CHAPTER 1

School mathematics in a changing world

LEARNING OUTCOMES

1.1 Connecting your disposition towards mathematics with your mathematical experiences.

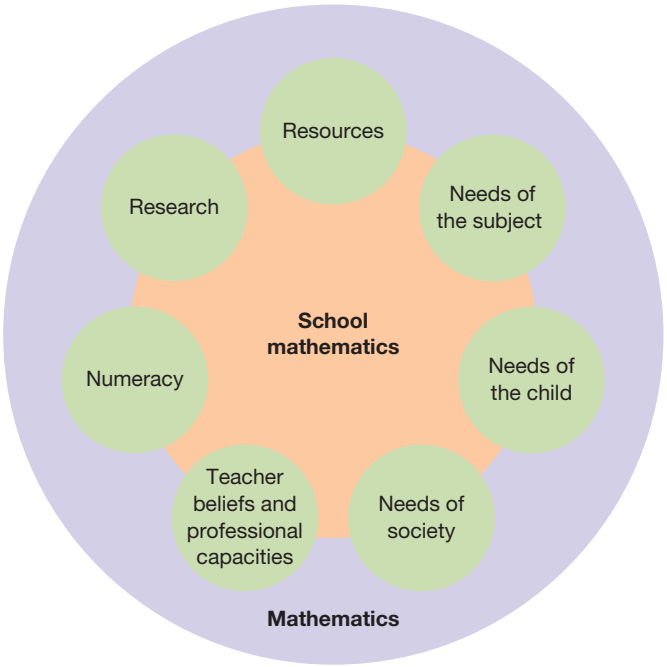
1.2 Recognising factors that determine the mathematics that is currently taught, and the role, rationale and aims of the Australian Curriculum in contemporary mathematics teaching and learning.

1.3 Evaluating resources to determine those of use in your continuing development of mathematics, mathematical learning and teaching mathematics.

‘Mathematics is a more powerful instrument of knowledge than any other that has been bequeathed to us by human agency.’

René Descartes (French philosopher and mathematician, 1596–1650)

Chapter 1 concept map



Introduction

Mathematics remains an essential tool for understanding the world, so mathematics education needs to keep pace with the advancements that our society makes at an ever-increasing rate. With the focus on Science, Technology, Engineering and Maths, or STEM, mathematics and mathematics education are now an even stronger focus (Office of the Chief Scientist, 2014a). Taken from a mathematics learning and teaching perspective, the words of Descartes are apt. As a teacher of mathematics, you have the opportunity to engage your students in the language of mathematics and to do this in a positive and meaningful way. The result has the potential to impact not just their education, but also their educational opportunities, employment and participation in society.

What is your vision of the mathematics you will be teaching? What is your vision of the classroom? Many of you will remember your experiences in primary school. Some of you will remember memorising multiplication tables, operating with fractions or doing long division. Others may remember exploring patterns, doing geometry projects or solving problems. Mathematics in primary school may have been a positive experience for some of you, but for others it was filled with anxiety and frustration. Why were you learning mathematics and when would you use it? Teachers want students to learn mathematics and to recognise that it is a useful subject. How can they make sure this happens?

This text is designed to expand your vision of mathematics teaching and learning and to help you help students learn, enjoy and engage with mathematics and mathematical thinking. The text interweaves three main themes.

- *Theme 1 — recommendations from national professional organisations and from research on learning and teaching mathematics.* These recommendations and research provide a basis for you to understand what mathematics children are expected to learn and how children learn mathematics.
- *Theme 2 — sense making*. Mathematics must make sense to children. If children make sense of the mathematics they are learning, they can build on this understanding to learn more mathematics and use the mathematics to solve problems.
- *Theme 3 — practical suggestions*. Learning to teach mathematics requires experience. This theme is explicated by including many suggestions from teachers and our own experiences as well as many ideas for you to use as you learn to teach and later when you become a teacher.

Learning to teach is a lifelong journey. During that journey, you will often ask questions such as the following.

- What mathematical knowledge and understandings does each student bring to the class?
- What mathematics do students need to learn?

- How can I teach each unique child so that he or she will learn?
- What is the impact of my own attitude towards mathematics?

