1
The Repricing Gap Model

1.1 INTRODUCTION

Among the models for measuring and managing interest rate risk, the *repricing gap* is certainly the best known and most widely used. It is based on a relatively simple and intuitive consideration: a bank's exposure to interest rate risk derives from the fact that interest-earning assets and interest-bearing liabilities show differing sensitivities to changes in market rates.

The repricing gap model can be considered an income-based model, in the sense that the target variable used to calculate the effect of possible changes in market rates is, in fact, an income variable: the net interest income (NII – the difference between interest income and interest expenses). For this reason this model falls into the category of "earnings approaches" to measuring interest rate risk. Income-based models contrast with equity-based methods, the most common of which is the *duration gap* model (discussed in the following chapter). These latter models adopt the market value of the bank's equity as the target variable of possible immunization policies against interest rate risk.

After analyzing the concept of *gap*, this chapter introduces *maturity-adjusted gaps*, and explores the distinction between marginal and cumulative gaps, highlighting the difference in meaning and various applications of the two risk measurements. The discussion then turns to the main limitations of the repricing gap model along with some possible solutions. Particular attention is given to the standardized gap concept and its applications.

1.2 THE GAP CONCEPT

The gap is a concise measure of interest risk that links changes in market interest rates to changes in NII. *Interest rate risk* is identified by possible unexpected changes in this variable. The gap (G) over a given time period t (*gapping period*) is defined as the difference between the amount of rate-sensitive assets (SA) and rate-sensitive liabilities (SL):

$$G_t = SA_t - SL_t = \sum_j sa_{t,j} - \sum_j sl_{t,j} \tag{1.1}$$

The term "sensitive" in this case indicates assets and liabilities that mature (or are subject to repricing) during period t. So, for example, to calculate the 6-month gap, one must take into account all fixed-rate assets and liabilities that mature in the next 6 months, as well as the variable-rate assets and liabilities to be repriced in the next 6 months. The gap, then, is a quantity expressed in monetary terms. Figure 1.1 provides a graphic representation of this concept.

By examining its link to the *NII*, we can fully grasp the usefulness of the gap concept. To do so, consider that *NII* is the difference between interest income (II) and interest expenses (IE). These, in turn, can be computed as the product of total financial assets (FA) and the average interest rate on assets (r_A) and total financial liabilities (FL) and average interest rate on liabilities (r_L) respectively. Using *NSA* and *NSL* as financial assets and

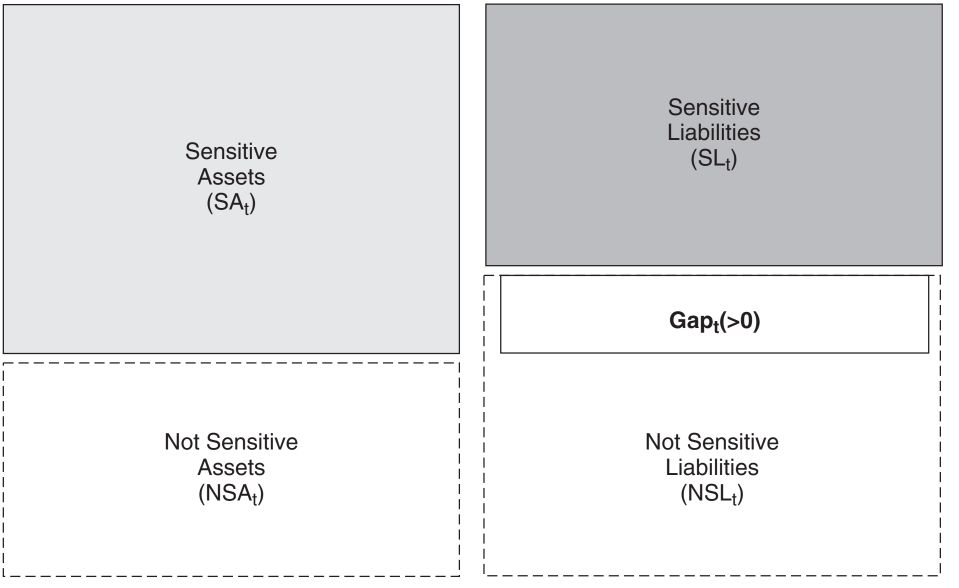

Figure 1 The repricing gap concept

liabilities which are not sensitive to interest rate fluctuations, and omitting t (which is considered given) for brevity's sake, we can represent the *NII* as follows:

$$NII = II - IE = r_A \cdot FA - r_L \cdot FL = r_A \cdot (SA + NSA) - r_L \cdot (SL + NSL) \tag{1.2}$$

from which:

$$\Delta NII = \Delta r_A \cdot SA - \Delta r_L \cdot SL \tag{1.3}$$

Equation (1.3) is based on the simple consideration that changes in market interest rates affect only rate-sensitive assets and liabilities. If, lastly, we assume that the change in rates is the same both for interest income and for interest expenses

$$\Delta r_A = \Delta r_L = \Delta r \tag{1.4}$$

the result is:

$$\Delta NII = \Delta r \cdot (SA - SL) = \Delta r \cdot \left(\sum_j sa_j - \sum_j sl_j \right) = \Delta r \cdot G \tag{1.5}$$

Equation (1.5) shows that the change in NII is a function of the gap and interest rate change. In other words, the gap represents the variable that links changes in NII to changes in market interest rates. More specifically, (1.5) shows that a rise in interest rates triggers an increase in the *NII* if the gap is positive. This is due to the fact that the quantity of rate-sensitive assets which will be renegotiated, resulting in an increase in interest income, exceeds rate-sensitive liabilities. Consequently, interest income grows faster than interest expenses, resulting in an increase of *NII*. Vice versa, if the gap is negative, a rise in interest rates leads to a lower *NII*.

Table 1.1 reports the possible combinations of the effects of interest rate changes on a bank's *NII*, depending on whether the gap is positive or negative and the direction of the interest rate change.

Table 1.1 Gaps, rate changes, and effects on *NII*

Gap / Δr	G > 0 positive net reinvestment	G < 0 positive net refinancing
> 0 higher rates	$\Delta NII > 0$	$\Delta NII < 0$
< 0 lower rates	$\Delta NII < 0$	$\Delta NII > 0$

The table also helps us understand the guidelines that may be inferred from gap analysis. When market rates are expected to increase, it is in the bank's best interest to reduce the value of a possible negative gap or increase the size of a possible positive gap and vice versa. Assuming that one-year rate-sensitive assets and liabilities are 50 and 70 million euros respectively, and that the bank expects a rise in interest rates over the coming year of 50 basis points (0.5 %),[1] the expected change in the *NII* would then be:

$$E(\Delta NII) = G \cdot E(\Delta r) = (-20,000,000) \cdot (+0.5\,\%) = -100,000 \qquad (1.6)$$

In a similar situation, the bank would be well-advised to cut back on its rate-sensitive assets, or as an alternative, add to its rate-sensitive liabilities. On the other hand, where there are no expectations about the future evolution of market rates, an immunization policy for safeguarding NII should be based on zero gap.

Some very common indicators in interest rate risk management can be derived from the gap concept. The first is obtained by comparing the gap to the bank's net worth. This allows one to ascertain the impact that a change in market interest rates would have on

[1] Expectations on the evolution of interest rates must be mapped out by bank management, which has various tools at its disposal in order to do so. The simplest one is the forward yield curve presented in Appendix 1B.

the NII/net worth ratio. This frequently-used ratio is an indicator of return on asset and liability management (ALM) – that is, traditional credit intermediation:

$$\Delta\left(\frac{NII}{NW}\right) = \frac{G}{NW}\cdot\Delta r \tag{1.7}$$

Applying (1.7) to a bank with a positive gap of 800 million euros and net worth of 400 million euros, for example, would give the following:

$$\Delta\left(\frac{NII}{NW}\right) = \frac{800}{400}\cdot\Delta r = 2\cdot\Delta r$$

If market interest rates drop by 50 basis points (0.5 %), the bank would suffer a reduction in its earnings from ALM of 1 %.

