motion of a pellet shot from a gun.

Conceptual Example 15 Taking Advantage of Symmetry

Figure 2.17*a* shows a pellet, having been fired from a gun, moving straight upward from the edge of a cliff. The initial speed of the pellet is 30 m/s. It goes up and then falls back down, eventually hitting the ground beneath the cliff. In Figure 2.17*b* the pellet has been fired straight downward at the same initial speed. In the absence of air resistance, does the pellet in part *b* strike the ground beneath the cliff with a smaller, a greater, or the same speed as the pellet in part *a*?

Reasoning and Solution Because air resistance is absent, the motion is that of free-fall, and the symmetry inherent in free-fall motion offers an immediate answer to the question. Figure 2.17*c* shows why. This part of the drawing shows the pellet after it has been fired upward and then fallen back down to its starting point. Symmetry indicates that the speed in part *c* is the same as in part *a*—namely, 30 m/s. Thus, part *c* is just like part *b*, where the pellet is actually fired downward with a speed of 30 m/s. Consequently, whether the pellet is fired as in part *a* or part *b*, it starts to move downward from the cliff edge at a speed of 30 m/s. In either case, there is the same acceleration due to gravity and the same displacement from the cliff edge to the ground below. Under these conditions, ***the pellet reaches the ground with the same speed no matter in which vertical direction it is fired initially.***

Related Homework: *Problems 43, 46*

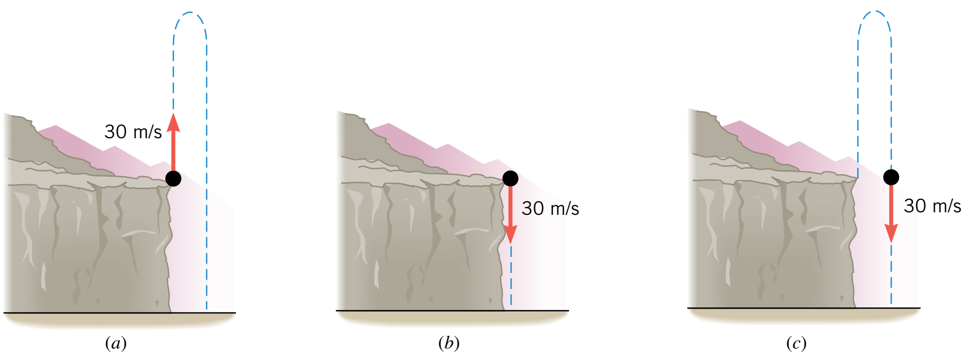


Figure 2.17 (*a*) From the edge of a cliff, a pellet is fired straight upward from a gun. The pellet's initial speed is 30 m/s. (*b*) The pellet is fired straight downward with an initial speed of 30 m/s. (*c*) In Conceptual Example 15 this drawing plays the central role in reasoning that is based on symmetry.

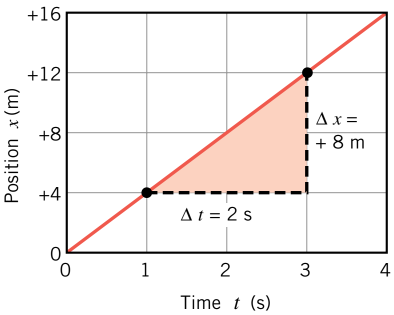


Figure 2.18 A graph of position vs. time for an object moving with a constant velocity of $v = \Delta x/\Delta t = +4$ m/s.

2.7 Graphical Analysis of Velocity and Acceleration

Graphical techniques are helpful in understanding the concepts of velocity and acceleration. Suppose a bicyclist is riding with a constant velocity of $v = +4$ m/s. The position x of the bicycle can be plotted along the vertical axis of a graph, while the time t is plotted along the horizontal axis. Since the position of the bike increases by 4 m every second, the graph of x versus t is a straight line. Furthermore, if the bike is assumed to be at $x = 0$ m when $t = 0$ s, the straight line passes through the origin, as Figure 2.18 shows. Each point on this line gives the position of the bike at a particular time. For instance, at $t = 1$ s the position is 4 m, while at $t = 3$ s the position is 12 m.

In constructing the graph in Figure 2.18, we used the fact that the velocity was +4 m/s. Suppose, however, that we were given this graph, but did not have prior knowledge of the velocity. The velocity could be determined by considering what happens to the bike between the times of 1 and 3 s, for instance. The change in time is $\Delta t = 2$ s. During this time interval, the position of the bike changes from +4 to +12 m, and the change in position is $\Delta x = +8$ m. The ratio $\Delta x/\Delta t$ is called the ***slope*** of the straight line.

$$\text{Slope} = \frac{\Delta x}{\Delta t} = \frac{+8\text{ m}}{2\text{ s}} = +4\text{ m/s}$$

Notice that the slope is equal to the velocity of the bike. This result is no accident, because $\Delta x/\Delta t$ is the definition of average velocity (see Equation 2.2). Thus, for an object moving with a constant velocity, the slope of the straight line in a position–time graph gives the velocity. Since the position–time graph is a straight line, any time interval Δt can be chosen to calculate the velocity. Choosing a different Δt will yield a different Δx, but the velocity $\Delta x/\Delta t$ will not change. In the real world, objects rarely move with a constant velocity at all times, as the next example illustrates.