Your answers to these questions will influence what you do when you are teaching. No matter what the age of the children you teach, you will probably have several general *goals* — for example:

- to help children make sense of specific mathematical content, including both procedures and concepts
- to help children learn how to apply mathematical ideas to solve problems
- to foster positive dispositions, such as persistence, flexibility, willingness to learn and an appreciation of the value of mathematics.

Developing ways to help you reach these goals is considered in later chapters of this text. This first chapter focuses on what mathematics is and what determines the mathematics that is taught in schools. We also share suggestions as to where you can turn for additional help.

1.1 What is mathematics?

LEARNING OUTCOME 1.1 Connecting your disposition towards mathematics with your mathematical experiences.

Frequently, people equate mathematics with arithmetic. In school mathematics, arithmetic is about numbers. At times, arithmetic, especially computation with numbers, has been the only focus in primary school. Currently, however, this limited view of mathematics is not prevalent. Table 1.1 provides an outline of the Australian Curriculum, Assessment and Reporting Authority's (ACARA) the *Australian Curriculum: Mathematics* content strands and sub-strands.

TABLE 1.1 Australian Curriculum content strands and sub-strands for mathematics

Number and Algebra	Measurement and Geometry	Statistics and Probability
Number and place value (F–8)	Using units of measurement (F–10)	Chance (1–10)
Fractions and decimals (1–6)	Shape (F–7)	Data representation and interpretation (F–10)
Real numbers (7–10)	Geometric reasoning (3–10)	
Money and financial mathematics (1–10)	Location and transformation (F–7)	
Patterns and algebra (F–10)	Pythagoras and trigonometry (9–10)	
Linear and nonlinear relationships (7–10)		

Source: © Australian Curriculum, Assessment and Reporting Authority, 2015a.

The three strands — Number and Algebra, Measurement and Geometry, and Statistics and Probability — are each composed of sub-strands. Most of these sub-strands are emphasised in the early school years. Within the Number and Algebra strand, the focus progresses from simple to more abstract number representations and computations, and then into algebraic relationships. In the Measurement and Geometry strand, a range of basic concepts and skills are developed across the primary school years, with more abstract and complex concepts related to reasoning and trigonometry beginning to be attended to in the middle school years. For the Statistics and Probability sub-strands, the emphasis across all of primary school through to the end of lower secondary school reflects an increased awareness of the importance of these areas of mathematics in daily life.

Although we can consider mathematics as a collection of separate areas such as geometry and algebra, this may not be the best way of looking at it. Charles (2005) considered the big ideas of mathematics as a way of providing opportunities to 'inform teaching and scaffold student learning' (p. 21). His approach has resonated with several researchers, including Siemon, Bleckly, and Neal (2012). Hurst and Hurrell (2014) also saw big ideas as a way to help teachers engage with mathematics themselves and with their students, as it would make the connections within mathematics evident. These approaches consider big ideas rather than focusing on individual subjects of mathematics. They show that it may be helpful to revisit established views of mathematics. The five views that follow give another perspective that may enable you to think of mathematics as something other than a collection of subjects.

1. *Mathematics is a study of patterns and relationships*. Children need to become aware of recurring ideas and of relationships between mathematical ideas. These relationships and ideas provide a unifying thread throughout the curriculum. Children should come to see how one idea is like or unlike other ideas. For example, children in Year 1 can see how one basic fact (say, $3 + 2 = 5$) is related to another basic fact (say, $5 - 3 = 2$). Older children can relate measuring to the nearest centimetre to rounding to the nearest hundred.
2. *Mathematics is a way of thinking*. Mathematics provides people with strategies for organising, analysing and synthesising information. Often symbolising a real-life problem reduces it to a well-known mathematical procedure, making the problem easier to solve.
3. *Mathematics is an art, characterised by order and internal consistency*. Many children think of mathematics as a confusing set of discrete facts and skills that must be memorised. Teachers tend to focus on developing the skills required to 'do' mathematics, and in doing so they may forget that children need guidance to recognise and appreciate the underlying orderliness and consistency as they construct their own understanding of mathematics.
4. *Mathematics is a language that uses carefully defined terms and symbols*. Learning these terms and symbols enhances our ability to communicate about science, real-life situations and mathematics itself. Like any language, you need to understand the meaning of these words and when it is appropriate to use them.
5. *Mathematics is a tool*. Not only do mathematicians use mathematics, but everyone in the course of daily life uses it. Seeing this will help children appreciate why they are learning mathematics. They, too, will be able to use mathematics to solve both abstract and practical problems, just as mathematicians and other people do. Mathematics has become an essential part of our world, both in everyday life and in the workplace.