In the same way, drawing a comparison between the gap and the total interest-earning assets (IEA), we come up with a measure of rate sensitivity of another profit ratio commonly used in bank management: the ratio of NII to interest-earning assets. In analytical terms:

$$\Delta\left(\frac{NII}{IEA}\right) = \frac{G}{IEA}\cdot\Delta r \tag{1.8}$$

A third indicator often used to make comparisons over time (evolution of a bank's exposure to interest rate risk) and in space (with respect to other banks) is the ratio of rate-sensitive assets to rate-sensitive liabilities, which is also called the *gap ratio*. Analytically:

$$GapRatio = \frac{SA}{SL} \tag{1.9}$$

Unlike the absolute gap, which is expressed in currency units, the gap ratio has the advantage of being unaffected by the size of the bank. This makes it particularly suitable as an indicator to compare different sized banks.

1.3 THE MATURITY-ADJUSTED GAP

The discussion above is based on the simple assumption that any changes in market rates translate into changes in interest on rate-sensitive assets and liabilities *instantaneously*, that is, affecting the entire gapping period. In fact, only in this way does the change in the annual NII correspond exactly to the product of the gap and the change in market rates.

In the case of the bank summarized in Table 1.2, for example, the "basic" gap computed as in (1.1), relative to a t of one year, appears to be zero (the sum of rate-sensitive assets, 500 million euros, looks identical to the total of rate-sensitive liabilities). However, over the following 12 months rate-sensitive assets will mature or be repriced at intervals which are not identical to rate-sensitive liabilities. This can give rise to interest rate risk that a rudimentary version of the repricing gap may not be able to identify.

One way of considering the problem (another way is described in the next section) hinges on the *maturity-adjusted gap*. This concept is based on the observation that when there is a change in the interest rate associated with rate-sensitive assets and liabilities,

Table 1.2 A simplified balance sheet

Assets	€ m	Liabilities	€ m
1-month interest-earning interbank deposits	200	1-month interest-bearing interbank deposits	60
3m gov't securities	30	Variable-rate CDs (next repricing in 3 months)	200
5yr variable-rate securities (next repricing in 6 months)	120	Variable-rate bonds (next repricing in 6 months)	80
5m consumer credit	80	1yr fixed-rate CDs	160
20yr variable-rate mortgages (next repricing in 1 year)	70	5yr fixed-rate bonds	180
5yr treasury bonds	170	10yr fixed-rate bonds	120
10yr fixed-rate mortgages	200	20yr subordinated securities	80
30yr treasury bonds	130	Equity	120
Total	1000	Totals	1000

this change is only felt from the date of maturity/repricing of each instrument to the end of the gapping period (usually a year). For example, in the case of the first item in Table 1.2, (interbank deposits with one-month maturity), the new rate would become effective only after 30 days (that is, at the point in time indicated by p in Figure 1.2) and would continue to impact the bank's profit and loss account for only 11 months of the following year.

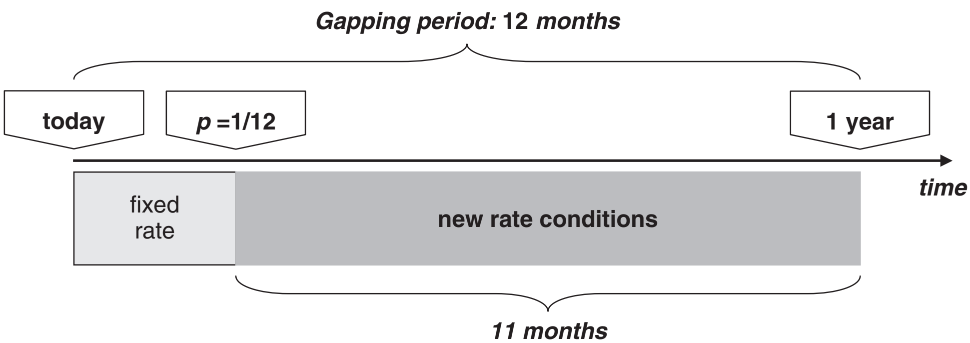

Figure 1.2 An example of repricing without immediate effect

More generally, in the case of any rate-sensitive asset j that yields an interest rate r_j, the interest income accrued in the following year would be:

$$ii_j = sa_j \cdot r_j \cdot p_j + sa_j \cdot (r_j + \Delta r_j) \cdot (1 - p_j) \tag{1.10}$$

where p_j indicates the period, expressed as a fraction of the year, from today until the maturity or repricing date of the j^{th} asset. The interest income associated with a generic rate-sensitive asset is therefore divided into two components: (i) a known component, represented by the first addendum of (1.10), and (ii) an unknown component, linked to future conditions of interest rates, represented by the second addendum of (1.10). Thus, the change in interest income is determined exclusively by the second component:

$$\Delta ii_j = sa_j \cdot (1 - p_j) \cdot \Delta r_j \tag{1.11}$$

If we wish to express the overall change of interest income associated with all the n rate-sensitive assets of the bank, we get:

$$\Delta II = \sum_{j=1}^{n} sa_j \cdot \Delta r_j \cdot (1 - p_j) \tag{1.12}$$

Similarly, the change in interest expenses generated by the k^{th} rate-sensitive liability can be expressed as follows:

$$\Delta ie_k = sl_k \cdot \Delta r_k \cdot (1 - p_k) \tag{1.13}$$

Furthermore, the overall change of interest expenses associated with all the m rate-sensitive liabilities of the bank comes out as:

$$\Delta IE = \sum_{k=1}^{m} sl_k \cdot \Delta r_k \cdot (1 - r_k) \tag{1.14}$$

Assuming a uniform change in the interest rates of assets and liabilities ($\Delta r_j = \Delta r_k = \Delta r$ $\forall j, \forall k$), the estimated change in the bank's NII simplifies to:

$$\Delta NII = \Delta II - \Delta IE = \left(\sum_j sa_j \cdot (1 - p_j) - \sum_j sl_j \cdot (1 - p_j) \right) \cdot \Delta r \equiv G^{MA} \cdot \Delta i \tag{1.15}$$

where G^{MA} stands for the maturity-adjusted gap, i.e. the difference between rate-sensitive assets and liabilities, each weighted for the time period from the date of maturity or repricing to the end of the gapping period, here set at one year.[2]

Using data from Table 1.2, and keeping the gapping period at one-year, we have:

$$\Delta II = \sum_j ir_j = \sum_j sa_j \cdot (1 - p_j) \cdot \Delta r = 312.5 \cdot \Delta r$$

$$\Delta IE = \sum_k ip_k = \sum_k sl_k \cdot (1 - p_k) \cdot \Delta r = 245 \cdot \Delta r$$

and finally

$$\Delta NII = G^{MA} \cdot \Delta r = (312.5 - 245) \cdot \Delta r = 67.5 \cdot \Delta r$$

[2] As Saita (2007) points out, by using (1.15), on the basis of the maximum possible interest rate variation (Δi_{wc}, 'worst case') it is also possible to calculate a measure of "*NII* at risk", i.e. the maximum possible decrease of the *NII*: $IMaR = G^{MA} \cdot \Delta i_{wc}$. This is somewhat similar to "Earnings at Risk" (see Chapter 23), although the latter refers to overall profits, not just to net interest income

Therefore, where the "basic" gap is seemingly zero, the maturity-adjusted gap is nearly 70 million euros. A drop in the market rates of 1 % would therefore cause the bank to earn 675,000 euros less. The reason for this is that in the following 12 months more assets are repriced earlier than liabilities.