Example 16 A Bicycle Trip

A bicyclist maintains a constant velocity on the outgoing leg of a trip, zero velocity while stopped, and another constant velocity on the way back. Figure 2.19 shows the corresponding position–time graph. Using the time and position intervals indicated in the drawing, obtain the velocities for each segment of the trip.

Reasoning The average velocity $\overline{v}$ is equal to the displacement Δx divided by the elapsed time Δt, $\overline{v} = \Delta x/\Delta t$. The displacement is the final position minus the initial position, which is a positive number for segment 1 and a negative number for segment 3. Note for segment 2 that $\Delta x = 0$ m, since the bicycle is at rest. The drawing shows values for Δx and Δt for each of the three segments.

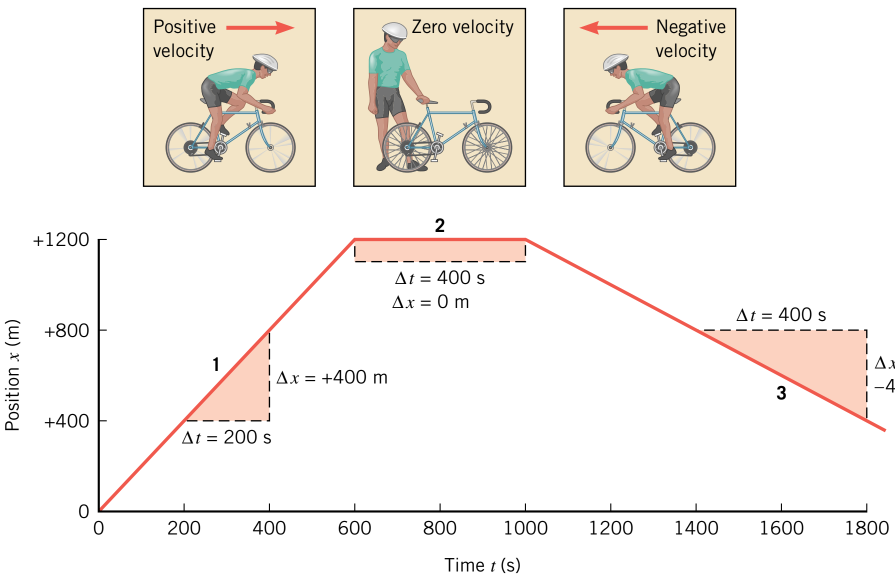


Figure 2.19 This position-vs.-time graph consists of three straight-line segments, each corresponding to a different constant velocity.

Solution The average velocities for the three segments are

Segment 1 $$\bar{v} = \frac{\Delta x}{\Delta t} = \frac{800\text{ m} - 400\text{ m}}{400\text{ s} - 200\text{ s}} = \frac{+400\text{ m}}{200\text{ s}} = \boxed{+2\text{ m/s}}$$

Segment 2 $$\bar{v} = \frac{\Delta x}{\Delta t} = \frac{1200\text{ m} - 1200\text{ m}}{1000\text{ s} - 600\text{ s}} = \frac{0\text{ m}}{400\text{ s}} = \boxed{0\text{ m/s}}$$

Segment 3 $$\bar{v} = \frac{\Delta x}{\Delta t} = \frac{400\text{ m} - 800\text{ m}}{1800\text{ s} - 1400\text{ s}} = \frac{-400\text{ m}}{400\text{ s}} = \boxed{-1\text{ m/s}}$$

In the second segment of the journey the velocity is zero, reflecting the fact that the bike is stationary. Since the position of the bike does not change, segment 2 is a horizontal line that has a zero slope. In the third part of the motion the velocity is negative, because the position of the bike decreases from $x = +800$ m to $x = +400$ m during the 400-s interval shown in the graph. As a result, segment 3 has a negative slope, and the velocity is negative.

If the object is accelerating, its velocity is changing. When the velocity is changing, the x-versus-t graph is not a straight line, but is a curve, perhaps like that in Figure 2.20. This curve was drawn using Equation 2.8 ($x = v_0 t + \frac{1}{2}at^2$), assuming an acceleration of $a = 0.26\text{ m/s}^2$ and an initial velocity of $v_0 = 0$ m/s. The velocity at any instant of time can be determined by measuring the slope of the curve at that instant. The slope at any point along the curve is defined to be the slope of the tangent line drawn to the curve at that point. For instance, in Figure 2.20 a tangent line is drawn at $t = 20.0$ s. To determine the slope of the tangent line, a triangle is constructed using an arbitrarily chosen time interval of $\Delta t = 5.0$ s. The change in x associated with this time interval can be read from the tangent line as $\Delta x = +26$ m. Therefore,

$$\text{Slope of tangent line} = \frac{\Delta x}{\Delta t} = \frac{+26\text{ m}}{5.0\text{ s}} = +5.2\text{ m/s}$$

The slope of the tangent line is the instantaneous velocity, which in this case is $v = +5.2$ m/s. This graphical result can be verified by using Equation 2.4 with $v_0 = 0$ m/s: $v = at = (+0.26\text{ m/s}^2)(20.0\text{ s}) = +5.2$ m/s.