1.2 What determines the mathematics being taught?

LEARNING OUTCOME 1.2 Recognising factors that determine the mathematics that is currently taught, and the role, rationale and aims of the Australian Curriculum in contemporary mathematics teaching and learning.

Mathematics is highly valued in society by parents, politicians and employers. However, many students do not share this view of mathematics due to a lack of interest and engagement in the subject. Often, this is because students are unable to see the relevance or purpose of the subject in real life or in a meaningful way. The *National Numeracy Review Report* (Council of Australian Governments, 2008, p. xii) suggests that this scenario is widely held and a risk to Australia achieving its human capital goals. Further, due to the high stakes nature of mathematics achievement as a selection and employment criterion, there is a direct relationship between the level of mathematics studied and students' aspirations and career prospects.

In December 2010, ACARA released the *Australian Curriculum: Mathematics* for implementation in all Australian schools by 2013. This marked the first time in Australia's history that there was a national approach to teaching and learning mathematics, from Foundation to Year 10, across all states and territories. However, in their review of the Australian Curriculum, Donnelly and Wiltshire (2014) stated 'the implementation picture is extremely confusing and provides little assurance that the Australian curriculum is being implemented, as intended, across the nation' (p. 106). They proposed several reasons for this, including disagreement between ACARA and state and territory governments, with 'a strident argument from states and territories that implementation is their domain and ACARA is not an accountable body for implementation' (p. 107). Donnelly and Wiltshire did find that the Australian Curriculum documents were being incorporated or supplemented by resources developed by the government body responsible for education in that state or territory.

In England, the national curriculum must be delivered by all schools that are maintained by the local authority (Department for Education [DfE], 2014). Mathematics, together with English and Science, is considered a core subject (DfE, 2014). Teachers are required to 'develop pupils' numeracy and mathematical reasoning in all subjects so that they understand and appreciate the importance of mathematics' (para. 5.2). The DfE created the National Centre for Excellence in the Teaching of Mathematics (NCETM) to enable mathematics teaching and learning to be improved as a means to stop the reduction of students studying mathematics. The NCETM addresses this through the provision of resources, professional learning for teachers, communities of educators, and funded projects (see www.ncetm.org.uk/ncetm/press).

In the United States, the National Council of Teachers of Mathematics (NCTM), the largest professional organisation of teachers of mathematics, developed standards for curriculum and for evaluation, teaching and assessment (NCTM, 1989, 1991, 1995). Because states and localities in the United States have the

right to determine their own school policies, these standards are not prescriptive, but they have provided vision and direction for schools in developing mathematics curricula. In Australia, the national professional body, the Australian Association of Mathematics Teachers (AAMT) has developed several statements and policies pertaining to mathematics learning and teaching. These include *Mathematical Knowledge and Understanding for Effective Participation in Australian Society* (1996a), *Statement on the Use of Calculators and Computers for Mathematics in Australian Schools* (1996b), *Policy on Numeracy Education in Schools* (1998), *Quality Mathematics in the Middle Years Communiqué* (2006a), *Standards for Excellence in Teaching Mathematics in Australian Schools* (2006b), *Position Paper on Early Childhood Mathematics* (2006), *Position Paper on the Practice of Assessing Mathematics Learning* (2017) and the *Position Paper on Consumer and Financial Literacy in Schools* (2012).

In 2000, the NCTM published an update of the standards in a document titled *Principles and Standards for School Mathematics* (NCTM, 2000). The principles represent fundamental beliefs about the characteristics of a high-quality, equitable mathematics program. The standards describe the mathematical content and mathematical processes that should be taught in school mathematics. Combined, the principles and standards present a vision for mathematics education programs in a changing world.