1.4 MARGINAL AND CUMULATIVE GAPS

To take into account the actual maturity profile of assets and liabilities within the gapping period, an alternative to the maturity-adjusted gap is the one based on marginal and cumulative gaps.

It is important to note that there is no such thing as an "absolute" gap. Instead, different gaps exist for different gapping periods. In this sense, then, we can refer to a 1-month gap, a 3-month gap, a 6-month gap, a 1-year gap and so on.[3]

An accurate interpretation of a bank's exposure to market rate changes therefore requires us to analyze several gaps relative to various maturities. In doing so, a distinction must be drawn between:

- *cumulative gaps* (G_{t1}, G_{t2}, G_{t3}), defined as the difference between assets and liabilities that call for renegotiation of interest rates *by* a set future date ($t1, t2 > t1, t3 > t2$, etc.)
- *period or marginal gaps* ($G'_{t1}, G'_{t2}, G'_{t3}$), defined as the difference between assets and liabilities that renegotiate rates *in* a specific period of time in the future (e.g. from 0 to $t1$, from $t1$ to $t2$, etc.)

Note that the cumulative gap relating to a given time period t is nothing more than the sum of all marginal gaps at t and previous time periods. Consequently, marginal gaps can also be calculated as the difference between adjacent cumulative gaps. For example:

$$G_{t2} = G'_{t1} + G'_{t2}$$

$$G'_{t2} == G_{t2} - G_{t1}$$

Table 1.3 provides figures for marginal and cumulative gaps computed from data in Table 1.2. Note that, setting the gapping period to the final maturity date of all assets and liabilities in the balance sheet (30 years), the cumulative gap ends up coinciding with the value of the bank's equity (that is, the difference between total assets and liabilities).

As we have seen in the previous section, the one-year cumulative gap suggests that the bank is fully covered from interest risk, that is, the *NII* is not sensitive to changes in market rates. We know, however, that this is misleading. In fact, the marginal gaps reported in Table 1.3 indicate that the bank holds a long position (assets exceed liabilities) in the first month and in the period from 3 to 6 months, which is set off by a short position from 1 to 3 months and from 6 to 12 months.

[3] For a bank whose non-financial assets (e.g. buildings and real estate assets) are covered precisely by its equity (so there is a perfect balance between interest-earning assets and interest-bearing liabilities), the gap calculated as the difference between all rate-sensitive assets and liabilities on an infinite time period ($t = \infty$) would realistically be zero. Extending this horizon indefinitely, in fact, all financial assets and liabilities prove sensitive to interest rate changes. Therefore, if financial assets and liabilities coincide, the gap calculated as the difference between the two is zero.

Table 1.3 marginal and cumulative gaps

Period	RATE-SENSITIVE ASSETS	RATE-SENSITIVE LIABILITIES	MARGINAL GAP G'_t	CUMULATIVE GAP G_t
0–1 months	200	60	140	140
1–3 months	30	200	−170	−30
3–6 months	200	80	120	90
6–12 months	70	160	−90	0
1–5 years	170	180	−10	−10
5–10 years	200	120	80	70
10–30 years	130	80	50	120
Total	1000	880	–	–

We see now how marginal gaps can be used to estimate the real exposure of the bank to future interest rate changes. To do so, for each time period indicated in Table 1.3 we calculate an average maturity (t_j^*), which is simply the date halfway between the start date ($t_j - 1$) and the end date (t_j) of the period:

$$t_j^* \equiv \frac{t_j + t_{j-1}}{2}$$

For example, for the second marginal gap (running from 1 to 3 months) the value of t^*_2 equals 2 months, or 2/12.

Using t^*_j to estimate the repricing date for all rate-sensitive assets and liabilities that fall in the marginal gap G'_{tj}, it is now possible to write a simplified version of (1.15) which does not require the knowledge of the actual repricing date of each rate-sensitive asset or liability, but only information on the value of various marginal gaps:

$$\Delta NII \cong \Delta r \cdot \sum_{j|t_j \leqslant 1} G'_{t_j}(1 - t_j^*) = \Delta r \cdot G_1^W \tag{1.16}$$

G_1^W represents the *one-year weighted cumulative gap*. This is an indicator (also called *NII duration*) of the sensitivity of the *NII* to changes in market rates, computed as the sum of marginal gaps, each one weighted by the average time left until the end of the gapping period (one year).

For the portfolio in Table 1.2, G_1^W is 45 million euros (see Table 1.4 for details on the calculation). As we can see, this number is different (and less precise) from the maturity-adjusted gap obtained in the preceding section (67.5). However, its calculation did not require to specify the repricing dates of the bank's single assets and liabilities (which can actually be much more numerous than those in the simplified example in Table 1.2). What is more, the "signal" this indicator transmits is similar to the one given by the maturity-adjusted gap. When rates fall by one percentage point, the bank risks a reduction in NII of approximately 450,000 euros.

Table 1.4 Example of a weighted cumulative gap calculation

Period	G'_t	t_j	$t*_j$	$1 - t*_j$	$G'_t \times (1 - t*_j)$
up to 1 month	140	1/12	1/24	23/24	134.2
up to 3 months	−170	3/12	2/12	10/12	−141.7
up to 6 months	120	6/12	9/24	15/24	75.0
up to 12 months	−90	1	9/12	3/12	−22.5
Total	0				45.0

Besides speeding up calculations by substituting the maturity-adjusted gap with the weighted cumulative gap, marginal gaps are well-suited to an additional application: they allow banks to forecast the impact on NII of several *infra-annual changes in interest rates*.

To understand how, consider the evolution of interest rates indicated in Table 1.5. Note that variation runs parallel in both rates (a decrease in rates during the first month, an increase during the second and third months, etc.), leaving the size of the spread unchanged. Moreover, changes in these rates with respect to the conditions at the starting point (in t_0) always have the opposite sign (+/−) than the marginal gap relative to the same period (as is also clear from Figure 1.3).

Table 1.5 Marginal gaps and interest rate changes

Period	INTEREST RATE ON ASSETS	INTEREST RATE ON LIABILITIES	Δi RELATIVE TO t_0 (BASIS POINTS)	G'_t. (€ MLN)	EFFECT ON NII
t_0	6.0 %	3 %			
1 month	5.5 %	2.5 %	−50	140	⇩
3 months	6.3 %	3.3 %	+30	−170	⇩
6 months	5.6 %	2.6 %	−40	120	⇩
12 months	6.6 %	3.6 %	+60	−90	⇩
Total					⇩

In this situation, the bank's *NII* is bound to fall monotonically. In each sub-period, in fact, changes in market rates are such that they adversely affect renegotiation conditions for assets and liabilities nearing maturity. At the end of the first month, for example, the bank will have more assets to reinvest than liabilities to refinance ($G'_{1/12}$ is positive at 140). Consequently, the *NII* will suffer due to the reduction in market rates. In three months' time, on the other hand, there will be more debt to refinance than investments to renew ($G'_{3/12} = -170$), so that a rise in returns will translate once again into a lower *NII*.