Insight into the meaning of acceleration can also be gained with the aid of a graphical representation. Consider an object moving with a constant acceleration of $a = +6\text{ m/s}^2$. If the object has an initial velocity of $v_0 = +5$ m/s, its velocity at any time is represented by Equation 2.4 as

$$v = v_0 + at = 5\text{ m/s} + (6\text{ m/s}^2)t$$

This relation is plotted as the velocity-versus-time graph in Figure 2.21. The graph of v versus t is a straight line that intercepts the vertical axis at $v_0 = 5$ m/s. The slope of this straight line can be calculated from the data shown in the drawing:

$$\text{Slope} = \frac{\Delta v}{\Delta t} = \frac{+12\text{ m/s}}{2\text{ s}} = +6\text{ m/s}^2$$

The ratio $\Delta v/\Delta t$ is, by definition, equal to the average acceleration (Equation 2.4), so the slope of the straight line in a velocity–time graph is the average acceleration.

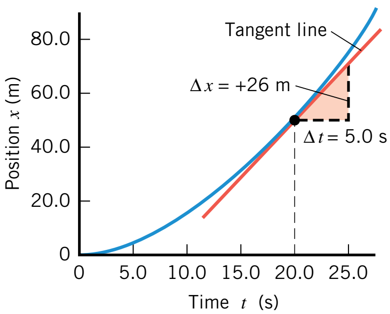


Figure 2.20 When the velocity is changing, the position-vs.-time graph is a curved line. The slope $\Delta x/\Delta t$ of the tangent line drawn to the curve at a given time is the instantaneous velocity at that time.

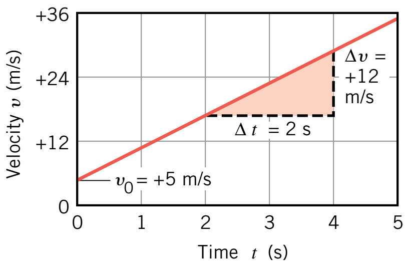


Figure 2.21 A velocity-vs.-time graph that applies to an object with an acceleration of $\Delta v/\Delta t = +6\text{ m/s}^2$. The initial velocity is $v_0 = +5$ m/s when $t = 0$ s.

Concept Summary

This summary presents an abridged version of the chapter, including the important equations and all available learning aids. For convenient reference, the learning aids (including the text's examples) are placed next to or immediately after the relevant equation or discussion. The following learning aids may be found on-line at **www.wiley.com/college/cutnell:**

Interactive LearningWare examples are solved according to a five-step interactive format that is designed to help you develop problem-solving skills.	**Concept Simulations** are animated versions of text figures or animations that illustrate important concepts. You can control parameters that affect the display, and we encourage you to experiment.
Interactive Solutions offer specific models for certain types of problems in the chapter homework. The calculations are carried out interactively.	**Self-Assessment Tests** include both qualitative and quantitative questions. Extensive feedback is provided for both incorrect and correct answers, to help you evaluate your understanding of the material.

Topic	***Discussion***	***Learning Aids***

2.1 Displacement

Displacement — Displacement is a vector that points from an object's initial position to its final position. The magnitude of the displacement is the shortest distance between the two positions.

2.2 Speed and Velocity

The average speed of an object is the distance traveled by the object divided by the time required to cover the distance:

Average speed

$$\text{Average speed} = \frac{\text{Distance}}{\text{Elapsed time}} \tag{2.1}$$

Example 1

The average velocity $\overline{\mathbf{v}}$ of an object is the object's displacement $\Delta\mathbf{x}$ divided by the elapsed time Δt:

Average velocity

$$\overline{\mathbf{v}} = \frac{\Delta\mathbf{x}}{\Delta t} \tag{2.2}$$

Example 2

Average velocity is a vector that has the same direction as the displacement. When the elapsed time becomes infinitesimally small, the average velocity becomes equal to the instantaneous velocity $\mathbf{v}$, the velocity at an instant of time:

Instantaneous velocity

$$\mathbf{v} = \lim_{\Delta t \to 0} \frac{\Delta\mathbf{x}}{\Delta t} \tag{2.3}$$

2.3 Acceleration

The average acceleration $\overline{\mathbf{a}}$ is a vector. It equals the change $\Delta\mathbf{v}$ in the velocity divided by the elapsed time Δt, the change in the velocity being the final minus the initial velocity:

Average acceleration

$$\overline{\mathbf{a}} = \frac{\Delta\mathbf{v}}{\Delta t} \tag{2.4}$$

Examples 3, 4, 17

When Δt becomes infinitesimally small, the average acceleration becomes equal to the instantaneous acceleration $\mathbf{a}$:

Interactive Solution 2.17

Instantaneous acceleration

$$\mathbf{a} = \lim_{\Delta t \to 0} \frac{\Delta\mathbf{v}}{\Delta t} \tag{2.5}$$

Acceleration is the rate at which the velocity is changing.

Use Self-Assessment Test 2.1 to evaluate your understanding of Sections 2.1–2.3.