In 2006, the NCTM published *Curriculum Focal Points for Prekindergarten through Grade 8 Mathematics* as one possible response to the question of how to organise curriculum standards within a coherent, focused curriculum by showing how to 'build on important mathematical content and connections identified for each grade level' (NCTM, 2006, p. 3). The tables of focal points and connections are reprinted in appendix B. The rationale, aims and content framework of the *Australian Curriculum: Mathematics* (ACARA, 2015) align with those of the NCTM *Principles and Standards for School Mathematics* (2000). Similarly, the guidelines in the *Position Paper on the Practice of Assessing Mathematics Learning* (AAMT, 2017) are consistent with those of the NCTM.

It is beneficial to consider the curriculum and standards documents developed in Australia, England and the US as this will enable you to develop a greater understanding of the content being addressed. It will also enable you to access resources created for each of these three locations and to use them in your mathematics programs (for example, the Australian website http://dimensions.aamt.edu.au; the UK website NRICH, https://nrich.maths.org/teacher-primary; and the US website NCTM Illuminations, https://illuminations.nctm.org). As you think about mathematics programs, you should give careful attention to three general factors: the needs of the subject, the needs of the child and the needs of society. Consideration of these factors highlights the importance of the guiding principles or the rationale of both the Australian (see figure 1.1) and North American systems. They have many similarities in approach and design.

FIGURE 1.1 Features of the *Australian Curriculum: Mathematics*

<table>
<tr>
<td>Content strands
• Number and Algebra
• Measurement and Geometry
• Statistics and Probability</td>
<td>Proficiency strands
• Understanding
• Fluency
• Problem Solving
• Reasoning</td>
</tr>
<tr>
<td colspan="2">Achievement standards
• Indicate the quality of learning that students should typically demonstrate by a particular point in their schooling</td>
</tr>
<tr>
<td colspan="2">Diversity of learners
• Special education needs
• English as an additional language or dialect
• Gifted and talented</td>
</tr>
<tr>
<td>General capabilities
• Literacy
• Numeracy
• Information and communication technology (ICT) capability
• Critical and creative thinking
• Ethical understanding
• Personal and social capability
• Intercultural understanding</td>
<td>Cross-curriculum priorities
• Aboriginal and Torres Strait Islander histories and cultures
• Asia and Australia's engagement with Asia
• Sustainability</td>
</tr>
</table>

Source: © Australian Curriculum, Assessment and Reporting Authority, 2015b.

Needs of the subject

The nature of mathematics helps determine what is taught and when it is taught in primary school years. For example, whole numbers are the basis for many mathematical ideas; moreover, experiences with whole numbers arise long before children come to school. Building on children's experiences with counting, the earlier years emphasise whole numbers. Work with fractions and decimals logically follows the work with whole numbers. Such seemingly natural sequences are the result of long years of curricular evolution. This process has involved much analysis of what constitutes a progression from easy to difficult, based in part on what is deemed necessary at one level for the development of ideas at later levels. Once a curriculum is in place for a long time, however, people tend to consider it the only proper sequence. Thus, omitting a topic or changing the sequence of topics often involves a struggle for acceptance. However, research shows that all students do not always learn in the sequence that has been ingrained in our curriculum. You need to be open to change so that each child's needs are met.

Mathematics is continually in flux; new mathematics is created, and new uses of mathematics are discovered. As part of this change, technology has seemingly de-emphasised some mathematics and has opened the door for other mathematics. The influence of technology and the importance of a flexible approach to curriculum are emphasised in the *Australian Curriculum: Mathematics* (ACARA, 2015, p. 11) as part of a student's information and communication technology (ICT) capability:

> Students develop ICT capability as they learn to use ICT effectively and appropriately to access, create and communicate information and ideas, solve problems and work collaboratively in all learning areas at school, and in their lives beyond school. ICT capability involves students in learning to make the most of the technologies available to them, adapting to new ways of doing things as technologies evolve.

You will teach at a time when technology dominates activities both in and out of school. Technology will continue to be important in teaching and learning mathematics, as long as it enhances what is being learned and how it is being taught. As you teach your classes, you should keep asking three questions.