The example illustrates that in order to actually quantify the effects of several infra-annual market changes on the bank's *NII*, we must consider the different time periods when the effects of these variations will be felt. Therefore, marginal gaps provide a way

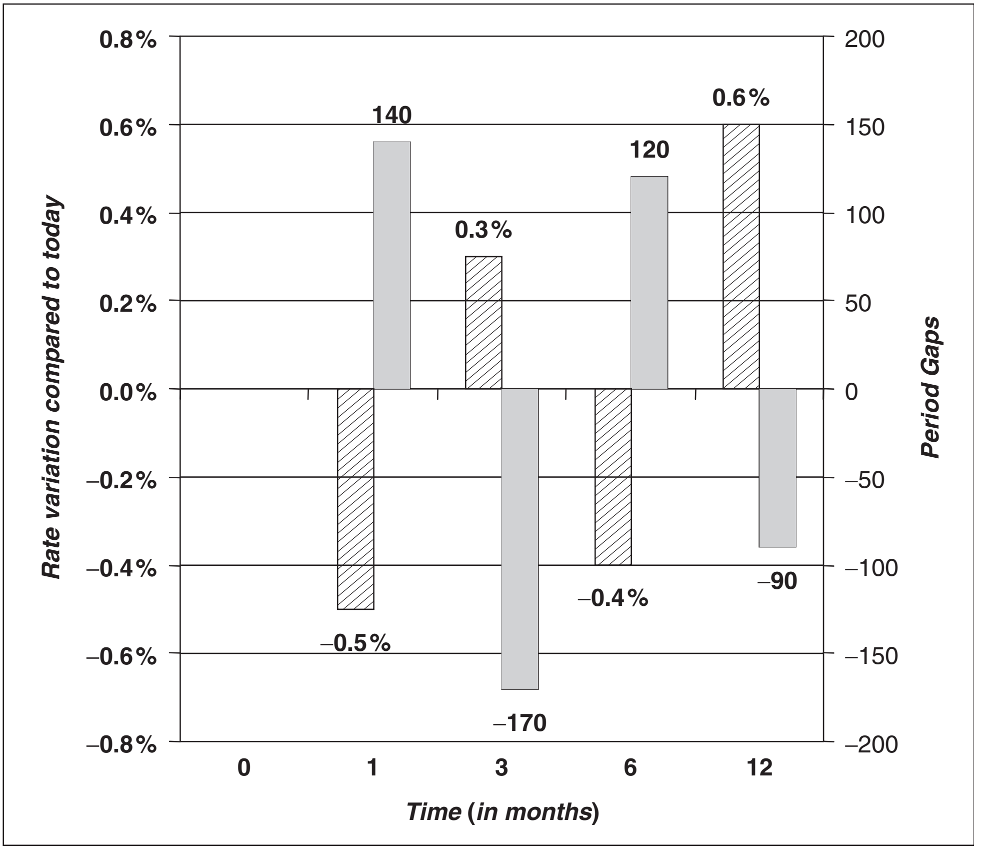

Figure 1.3 Marginal gaps and possible rate variations

to analyze the impact of a possible *time path of market rates* on margins, rather than a simple isolated change.

Summing up, there are two primary reasons why non zero marginal gaps can generate a change in *NII* even when there is a zero cumulative gap:

(1) a *single change* of market rates has different effects on the *NII* generated by rate-sensitive assets and rate-sensitive liabilities that form the basis of single period gaps (the case of weighted cumulative gap not equal to zero[4], see formula 1.16);
(2) the possibility that within this timeframe *several changes of market rates* come into play with opposite signs (+/−) than the marginal gaps (the case of marginal gaps not equal to zero, see Table 1.5 and Figure 1.3).[5]

[4] Note that where there are infra-annual marginal gaps that are all equal to zero, even the weighted cumulative gap calculated with (1.16) would be zero. It would be then logical to conclude that the bank is immunized against possible market rate changes.
[5] We can see that in theory situations could arise in which the infra-annual marginal gaps not equal to zero bring about a cumulative marginal gap of zero. It is possible then that case number 2 may occur (losses when there are several infra-annual changes) but not number 1 (losses when there is only one rate change).

At this point, it is clear that an immunization policy to safeguard NII against market rate changes (in other words, the complete immunization of interest risk following a repricing gap logic) requires that marginal gaps of every individual period be zero. However, even quarterly or monthly gaps could be disaggregated into shorter ones, just like the one-year has been decomposed into shorter-term gaps in Table 1.5. Hence, a perfect hedging from interest risk would imply that all marginal gaps, even for very short time periods, be equal to zero.

A bank should therefore equate all daily marginal gaps to zero (that is, the maturity of all assets and liabilities should be perfectly matched, with every asset facing a liability of equal value and duration). Given a bank's role in transforming maturities, such a requirement would be completely unrealistic.

Moreover, although many banks have information on marginal gaps relating to very short sub-periods, still they prefer to manage and hedge only a small set of gaps relative to certain standard periods (say: 0–1 month, 1–3 months, 3–6 months, 6–12 months, 1–3 years, 3–5 years, 5–10 years, 10–30 years, over 30 years). As we will see further on, the reason for this standardization (beyond the need for simplification) is mainly related to the presence of some hedging instruments that are available only for some standard maturities.

1.5 THE LIMITATIONS OF THE REPRICING GAP MODEL

Measuring interest risk with the repricing gap technique, as common as this practice is among banks, involves several problems.

1. *The assumption of uniform changes of interest rates of assets and liabilities and of rates for different maturities*

   The gap model gives an indication of the impact that changes in market interest rates have on the bank's *NII* in a situation where the change of interest rates on assets is equal to the one of liabilities. In practice, some assets or liabilities negotiated by the bank are likely to readjust more noticeably than others. In other words, the different assets and liabilities negotiated by the bank can have differing degrees of sensitivity to relative interest rates. This, in turn, can be caused by the different bargaining power the bank may enjoy with various segments of its clientele. Generally speaking, therefore, the degree of sensitivity of interest rates of assets and liabilities to changes in market rates is not necessarily constant across-the-board. In addition to this, the repricing gap model assumes that rates of different maturities within the same gapping period are subject to the same changes. This is clearly another unrealistic assumption.

2. *Treatment of demand loans and deposits*

   One of the major problems associated with measuring repricing gaps (and interest risk as a whole) arises from on-demand assets and liabilities, i.e. those instruments that do not have a fixed maturity date. Examples are current account deposits or credit lines. Following the logic used to this point, these items would be assigned a very short (even daily) repricing period. In fact, where there is a rise in market rates, an account holder could in principle ask immediately for a higher interest rate (and if this request is denied, transfer her funds to another bank). In the same way, when a drop in market rates occurs, customers might immediately ask for a rate reduction on their financing (and again, if this request is not granted, they may pay back their loans and turn to another bank). In practice, empirical analysis demonstrates that interest rates relative to on-demand instruments do not immediately respond to market rate changes. Various factors account for this delay, such as: (a) transaction costs that retail customers or companies must inevitably sustain to transfer their financial dealings to another bank, (b) the fact that the terms a bank might agree to for a loyal business customer often result from a credit standing assessment based on a long-term relationship (so the company in question would not easily obtain the same conditions by going to a new bank), (c) the fact that some companies' creditworthiness would not allow them to easily get a credit line from another bank. We can also see that, in addition to being sticky, returns from on-demand instruments also tend to adjust asymmetrically. In other words, adjustments happen more quickly for changes that give the bank an immediate economic advantage (e.g. increases in interest income, decreases in interest expenses). This stickiness and lack of symmetry can be stronger or weaker for customers with different bargaining power. For example, one can expect decreases in market rates to take longer to reflect on interest rates paid on retail customers' deposits. On the other hand, rate drops would be quicker to impact interest rates applied to deposits for large businesses.

3. *The effects of interest rate changes on the amount of intermediated funds are disregarded*

   The gap model focuses on the effects that changes in market interest rates produce on the bank's *NII*, i.e. on interest income and expenses. The attention is concentrated on flows only, without any consideration of possible effects on stocks, that is, on the value of assets and liabilities of the bank. However, a reduction in market interest rates could, for example, prompt the customer to pay off fixed-rate financing and to increase demand for new financing. In the same way, an increase in market rates would encourage depositors to look for more profitable forms of savings than deposits in current accounts, and as a result the bank's on-demand liabilities would shrink.

4. *The effects of rate changes on market values are disregarded.*

   A further problem of the repricing gap model is that the impact of interest rate changes on the market value of assets and liabilities is not taken into account. Indeed, an increase in interest rates has effects which are not limited exclusively to income flows associated with interest-earning assets or interest-bearing liabilities – the market value of these instruments is also modified. So, for example, a rise in market rates leads to a decrease in the market value of a fixed-rate bond or mortgage. These effects are, for all practical purposes, ignored by the repricing gap.