2.4 Equations of Kinematics for Constant Acceleration

2.5 Applications of the Equations of Kinematics

The equations of kinematics apply when an object moves with a constant acceleration along a straight line. These equations relate the displacement $x - x_0$, the acceleration a, the final velocity v, the initial velocity v_0, and the elapsed time $t - t_0$. Assuming that $x_0 = 0$ m at $t_0 = 0$ s, the equations of kinematics are

Equations of kinematics

$$v = v_0 + at \tag{2.4}$$

$$x = \tfrac{1}{2}(v_0 + v)t \tag{2.7}$$

$$x = v_0 t + \tfrac{1}{2}at^2 \tag{2.8}$$

$$v^2 = v_0^2 + 2ax \tag{2.9}$$

Examples 5–9, 18

Concept Simulations 2.1, 2.2

Interactive LearningWare 2.1, 2.2

Interactive Solutions 2.29, 2.31

Topic	Discussion	Learning Aids
	Use Self-Assessment Test 2.2 to evaluate your understanding of Sections 2.4 and 2.5.	
	2.6 Freely Falling Bodies	
	In free-fall motion, an object experiences negligible air resistance and a constant acceleration due to gravity. All objects at the same location above the earth have the same acceleration due to gravity. The acceleration due to gravity	**Examples 10–15** **Concept Stimulation 2.3**
Acceleration due to gravity	is directed toward the center of the earth and has a magnitude of approximately $9.80\ m/s^2$ near the earth's surface.	**Interactive Solutions 2.47, 2.49**
	2.7 Graphical Analysis of Velocity and Acceleration	
	The slope of a plot of position versus time for a moving object gives the object's velocity. The slope of a plot of velocity versus time gives the object's acceleration.	**Example 16** **Concept Simulation 2.4**
	Use Self-Assessment Test 2.3 to evaluate your understanding of Sections 2.6 and 2.7.	

Problems

ssm **Solution is in the Student Solutions Manual.** **www** **Solution is available on the World Wide Web at www.wiley.com/college/cutnell**
This icon represents a biomedical application.

Section 2.1 Displacement, Section 2.2 Speed and Velocity

1. **ssm** A plane is sitting on a runway, awaiting takeoff. On an adjacent parallel runway, another plane lands and passes the stationary plane at a speed of 45 m/s. The arriving plane has a length of 36 m. By looking out of a window (very narrow), a passenger on the stationary plane can se the moving plane. For how long a time is the moving plane visible?

2. One afternoon, a couple walks three-fourths of the way around a circular lake, the radius of which is 1.50 km. They start at the west side of the lake and head due south to begin with. (a) What is the distance they travel? (b) What are the magnitude and direction (relative to due east) of the couple's displacement?

3. **ssm** A whale swims due east for a distance of 6.9 km, turns around and goes due west for 1.8 km, and finally turns around again and heads 3.7 km due east. (a) What is the total distance traveled by the whale? (b) What are the magnitude and direction of the displacement of the whale

4. The Space Shuttle travels at a speed of about 7.6×10^3 m/s. The blink of an astronaut's eye lasts about 110 ms. How many football fields (length = 91.4 m) does the Shuttle cover in the blink of an eye?

5. As the earth rotates through one revolution, a person standing on the equator traces out a circular path whose radius is equal to the radius of the earth (6.38×10^6 m). What is the average speed of this person in (a) meters per second and (b) miles per hour?

6. In 1954 the English runner Roger Bannister broke the four-minute barrier for the mile with a time of 3:59.4 s (3 min and 59.4 s). In 1999 the Moroccan runner Hicham el-Guerrouj set a record of 3:43.13 s for the mile. If these two runners had run in the same race, each running the entire race at the average speed that earned him a place in the record books, el-Guerrouj would have won. By how many meters?

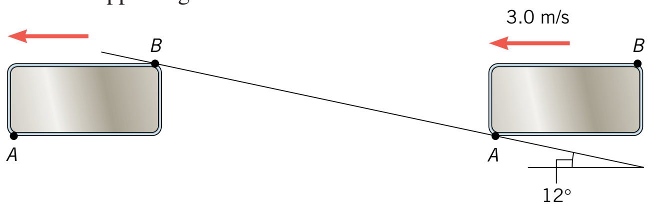
7. A tourist being chased by an angry bear is running in a straight line toward his car at a speed of 4.0 m/s. The car is a distance *d* away. The bear is 26 m behind the tourist and running at 6.0 m/s. The tourist reaches the car safely. What is the maximum possible value for *d*?

* **8.** In reaching her destination, a backpacker walks with an average velocity of 1.34 m/s, due west. This average velocity results because she hikes for 6.44 km with an average velocity of 2.68 m/s, due west, turns around, and hikes with an average velocity of 0.447 m/s, due east. How far east did she walk?

* **9.** **ssm** **www** A woman and her dog are out for a morning run to the river, which is located 4.0 km away. The woman runs at 2.5 m/s in a straight line. The dog is unleashed and runs back and forth at 4.5 m/s between his owner and the river, until she reaches the river. What is the total distance run by the dog?

* **10.** A car makes a trip due north for three-fourths of the time and due south one-fourth of the time. The average northward velocity has a magnitude of 27 m/s, and the average southward velocity has a magnitude of 17 m/s. What is the average velocity, magnitude and direction, for the entire trip?