1. How can I help children use technology appropriately?
2. What mathematics do children need in order to use technology wisely?
3. What mathematics do children need to know differently because of technology?

Some parents continue to be concerned about the use of calculators in learning mathematics in primary schools. A meta-analysis of 54 research studies on the use of and attitudes towards calculators (Ellington, 2003) suggests that using calculators does not hinder the development of mathematical skills and that students who used calculators had better attitudes towards mathematics than those who did not. Of course, children need to learn to use calculators appropriately, as they do any other tool.

No one knows exactly what mathematics will be needed as the twenty-first century progresses, but it is clear that students will need to know how to reason mathematically and how to apply mathematical thinking to a wide range of situations. How you view mathematics will determine how you view teaching mathematics. If you view mathematics as a collection of facts to learn and procedures to practise, then you will teach that to your students. If you view mathematics as a logical body of knowledge, you will adopt teaching strategies that let you focus on guiding children to make sense of mathematics.

Children need a curriculum that does more than represent mathematics as a collection of isolated skills and fun activities. The Australian Curriculum is designed so that 'links between various components of mathematics, as well as the relationship between mathematics and other disciplines, are made clear' (ACARA, 2012, p. 3).

Throughout the remaining chapters of this text, you will investigate topics that you can incorporate into the mathematics curriculum for your students.

Needs of the child

The mathematics curriculum has been influenced by beliefs about how children learn and, ultimately, about how they should be taught. Before the early years of the twentieth century, mathematics was taught to train 'mental faculties' or provide 'mental discipline'. Struggling with mathematical procedures was thought to exercise the mind (like muscles are exercised), helping children's brains work more effectively. Around the turn of the twentieth century, 'mental discipline' was replaced by *connectionism*, the belief that learning established bonds, or connections, between a stimulus and responses. This led teachers to the endless use of drills aimed at establishing important mathematical connections.

In the 1920s, the Progressive movement advocated *incidental learning*, reflecting the belief that children would learn as much arithmetic as they needed and would learn it better if it was not systematically taught.

The teacher's role was to take advantage of situations as they occurred to teach arithmetic, as well as to create situations in which arithmetic would arise.

During the late 1920s, the Committee of Seven, a committee of school superintendents and principals from midwestern US cities, surveyed pupils to find out when they mastered various topics (Washburne, 1931). Based on that survey, the committee recommended teaching mathematics topics according to students' mental age. For example, subtraction facts under 10 were to be taught to children with a mental age of 6 years 7 months, and facts over 10 at 7 years 8 months; subtraction with borrowing or carrying was to be taught at 8 years 9 months. The recommendations of the Committee of Seven had a strong impact on the sequencing of the curriculum in the United States and elsewhere for years afterward.

Another change in thinking occurred in the mid-1930s, under the influence of *field theory*, or *Gestalt theory*. With William A. Brownell (2006) as a prominent spokesperson, this approach placed greater emphasis on a planned program to encourage the development of insight and the understanding of relationships, structures, patterns, interpretations and principles. It contributed to an increased focus on learning as a process that led to *meaning and understanding*. The value of drill was acknowledged, but it was given less importance than understanding; drill was no longer the major means of providing instruction.

The relative importance of speed and understanding is still debated today. In this debate, people often treat understanding and learning skills as if they were opposites, but this is not the case. Clearly, there is a need to build speed and accuracy and to make skills automatic. However, equally clearly, you need to know *why* as well as *how*. Both skills and understanding must be developed, and they can be developed together. The importance of both of these skills is evident in the *Australian Curriculum: Mathematics* proficiency strands.

Changes in the field of psychology have continued to affect education. During the second half of the twentieth century, educators came to understand that the developmental level of the child is a major factor in determining the sequence of the curriculum. Topics cannot be taught until children are developmentally ready to learn them. Or, from another point of view, topics must be taught in such a way that children at a given developmental level are ready to learn them.