Each of these problems is addressed in the following section, and solutions are described whenever possible.

1.6 SOME POSSIBLE SOLUTIONS

1.6.1 Non-uniform rate changes: the standardized gap

One way to overcome the first problem mentioned above (different sensitivity of interest rates on assets and liabilities to changes in market rates) is based on an attempt to estimate this sensitivity and to use these sensitivities when calculating the gap. More specifically, the method of analysis is based on three different phases:

- √ Identifying a reference rate, such as the 3-month interbank rate (Euribor 3m).
- √ Estimating the sensitivity of various interest rates of assets and liabilities with respect to changes in the reference rate.
- √ Calculating an "adjusted gap" that can be used to estimate the actual change that the bank's *NII* would undergo when there is a change in the market reference rate.

At this point, let us assume that an estimation has been made[6] of the sensitivity of interest rates of assets and liabilities with respect to changes in the 3 month interbank rate, and that the results obtained are those reported in Table 1.6. This table shows the case of a short-term bank which, besides its equity, only holds assets and liabilities sensitive to a one-year gapping period. For these instruments, the relative Euribor sensitivity coefficients are also included in the table (indicated by β_j and γ_k respectively for assets and liabilities).

Table 1.6 Example of a simplified balance sheet structure

Assets	€ m	β	Liabilities	€ m	γ
On-demand credit lines	460	0.95	Clients' deposits	380	0.8
Interbank 1 m deposits	80	1.1	1m interbank deposits	140	1.1
3 month gov't. securities	60	1.05	Variable-rate CDs (next repricing in 3m)	120	0.95
5yr variable-rate consumer credit (repricing in 6m)	120	0.9	10yr variable-rate bonds (*euribor* + 50 bp, repricing in 6m)	160	1
10yr variable-rate mortgages (Euribor+100 basis points, repricing in 1yr)	280	1	1yr fixed-rate CDs	80	0.9
			Equity	120	
Total / average	1000	0.98	Total / average	1000	0.91

On the basis of the data in the table, it is possible to calculate a gap that takes into account the different sensitivity of assets and liabilities rates to changes in the reference market rate by simply multiplying each one by the relative sensitivity coefficient.

In fact, if on-demand loans show a rate-sensitivity coefficient of 0.95, this means that when there is a change of one percentage point of the three-month Euribor rate the relative interest rate varies on average by 0.95 % (see Figure 1.4). It follows that interest rates

[6] This can be done, e.g., by ordinary least squares (OLS). Readers who are not familiar with OLS and other simple statistical estimation techniques will find a comprehensive presentation, e.g., in Mood et al. (1974) or Greene (2003).

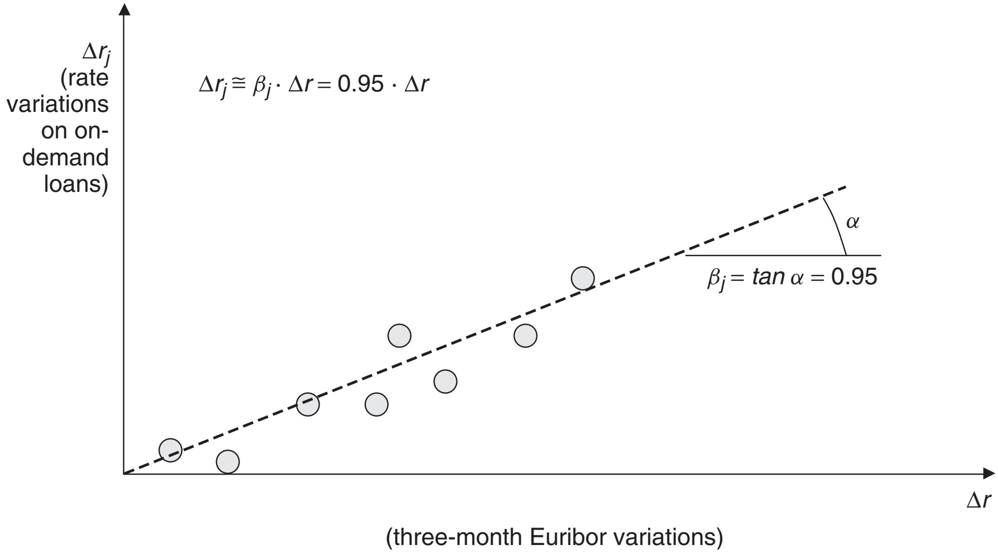

Figure 1.4 example of an estimate of the beta of a rate-sensitive asset

on these loans also undergo a similar change. Since this is true for all rate-sensitive n assets and m liabilities, we can rewrite the change in *NII* following changes in the three-month Euribor as:

$$\Delta NII = \sum_{j=1}^{n} sa_j \cdot \Delta r_j - \sum_{k=1}^{m} sl_k \cdot \Delta r_k \cong \sum_{j=1}^{n} sa_j \cdot \beta_j \cdot \Delta r - \sum_{k=1}^{m} sl_k \cdot \gamma_k \cdot \Delta r =$$

$$= \left(\sum_{j=1}^{n} sa_j \cdot \beta_j - \sum_{k=1}^{m} sl_k \cdot \gamma_k \right) \cdot \Delta r \equiv G^s \cdot \Delta r \tag{1.17}$$

The quantity in parenthesis is called the standardized gap:

$$G^s = \sum_{j=1}^{n} sa_j \cdot \beta_j - \sum_{k=1}^{m} sl_k \cdot \gamma_k \tag{1.18}$$

and represents the repricing gap adjusted for the different degrees of sensitivity of assets and liabilities to market rate changes.

Applying (1.18) to the example in the table we come up with a standardized one-year gap of 172 (See Table 1.7 for details on the calculation). This value is greater than the gap we would have gotten without taking into consideration the different rate-sensitivities of assets and liabilities (120). This is due to the fact that rate-sensitive assets, beyond being larger than liabilities, are also more sensitive to Euribor changes (the average weighted value of β, in the last line of Table 1.6, exceeds the average weighted value of γ by 7 percentage points.)

Table 1.7 Details of the standardized gap calculation

Assets	as_j	β_j	$as_j \times \beta_j$
On-demand credit lines	460	95 %	437
1m interbank deposits	80	110 %	88
3m government securities	60	105 %	63
5yr variable-rate consumer credit (repricing in 6m)	120	90 %	108
10yr variable-rate mortgages (Euribor + 100 basis points, repricing in 1 yr)	280	100 %	280
Total assets			976
Liabilities	ps_k	γk	$ps_k \times \gamma_k$
Customers' deposits	380	80 %	304
1m interbank deposits	140	110 %	154
Variable-rate CDs (next repricing in 3m)	120	95 %	114
10yr variable-rate bonds (Euribor + 50 bp, repricing in 6m)	160	100 %	160
1yr fixed-rate CDs	80	90 %	72
Total liabilities			804
Assets / liabilities imbalance (gap)			*172*

Given the positive value of the standardized gap, formula (1.18) suggests that in the event of a rise in market rates, the bank will experience an increase in its net interest income. The size of this increase is obviously greater than that estimated with the simple repricing gap. The same is true when market interest rates fall (the bank's *NII* undergoes a greater decrease than that estimated with a non-standardized gap.)