** **11.** You are on a train that is traveling at 3.0 m/s along a level straight track. Very near and parallel to the track is a wall that slopes upward at a 12° angle with the horizontal. As you face the window (0.90 m high, 2.0 m wide) in your compartment, the train is moving to the left, as the drawing indicates. The top edge of the wall first appears at window corner A and eventually disappears at window corner B. How much time passes between appearance and disappearance of the upper edge of the wall?



Section 2.3 Acceleration

12. For a standard production car, the highest road-tested acceleration ever reported occurred in 1993, when a Ford RS200 Evolution went from zero to 26.8 m/s (60 mi/h) in 3.275 s. Find the magnitude of the car's acceleration.

13. **ssm** A motorcycle has a constant acceleration of 2.5 m/s^2. Both the velocity and acceleration of the motorcycle point in the same direction. How much time is required for the motorcycle to change its speed from (a) 21 to 31 m/s, and (b) 51 to 61 m/s?

14. NASA has developed *Deep-Space 1* (DS-1), a spacecraft that is scheduled to rendezvous with the asteroid named 1992 KD (which orbits the sun millions of miles from the earth). The propulsion system of DS-1 works by ejecting high-speed argon ions out the rear of the engine. The engine slowly increases the velocity of DS-1 by about +9.0 m/s per day. (a) How much time (in days) will it take to increase the velocity of DS-1 by +2700 m/s? (b) What is the acceleration of DS-1 (in m/s^2)?

15. **ssm** A runner accelerates to a velocity of 5.36 m/s due west in 3.00 s. His average acceleration is 0.640 m/s^2, also directed due west. What was his velocity when he began accelerating?

16. The land speed record of 13.9 m/s (31 mi/h) for birds is held by the Australian emu. An emu running due south in a straight line at this speed slows down to a speed of 11.0 m/s in 3.0 s. (a) What is the direction of the bird's acceleration? (b) Assuming that the acceleration remains the same, what is the bird's velocity after an additional 4.0 s has elapsed?

* **17.** Consult **Interactive Solution 2.17** at **www.wiley.com/college/cutnell** before beginning this problem. A car is traveling along a straight road at a velocity of +36.0 m/s when its engine cuts out. For the next twelve seconds the car slows down, and its average acceleration is $\bar{a}_1$. For the next six seconds the car slows down further, and its average acceleration is $\bar{a}_2$. The velocity of the car at the end of the eighteen-second period is +28.0 m/s. The ratio of the average acceleration values is $\bar{a}_1/\bar{a}_2 = 1.50$. Find the velocity of the car at the end of the initial twelve-second interval.

** **18.** Two motorcycles are traveling due east with different velocities. However, four seconds later, they have the same velocity. During this four-second interval, motorcycle A has an average acceleration of 2.0 m/s^2 due east, while motorcycle B has an average acceleration of 4.0 m/s^2 due east. By how much did the speeds *differ* at the beginning of the four-second interval, and which motorcycle was moving faster?

Section 2.4 Equations of Kinematics for Constant Acceleration, Section 2.5 Applications of the Equations of Kinematics

19. In getting ready to slam-dunk the ball, a basketball player starts from rest and sprints to a speed of 6.0 m/s in 1.5 s. Assuming that the player accelerates uniformly, determine the distance he runs.

20. Review Conceptual Example 7 as background for this problem. A car is traveling to the left, which is the negative direction. The direction of travel remains the same throughout this problem. The car's initial speed is 27.0 m/s, and during a 5.0-s interval, it changes to a final speed of (a) 29.0 m/s and (b) 23.0 m/s. In each case, find the acceleration (magnitude and algebraic sign) and state whether or not the car is decelerating.

21. **ssm** A VW Beetle goes from 0 to 60.0 mi/h with an acceleration of +2.35 m/s^2. (a) How much time does it take for the Beetle to reach this speed? (b) A top-fuel dragster can go from 0 to 60.0 mi/h in 0.600 s. Find the acceleration (in m/s^2) of the dragster.

22. (a) What is the magnitude of the average acceleration of a skier who, starting from rest, reaches a speed of 8.0 m/s when going down a slope for 5.0 s? (b) How far does the skier travel in this time?

23. The left ventricle of the heart accelerates blood from rest to a velocity of +26 cm/s. (a) If the displacement of the blood during the acceleration is +2.0 cm, determine its acceleration (in cm/s^2). (b) How much time does blood take to reach its final velocity?

24. Consult **Concept Simulation 2.1** at **www.wiley.com/college/cutnell** for help in preparing for this problem. A cheetah is hunting. Its prey runs for 3.0 s at a constant velocity of +9.0 m/s. Starting from rest, what constant acceleration must the cheetah maintain in order to run the same distance as its prey runs in the same time?

25. **ssm** A jetliner, traveling northward, is landing with a speed of 69 m/s. Once the jet touches down, it has 750 m of runway in which to reduce its speed to 6.1 m/s. Compute the average acceleration (magnitude and direction) of the plane during landing.