1.6.2 Changes in rates of on-demand instruments

The standardized gap method can be fine-tuned even further to deal with on-demand instruments which have no automatic indexing mechanism.[7]

First of all, for each of these instruments we need to estimate the structure of average delays in rate adjustments with respect to the point in time when a market rate change occurs. This can be done by means of a statistical analysis of past data, as shown in Table 1.8, which gives an example relating to customers' deposits. Here the overall sensitivity coefficient (γ_k) to the 3-month Euribor rate is 80 %, which tells us that for a change of one percentage point of the Euribor, the average return on demand deposits

[7] If in fact such a mechanism were present, as is often the case for current account deposits with standard conditions granted to customers belonging to particular groups with high bargaining power (employees of the same company, members of an association, etc.) or for opening a credit line on an account for companies with a high credit standing, the relative instruments would be treated as if they had a maturity date equal to the indexing delay.

only varies by 80 basis points.[8] Moreover, of these 80 basis points only 10 appear within a month of the Euribor variation, while for the next 50, 12 and 8 there is a delay of 3, 6, and 12 months respectively. In this case, not only in calculating the standardized gap, the bank's demand deposits (380 million euros in the example in Table 1.6) are multiplied by γ_k and counted only for 304 (380·0.80) million. This amount is allocated to the various marginal gaps on the basis of delays which were found in past repricing. This means that 38 million (380·0.10) will be placed in the one-month maturity bracket, while 190 million (380·0.50) will be positioned in the three-month bracket and so on (see the right hand column in Table 1.8.)

Table 1.8 Example of progressive repricing of demand deposits

Timeframe	Percentage of variation absorbed	Funding allocated in different time periods (millions of €)
On-demand	0 %	0.0
1 month	10 %	38.0
3 month	50 %	190.0
6 month	12 %	45.6
1 year	8 %	30.4
Total	80 %	304.0

In the previous section we mentioned the fact that on-demand loans and deposits adjust to changes in benchmark rates asymmetrically (that is, banks are quicker at reducing rates on deposits and increasing rates on earning assets). If this is so, Table 1.8 should be split into two versions, measuring the progressive repricing of demand deposits when Euribor rates rise or fall. This implies that the of earnings on on-demand instruments to different marginal gaps (last column in the table) will be done differently if we want to predict the effects of positive, rather than negative, interest rate changes on NII. Following this logic (which applies to all on-demand instruments, both interest-earning and interest-bearing) leads us to calculate two different repricing gaps the bank can use to measure the sensitivity of *NII* to increases and decreases of market rates.

1.6.3 Price and quantity interaction

In principle, the coefficients β and γ used in the calculation of the standardized gap could be modified to take into account the elasticity of quantities relative to prices: if, for example, given a 1 % change of benchmark rates, a given rate-sensitive asset undergoes a rate change of β, but at the same time records- a volume change of x%, a modified β equal to $\beta' = \beta{\cdot}(1 + x\%)$ would be enough to capture the effect on expected interest income flows both of unit yields as well as intermediate quantities. The γ coefficients of

[8] This reduced rate-sensitivity comes from the fact that the public does not hold current accounts for investment purposes only, but also for liquidity purposes. For this reason, the return on deposits is not particularly sensitive to changes in market rates.

rate-sensitive liabilities could likewise be modified. The change of the *NII* estimated in this way would then be adjusted to make allowances for the value of funds bought or sold on the interbank market as a result of possible imbalances between new volumes of assets and liabilities.

In practice, this type of correction would prove extremely arbitrary, since demand for bank assets and liabilities does not react only to interest rates, but also to a number of other factors (state of the economic cycle, preference for liquidity, returns on alternative investments). In addition, this would distance the model from its original significance, making its conclusions less readable and less transparent. For this reason, in calculating the interest rate risk on the balance sheet of a bank, the effect associated with the interaction of prices and quantities is usually ignored.

1.6.4 Effects on the value of assets and liabilities

As mentioned above, a change in market rates can cause changes in the value of assets and liabilities that go beyond the immediate effects on the *NII*. The repricing gap, being an income-based method anchored to a target variable taken from the profit and loss account, is intrinsically unsuitable for measuring such changes. To do so, we have to take on a different perspective and adopt an equity method, such as the duration gap presented in the next chapter.

SELECTED QUESTIONS AND EXERCISES

1. What is a “sensitive asset” in the repricing gap model?
   (A) An asset maturing within one year (or renegotiating its rate within one year);
   (B) An asset updating its rate immediately when market rates change;
   (C) It depends on the time horizon adopted by the model;
   (D) An asset the value of which is sensitive to changes in market interest rates.

2. The assets of a bank consist of € 500 of floating-rate securities, repriced quarterly (and repriced for the last time 3 months ago), and of € 1,500 of fixed-rate, newly issued two-year securities; its liabilities consist of € 1,000 of demand deposits and of € 400 of three-year certificates of deposit, issued 2,5 years ago.
   Given a gapping period of one year, and assuming that the four items mentioned above have a sensitivity (“beta”) to market rates (e.g, to three-month interbank rates) of 100 %, 20 %, 30 % and 110 % respectively, state which of the following statements is correct:
   (A) The gap is negative, the standardized gap is positive;
   (B) The gap is positive, the standardized gap is negative;
   (C) The gap is negative, the standardized gap is negative;
   (D) The gap is positive, the standardized gap is positive.

3. Bank Omega has a maturity structure of its own assets and liabilities like the one shown in the Table below. Calculate:
   (A) Cumulated gaps relative to different maturities;
   (B) Marginal (periodic) gaps relative to the following maturities: (i) 0–1 month, (ii) 1–6 months, (iii) 6 months–1 year, (iv) 1–2 years, (v) 2–5 years, (vi) 5–10 years, (vii) beyond 10 years;

(C) The change experienced by the bank's net interest income next year if lending and borrowing rates increase, for all maturities, by 50 basis points, assuming that the rate repricing will occur exactly in the middle of each time band (e.g., after 15 days for the band between 0 and 1 month, 3.5 months for the band 1–6 months, etc.).

Sensitive assets and liabilities for Bank Omega (data in million euros)

	1 month	6 months	1 year	2 years	5 years	10 years	Beyond10 years
Total sensitive assets	5	15	20	40	55	85	100
Total sensitive liabilities	15	40	60	80	90	95	100

4. The interest risk management scheme followed by Bank Lambda requires it to keep all marginal (periodic) gaps at zero, for any maturity band. The Chief Financial Officer states that, accordingly, the bank's net interest income is immune for any possible change in market rates. Which of the following events could prove him wrong?
   (I) a change in interest rates not uniform for lending and borrowing rates;
   (II) a change in long term rates which affects the market value of items such as fixed-rate mortgages and bonds;
   (III) the fact that borrowing rates are stickier than lending rates;
   (IV) a change in long term rates greater than the one experienced by short-term rates.

   (A) I and III;
   (B) I, III and IV;
   (C) I, II and III;
   (D) All of the above.

5. Using the data in the Table below (and assuming, for simplicity, a 360-day year made of twelve 30-day months):
   (i) compute the one-year repricing gap and use it to estimate the impact, on the bank's net interest income, of a 0.5 % increase in market rates;
   (ii) compute the one-year maturity-adjusted gap and use it to estimate the effect, on the bank's net interest income, of a 0.5 % increase in market rates;
   (iii) compute the one-year standardised maturity-adjusted gap and use it to estimate the effect, on the bank's net interest income, of a 0.5 % increase in market rates;
   (iv) compare the differences among the results under (i), (ii) and (iii) and provide an explanation.

Assets	Amount	Days to maturity/ repricing	β
Demand loans	1000	0	90 %
Floating rate securities	600	90	100 %
Fixed-rate instalment loans	800	270	80 %
Fixed-rate mortgages	1200	720	100 %
Liabilities	Amount	Days to maturity/ repricing	γ
Demand deposits	2000	0	60 %
Fixed-rate certificates of deposit	600	180	90 %
Floating-rate bonds	1000	360	100 %

6. Which of the following represents an advantage of the zero-coupon rates curve relative to the yield curve?
   (A) The possibility to take into account the market expectations implied in the interest rates curve
   (B) The possibility of assuming non parallel shifts of the interest rates curve;
   (C) The possibility of associating each cash flow with its actual return;
   (D) The possibility of achieving a more accurate pricing of stocks.

Appendix 1A
The Term Structure of Interest Rates

1A.1 FOREWORD

The term structure of interest rates is usually represented by means of a curve (yield curve) indicating market rates for different maturities. This is usually based on rates paid on Treasury bonds, where default risk can be considered negligible (especially if the issuer is a sovereign state belonging to the G-10 and having a high credit rating); different rates (like those on interbank deposits, up to one year, and interest rate swaps, for longer maturities) may also be used. However, the rates must refer to securities that are homogeneous in all main characteristics (such as default risk, the size of any coupons, etc.) except their time-to-maturity. When based on zero-coupon securities, the yield curve is usually called *zero-coupon curve*.

The yield curve may take different shapes: it can be upward sloping (if short-term rates are lower than long-term ones), downward sloping, flat or hump-shaped see Figure B.4) when rates first increase (decrease) and then decrease (increase) as maturities rise further.

Four main theories try to explain the shape of the yield curve: (i) the theory of expectations, originally due to Fischer,[9] back in 1930; (ii) the theory of the preference for liquidity, proposed by Hicks[10] in 1946; (iii) the theory of the preferred habitat, due to Modigliani and Sutch in 1966 and (iv) the theory of market segmentation. Generally speaking, all these theories acknowledge the role of market expectations in shaping forward rates, hence the slope of the yield curve. They differ in that they may or may not assign a role to other factors, such as liquidity premiums, institutional factors preventing the free flow of funds across maturities, and so on.

1A.2 THE THEORY OF UNBIASED EXPECTATIONS

Based on the expectations theory, the shape of the yield curve depends only on market expectations on the future level of short-term rates. According to this theory, long term rates are simply the product of current short term rates and of the short term rates expected in the future. Hence if, for example, the one-year rate is lower than the two-year rate, this is due to the fact that the market expects one-year rates to increase in the future.

More formally, investors are supposed to be equally well off either investing over long maturities or rolling over a series of short term investments. Formally:

$$(1 + r_T)^T = \prod_{j=0}^{T-1} \left[1 + E({}_j r_1)\right] \tag{1A.1}$$

where r_T is the rate on a T-year investment and $E({}_j r_1)$ denotes the expected rate on a one-year investment starting at time j.

[9] Fischer (1965).
[10] Hicks (1946).

Long-term rates like r_T therefore depend on expected future short-term rates:

$$r_T = \left\{ \prod_{j=0}^{T-1} \left[1 + E({}_j r_1) \right] \right\}^{1/T} - 1 \qquad (1A.2)$$

Suppose, e.g., that the one-year return on Treasury bills is 3 %, while the expected one-year returns for the following four years are $E({}_1r_1) = 3.5\%$, $E({}_2r_1) = 4\%$, $E({}_3r_1) = 4.5\%$ and $E({}_4r_1) = 5\%$.

According to the expectations theory, the spot five-year rate would be

$$r_5 = \left\{ \prod_{j=0}^{4} \left[1 + E({}_j r_1) \right] \right\}^{1/5} - 1 =$$
$$= \sqrt[5]{(1 + 3\,\%) \cdot (1 + 3.5\,\%) \cdot (1 + 4\,\%) \cdot (1 + 4.5\,\%) \cdot (1 + 5\,\%)} - 1 = 4.00\,\%$$

Similarly, rates for shorter maturities could be found as:

$$r_4 = \sqrt[4]{(1 + 3\,\%)(1 + 3.5\,\%)(1 + 4\,\%)(1 + 4.5\,\%)} - 1 = 3.75\,\%$$
$$r_3 = \sqrt[3]{(1 + 3\,\%)(1 + 3.5\,\%)(1 + 4\,\%)} - 1 = 3.50\,\%$$
$$r_2 = \sqrt{(1 + 3\,\%)(1 + 3.5\,\%)} - 1 = 3.25\,\%$$

These results are shown in Figure A.1:

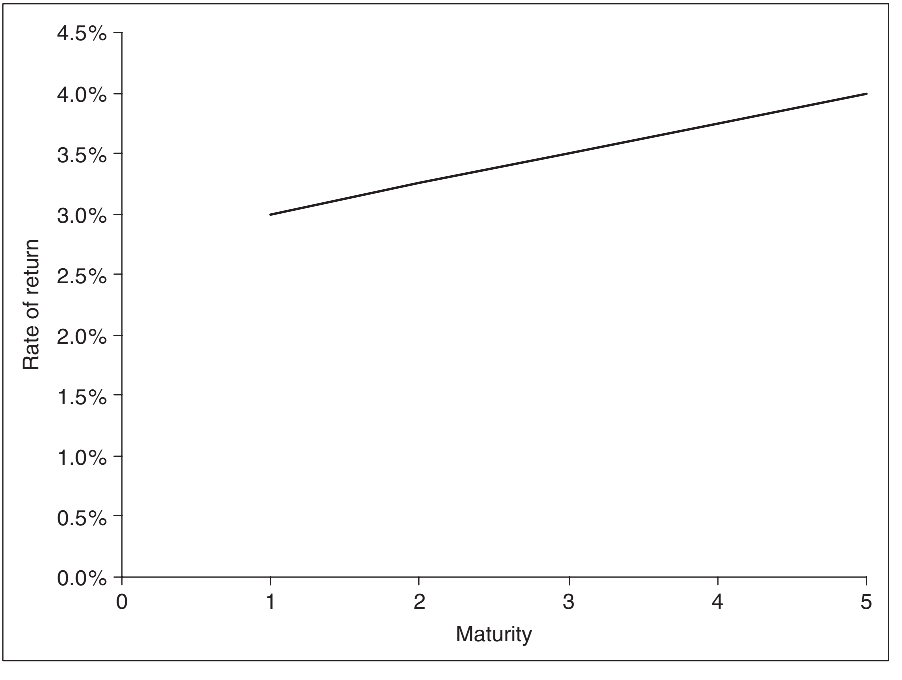

Figure A.1 An example of yield curve

According to the expectations theory, an upward-sloped yield curve denotes expectations of an increase in short term interest rates. The opposite is true when the curve is downward-sloped.

The expectations theory is based on a very demanding hypothesis: that investors are risk neutral and decide how to invest based on their expectations. To appreciate the implications in this hypothesis, consider the fair two-year spot rate found in the example above (3.25 %, following from a current one-year rate of 3 % and an expected one-year rate, after one year, of 3.5 %). Now, suppose that the actual two-year rate is 3.2 %, that is, lower than this equilibrium value. To get the higher rate (3.25 %), a rational investor would invest for the first year at 3 %, then roll over its investment at a rate that, based on his/her expectations, should be 3.5 %. However, this strategy is not risk-free, since the rate for the second year (3.5 %) is not certain, but only represents an expectation; it could turn out to be lower than expected, driving the average return on the two-year strategy well below the expected 3.25 %. Hence, a risk-averse investor could abandon this strategy and accept a risk-free 2-year investment offering 3.2 %. In this case, long-term rates would differ from the equilibrium values dictated by market expectations.

1A.3 THE LIQUIDITY PREFERENCE THEORY

Empirical evidence shows that the yield curve is usually positively sloped. If market expectations were the only driver of the curve, this would imply that short-term rates are always expected to increase. Hence, some other factor must be invoked, besides market expectation, to explain this upward slope.

The liquidity preference theory states that investors tend to prefer investments that, all other things being equal, have a shorter maturity and therefore are more liquid. This is mainly due to the fact that, when underwriting a long-term investment, investors commit their money over a longer horizon and "lock" a rate of return that cannot be subsequently modified. Hence, investors are willing to invest over longer maturities only if they get compensated through higher returns.