26. Consult **Concept Simulation 2.1** at **www.wiley.com/college/cutnell** before starting this problem. The Kentucky Derby is held at the Churchill Downs track in Louisville, Kentucky. The track is one and one-quarter miles in length. One of the most famous horses to win this event was Secretariat. In 1973 he set a Derby record that has never been broken. His average acceleration during the last four quarter-miles of the race was +0.0105 m/s^2. His velocity at the start of the final mile ($x = +1609$ m) was about +16.58 m/s. The acceleration, although small, was very important to his victory. To assess its effect, determine the difference between the time he would have taken to run the final mile at a constant velocity of +16.58 m/s and the time he actually took. Although the track is oval in shape, assume it is straight for the purpose of this problem.

27. **ssm** **www** A speed ramp at an airport is basically a large conveyor belt on which you can stand and be moved along. The belt of one ramp moves at a constant speed such that a person who stands still on it leaves the ramp 64 s after getting on. Clifford is in a real hurry, however, and skips the speed ramp. Starting from rest with an acceleration of 0.37 m/s^2, he covers the same distance as the ramp does, but in one-fourth the time. What is the speed at which the belt of the ramp is moving?

* **28.** A drag racer, starting from rest, speeds up for 402 m with an acceleration of +17.0 m/s^2. A parachute then opens, slowing the car down with an acceleration of −6.10 m/s^2. How fast is the racer moving 3.50×10^2 m after the parachute opens?

* **29.** Review **Interactive Solution 2.29** at **www.wiley.com/college/cutnell** in preparation for this problem. Suppose a car is traveling at 20.0 m/s, and the driver sees a traffic light turn red. After 0.530 s has elapsed (the reaction time), the driver applies the brakes, and the car decelerates at 7.00 m/s^2. What is the stopping distance of the car, as measured from the point where the driver first notices the red light?

* **30.** A speedboat starts from rest and accelerates at +2.01 m/s^2 for 7.00 s. At the end of this time, the boat continues for an additional 6.00 s with an acceleration of +0.518 m/s^2. Following this, the boat accelerates at −1.49 m/s^2 for 8.00 s. (a) What is the velocity of the boat at $t = 21.0$ s? (b) Find the total displacement of the boat.

* **31.** **Interactive Solution 2.31** at **www.wiley.com/college/cutnell** offers help in modeling this problem. A car is traveling at a constant speed of 33 m/s on a highway. At the instant this car passes an entrance ramp, a second car enters the highway from the ramp. The second car starts from rest and has a constant acceleration. What acceleration must it maintain, so that the two cars meet for the first time at the next exit, which is 2.5 km away?

* **32.** A cab driver picks up a customer and delivers her 2.00 km away, on a straight route. The driver accelerates to the speed limit and, on reaching it, begins to decelerate at once. The magnitude of the deceleration is three times the magnitude of the acceleration. Find the lengths of the acceleration and deceleration phases.

* **33.** Along a straight road through town, there are three speed-limit signs. They occur in the following order: 55, 35, and 25 mi/h, with the 35-mi/h sign being midway between the other two. Obeying these speed limits, the smallest possible time t_A that a driver can spend on this part of the road is to travel between the first and second signs at 55 mi/h and between the second and third signs at 35 mi/h. More realistically, a driver could slow down from 55 to 35 mi/h with a constant deceleration and then do a similar thing from 35 to 25 mi/h. This alternative requires a time t_B. Find the ratio t_B/t_A.

** **34.** A Boeing 747 "Jumbo Jet" has a length of 59.7 m. The runway on which the plane lands intersects another runway. The width of the intersection is 25.0 m. The plane decelerates through the intersection at a rate of 5.70 m/s^2 and clears it with a final speed of 45.0 m/s. How much time is needed for the plane to clear the intersection?

** **35.** ssm A train has a length of 92 m and starts from rest with a constant acceleration at time $t = 0$ s. At this instant, a car just reaches the end of the train. The car is moving with a constant velocity. At a time $t = 14$ s, the car just reaches the front of the train. Ultimately, however, the train pulls ahead of the car, and at time $t = 28$ s, the car is again at the rear of the train. Find the magnitudes of (a) the car's velocity and (b) the train's acceleration.

** **36.** In the one-hundred-meter dash a sprinter accelerates from rest to a top speed with an acceleration whose magnitude is 2.68 m/s^2. After achieving top speed, he runs the remainder of the race without speeding up or slowing down. If the total race is run in 12.0 s, how far does he run during the acceleration phase?

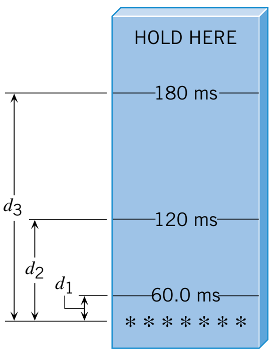
Section 2.6 Freely Falling Bodies

37. ssm A penny is dropped from rest from the top of the Sears Tower in Chicago. Considering that the height of the building is 427 m and ignoring air resistance, find the speed with which the penny strikes the ground.

38. In preparation for this problem, review Conceptual Example 7. From the top of a cliff, a person uses a slingshot to fire a pebble straight downward, which is the negative direction. The initial speed of the pebble is 9.0 m/s. (a) What is the acceleration (magnitude and direction) of the pebble during the downward motion? Is the pebble decelerating? Explain. (b) After 0.50 s, how far beneath the cliff top is the pebble?

39. Concept Simulation 2.3 at **www.wiley.com/college/cutnell** offers a useful review of the concepts central to this problem. An astronaut on a distant planet wants to determine its acceleration due to gravity. The astronaut throws a rock straight up with a velocity of +15 m/s and measures a time of 20.0 s before the rock returns to his hand. What is the acceleration (magnitude and direction) due to gravity on this planet?

40. The drawing shows a device that you can make with a piece of cardboard, which can be used to measure a person's reaction time. Hold the card at the top and suddenly drop it. Ask a friend to try to catch the card between his or her thumb and index finger. Initially, your friend's fingers must be level with the asterisks at the bottom. By noting where your friend catches the card, you can determine his or her reaction time in milliseconds (ms). Calculate the distances d_1, d_2, and d_3.



41. ssm From her bedroom window a girl drops a water-filled balloon to the ground, 6.0 m below. If the balloon is released from rest, how long is it in the air?

42. Review Concept Simulation 2.3 at **www.wiley.com/college/cutnell** before attempting this problem. At the beginning of a basketball game, a referee tosses the ball straight up with a speed of 4.6 m/s. A player cannot touch the ball until after it reaches its maximum height and begins to fall down. What is the minimum time that a player must wait before touching the ball?

43. Review Conceptual Example 15 before attempting this problem. Two identical pellet guns are fired simultaneously from the edge of a cliff. These guns impart an initial speed of 30.0 m/s to each pellet. Gun A is fired straight upward, with the pellet going up and then falling back down, eventually hitting the ground beneath the cliff. Gun B is fired straight downward. In the absence of air resistance, how long after pellet B hits the ground does pellet A hit the ground?

44. A diver springs upward with an initial speed of 1.8 m/s from a 3.0-m board. (a) Find the velocity with which he strikes the water. *[Hint: When the diver reaches the water, his displacement is $y = -3.0$ m (measured from the board), assuming that the downward direction is chosen as the negative direction.]* (b) What is the highest point he reaches above the water?

45. ssm A wrecking ball is hanging at rest from a crane when suddenly the cable breaks. The time it takes for the ball to fall halfway to the ground is 1.2 s. Find the time it takes for the ball to fall from rest all the way to the ground.

46. Before working this problem, review Conceptual Example 15. A pellet gun is fired straight downward from the edge of a cliff that is 15 m above the ground. The pellet strikes the ground with a speed of 27 m/s. How far above the cliff edge would the pellet have gone had the gun been fired straight upward?

47. Consult Interactive Solution 2.47 at **www.wiley.com/college/cutnell** before beginning this problem. A ball is thrown straight upward and rises to a maximum height of 12.0 m above its launch point. At what height above its launch point has the speed of the ball decreased to one-half of its initial value?

* **48.** Two arrows are shot vertically upward. The second arrow is shot after the first one, but while the first is still on its way up. The initial speeds are such that both arrows reach their maximum heights at the same instant, although these heights are different. Suppose that the initial speed of the first arrow is 25.0 m/s and that the second arrow is fired 1.20 s after the first. Determine the initial speed of the second arrow.

* **49.** Review Interactive Solution 2.49 at **www.wiley.com/college/cutnell** before beginning this problem. A woman on a bridge 75.0 m high sees a raft floating at a constant speed on the river below. She drops a stone from rest in an attempt to hit the raft. The stone is released when the raft has 7.00 m more to travel before passing under the bridge. The stone hits the water 4.00 m in front of the raft. Find the speed of the raft.

* **50.** Consult Concept Simulation 2.3 at **www.wiley.com/college/cutnell** to review the concepts on which this problem is based. Two students, Anne and Joan, are bouncing straight up and down on a trampoline. Anne bounces twice as high as Joan does. Assuming both are in free-fall, find the ratio of the time Anne spends between bounces to the time Joan spends.

* **51.** ssm A log is floating on swiftly moving water. A stone is dropped from rest from a 75-m-high bridge and lands on the log as it passes under the bridge. If the log moves with a constant speed of 5.0 m/s, what is the horizontal distance between the log and the bridge when the stone is released?

* **52.** (a) Just for fun, a person jumps from rest from the top of a tall cliff overlooking a lake. In falling through a distance H, she acquires a certain speed v. Assuming free-fall conditions, how much farther must she fall in order to acquire a speed of $2v$? Express your answer in terms of H. (b) Would the answer to part (a) be different if this event were to occur on another planet where the acceleration due to gravity had a value other than $9.80\ \text{m/s}^2$? Explain.

* **53.** **ssm** **www** A spelunker (cave explorer) drops a stone from rest into a hole. The speed of sound is 343 m/s in air, and the sound of the stone striking the bottom is heard 1.50 s after the stone is dropped. How deep is the hole?

* **54.** A ball is thrown upward from the top of a 25.0-m-tall building. The ball's initial speed is 12.0 m/s. At the same instant, a person is running on the ground at a distance of 31.0 m from the building. What must be the average speed of the person if he is to catch the ball at the bottom of the building?