Due to such liquidity premiums, the curve might be upward sloped even if expectations on future short-term rates were steady. More formally, indicating by ${}_jL_1$ the liquidity premium required by investors to face the opportunity cost linked to the uncertainty surrounding the level of future one-year rates at time j, we get:

$$r_T = \left\{ \prod_{j=0}^{T-1} \left[1 + E({}_jr_1) + {}_jL_1\right] \right\}^{1/T} - 1 \qquad (1A.3)$$

As the uncertainty in the level of future short-term rates increases with time, we have that ${}_1L_1 < {}_2L_1 < \ldots < {}_{T-1}L_1$. This causes the curve to be positively sloped even when expectations on short rates are steady.

1A.4 THE THEORY OF PREFERRED HABITATS

The theory of preferred habitats assumes that different classes of investors are characterised by different investment horizons. Accordingly, families and individuals tend to prefer short maturities, whereas institutional investors like insurance companies funding life insurance policies and pension funds tend to have a longer investment horizon.

Therefore, different maturity brackets or “habitats” exist, where different investors can be found. Investors are reluctant to pursue arbitrage strategies that would involve leaving their preferred habitat and tend to do so only if the gains implied by such strategies are large enough to compensate them.

While the liquidity preference theory dictates that rates increase with maturities, the preferred habitat hypothesis may also be compatible with a market where long-term rates are lower. This would simply imply that, due to demand and supply conditions, issuers of short-term securities have to offer a premium to lure long-term-minded investors out of their preferred habitat, and that such a premium is higher than any liquidity premium associated with longer maturities.

1A.5 THE MARKET SEGMENTATION HYPOTHESIS

This is the only approach that does not include expectations on future rates as a driver of the current yield curve. In fact, this theory states that maturity brackets represent separate markets, where rates of return are determined independently, based on supply and demand conditions, as well as on some macroeconomic variables. Namely, while monetary variables tend to affect rates on short term maturities, long-term rates are mainly driven by the state of the real economy. Similar to the preferred habitats theory, the segmented market hypothesis acknowledges the fact that different types of investors operate in different maturity segments.

Appendix 1B Forward Rates

Consider an investor wishing to invest 10,000 euros for two years. She may either buy a one-year Treasury bill (paying a yield of 3.5 %) or a two-year Treasury bond (offering a return of 3.8 %). What alternative would be more attractive? The answer clearly depends on the future value of the one-year rates. In fact, while the two-year bond offers a given return for the whole investment period, the one-year T-bill creates a reinvestment risk for the second year.

Now, suppose that it is possible, today, to lock the rate on a one-year investment starting in one year. Such a rate, fixed today but referred to a future investment, is called a *forward rate* (as opposed to "spot rates", that is, rates for "normal" investments starting immediately).

The two investment strategies would be equivalent only if they gave rise to the same per-euro outcome at the end of the second year:

$$(1 + r_2)^2 = (1 + r_1)(1 + {}_1r_1) \tag{1B.1}$$

where ${}_jr_t$ denotes the forward rate on a t-year investment starting at time j. Note that all rates in equation 1B.1 are assumed to be known with certainty. In fact, the forward rate ${}_1r_1$, while related to an investment taking place in the future, is agreed upon today and cannot be changed subsequently.

From (1B.1) it follows that the "fair" forward rate can be computed as

$${}_1r_1 = \frac{(1 + r_2)^2}{(1 + r_1)} - 1 \tag{1B.2}$$

More generally:

$${}_jr_t = \frac{(1 + r_{t+j})^{t+j}}{(1 + r_j)^j} - 1 \tag{1B.3}$$

Applying equation (1B.3) to our example, we get a forward rate of:

$${}_1r_1 = \frac{(1 + 3.8\,\%)^2}{1 + 3.5\,\%} - 1 \cong 4.10\,\%$$

Note that this forward rate is higher than the two spot rates. This is logical: forward rates can be seen as estimates of the expected future rates and the fact that r_2 is greater than r_1 suggests that future rates are expected to rise.

Note that any value for the forward rate above or below the one dictated by equations 1B.2 and 1B.3 would immediately give rise to arbitrage strategies. Suppose, e.g., that the forward rate in our example be lower than 4.10 % (for instance, 4 %). In this case, an investor could invest, say, 1,000 euros in the two-year bond (yielding 3.8 % per year) while financing herself with two one-year loans at 3.5 % and 4 %, respectively. The final value of the bond would be $1{,}000 \cdot (1 + 3.8\%)^2 \cong 1{,}077$ euros, while the final value of the loan would be $1{,}000 \cdot (1 + 3.5\%) \cdot (1 + 4\%) \cong 1{,}076$ euros. Hence, a risk-less profit of one euro profit could be achieved. Such arbitrage schemes would of course increase the demand for two-year investments, as well as the demand for one-year loans (both spot and forward). The rates on the former would then decrease, while the cost of one-year loans would rise, until any arbitrage opportunities have disappeared, and market rates are consistent with equations 1B.2 and 1B.3.

Based on (1B.3), forward rates can be computed, starting from spot rates, for any future time window. Suppose, e.g., that the three-year spot rate is 4.5 %. Then, the one-year forward rate for investments taking place after two years (${}_2r_1$) can be found as:

$${}_2r_1 = \frac{(1 + 4.05\,\%)^3}{(1 + 3.8\,\%)^2} - 1 = 4.552\,\%$$

Once again, as the three-year rate exceeds the two-year one, the forward rate is higher than both of them. Generally speaking, when the yield curve is positively sloped, forward rates stay above spot rates; the opposite occurs when spot rates decrease with maturities. Again, this is quite logical if we follow the expectations theory and if forward rates can be interpreted as an estimate of expected spot rates in the future. In fact, if forward rates are above spot rates, then short-term rates are expected to increase, and this must be reflected in higher long-term rates (see Figure 1B.1, panel I). If, on the other hand, lower forward rates signal an expected reduction in future short-term rates, spot rates will decrease as maturities increase, accounting for these expectations (Figure 1B.1, panel II). Finally, if forward rates were to be equal to spot rates with the same maturity, this would signal that rates are expected to stay constant, and the spot curve would be flat (Figure 1B.1, panel III).

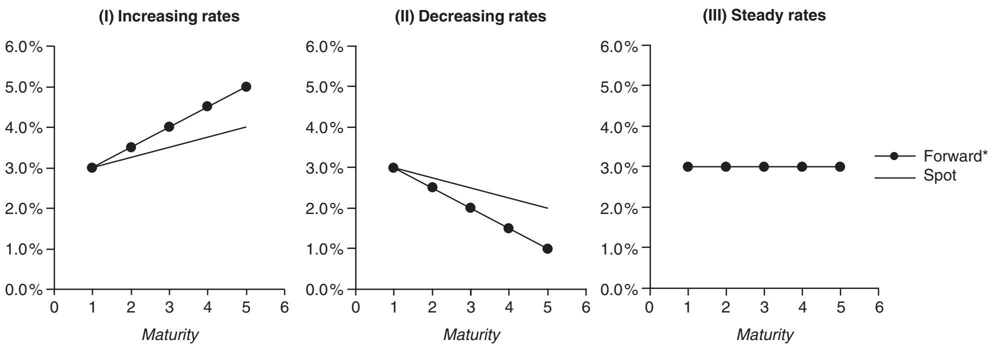

Figure A.1 Spot and forward curves

Note that in this appendix we used annual compounding, as this is the most widely used approach for banks and financial markets. When using continuously-compound rates, the relationship between spot and forward rates becomes even more straightforward. In fact, (1B.1) becomes

$$e^{2r_2} = e^{r_1} e^{{}_1r_1} = e^{r_1 + {}_1r_1} \tag{1B.4}$$

and equations (1B.2) and (1B.3) simplify to:

$${}_1r_1 = 2r_2 - r_1 \tag{1B.5}$$

$${}_jr_t = (j + t)r_{j+t} - jr_j \tag{1B.6}$$