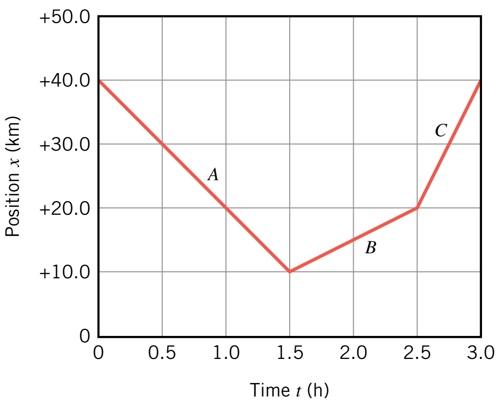
** **55.** A ball is dropped from rest from the top of a cliff that is 24 m high. From ground level, a second ball is thrown straight upward at the same instant that the first ball is dropped. The initial speed of the second ball is exactly the same as that with which the first ball eventually hits the ground. In the absence of air resistance, the motions of the balls are just the reverse of each other. Determine how far below the top of the cliff the balls cross paths.

** **56.** Review **Interactive LearningWare 2.2** at **www.wiley.com/college/cutnell** as an aid in solving this problem. A hot air balloon is ascending straight up at a constant speed of 7.0 m/s. When the balloon is 12.0 m above the ground, a gun fires a pellet straight up from ground level with an initial speed of 30.0 m/s. Along the paths of the balloon and the pellet, there are two places where each of them has the same altitude at the same time. How far above ground level are these places?

Section 2.7 Graphical Analysis of Velocity and Acceleration

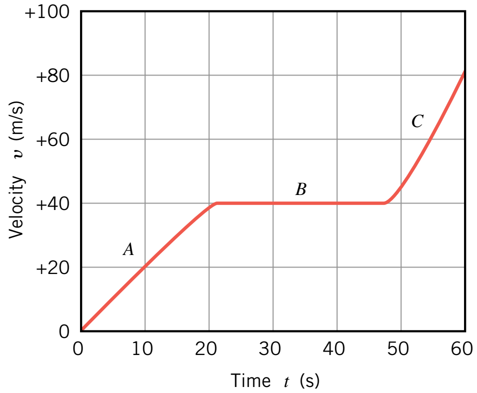
57. **ssm** For the first 10.0 km of a marathon, a runner averages a velocity that has a magnitude of 15.0 km/h. For the next 15.0 km, he averages 10.0 km/h, and for the last 15.0 km, he averages 5.0 km/h. Construct, to scale, the position–time graph for the runner.

58. A bus makes a trip according to the position–time graph shown in the drawing. What is the average velocity (magnitude and direction) of the bus during each of the segments labeled A, B, and C? Express your answers in km/h.



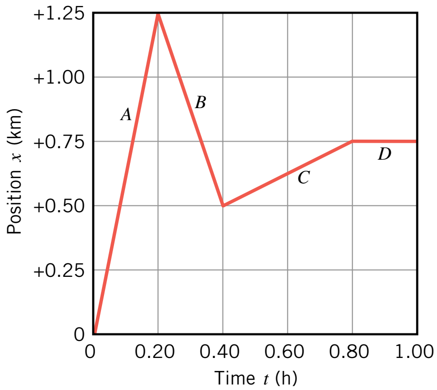
59. **Concept Simulation 2.5** at **www.wiley.com/college/cutnell** provides a review of the concepts that play a role in this problem. A snowmobile moves according to the velocity–time graph shown in the drawing (see top of right column). What is the snowmobile's average acceleration during each of the segments A, B, and C?

60. A person who walks for exercise produces the position–time graph given with this problem. (a) Without doing any calculations, decide which segments of the graph (A, B, C, or D) indicate positive,

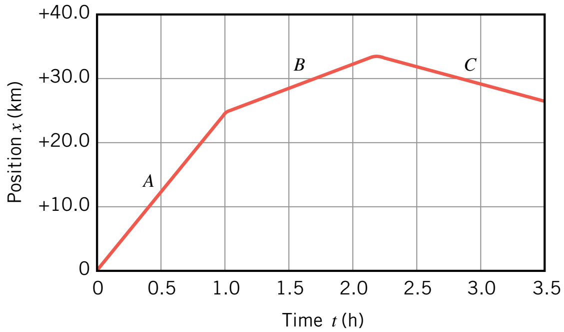


Problem 59

negative, and zero average velocities. (b) Calculate the average velocity for each segment to verify your answers to part (a).



* **61.** **ssm** A bus makes a trip according to the position–time graph shown in the illustration. What is the average acceleration (in km/h^2) of the bus for the entire 3.5-h period shown in the graph?



* **62.** A runner is at the position $x = 0$ m when time $t = 0$ s. One hundred meters away is the finish line. Every ten seconds, this runner runs half the remaining distance to the finish line. During each ten-second segment, the runner has a constant velocity. For the first forty seconds of the motion, construct (a) the position–time graph and (b) the velocity–time graph.

** **63.** Two runners start one hundred meters apart and run toward each other. Each runs ten meters during the first second. During each second thereafter, each runner runs ninety percent of the distance he ran in the previous second. Thus, the velocity of each person changes from second to second. However, during any one second, the velocity remains constant. Make a position–time graph for one of the runners. From this graph, determine (a) how much time passes before the runners collide and (b) the speed with which each is running at the moment of collision.