Additionally, educators' attention is being drawn to the increasing evidence that children *construct* their own knowledge. Jean Piaget and Lev Vygotsky are famous theorists in this area. Thus, helping children learn mathematics means being aware of how children construct mathematics from their experiences both in and out of school. Such considerations are now routinely taken into account in developing mathematics curricula.

Finally, the focus on the child has prompted investigations into how each child might position themselves in terms of mathematics. Markku Hannula has researched how emotions and beliefs (often referred to as *affect*) has an impact on mathematics. Jo Boaler built on the growth mindset work of Carol Dweck to develop mathematical mindsets. As a teacher, it might help you engage children with mathematics if you also consider their mathematical affect and their mathematical mindset.

The Australian Curriculum proficiency strands 'describe the actions in which students can engage when learning and using the content' of mathematics and 'indicate the breadth of mathematical actions that teachers can emphasise' (ACARA, 2012, p. 4). Table 1.2 outlines the key student actions that comprise the foci of these strands. These strands should be viewed as interdependent and hence they need to be taught in an interwoven manner.

TABLE 1.2 Student actions as they relate to proficiency strands

Proficiency strand	Key ideas
Understanding	• Developing a robust knowledge of adaptable and transferable mathematical concepts • Making connections between related concepts and applying the familiar to develop new ideas.
Fluency	• Developing skills in choosing appropriate procedures • Carrying out procedures flexibly, accurately, efficiently and appropriately • Recalling factual knowledge and concepts readily
Problem Solving	• Making choices • Interpreting, formulating, modelling and investigating problem situations • Communicating solutions effectively
Reasoning	• Developing a capacity for logical thought and actions, such as analysing, proving, evaluating, explaining, inferring, justifying and generalising

Source: Adapted from © Australian Curriculum, Assessment and Reporting Authority, 2015c.

What it means to learn mathematics has changed a great deal over the past century. In a changing world, learning mathematics with understanding is essential in order to meet the goal of mathematical proficiency. Students must actively build knowledge of mathematics from their personal experiences and prior knowledge. Research has shown that if children are able to make sense of the mathematics they are learning, they can build on this understanding to learn more mathematics and use that mathematics to solve problems in order to become mathematically proficient.

Ideas about developing mathematical proficiency are considered in more depth in the next chapter, but here you can consider some initial important questions. What does it mean to learn mathematics with understanding? How did you learn mathematics? How can you learn to understand what children know? Where can you turn to find out what mathematics children need to learn and how you can challenge and support them in learning that mathematics?

To teach mathematics effectively, teachers must know more than just mathematics. They need to know their students as learners and they must adjust their pedagogical strategies in response to students' varying experiences and dispositions. Teachers must design lessons that reveal to them what students already know, that reveal students' misunderstandings and that guide students to construct more complex understandings of mathematics. Teachers must create challenging and supportive classroom learning environments that help children make sense of mathematics. Teachers must also encourage students to think, question, solve problems and discuss their ideas. 'Effective mathematics teaching requires understanding what students know and need to learn and then challenging and supporting them to learn it well' (NCTM, 2000, p. 16). Teachers can help children make sense of mathematics in many ways. The chapter on planning and teaching initiates the discussion of teaching, and subsequent chapters focus on ways to teach and on useful types of activity.

Needs of society

The usefulness of mathematics in everyday life and in many vocations has also affected what is taught and when it is taught. In earlier times, mathematics was considered necessary primarily for clerks and bookkeepers. The curriculum was limited to counting; the simpler procedures for addition, subtraction and multiplication; and some facts about measures and fractions. By the late nineteenth century, business and commerce had advanced to the point where mathematics was considered important for everyone. The arithmetic curriculum expanded to include such topics as percentage, ratio and proportion, powers, roots and series.

This emphasis on *social utility*, on teaching what was needed for use in occupations, continued into the twentieth century. The proponents of social utility approaches focused on the essential skills and arithmetic used by carpenters, shopkeepers and other workers as the foundation for teaching school mathematics. They thought that school mathematics programs should be limited to teaching only these skills.

The outburst of public concern in the 1950s over the 'space race' resulted in a wave of research and development in mathematics curricula. Much of this effort was focused on teaching the mathematically talented student. By the mid-1960s, however, concern was also being expressed for the disadvantaged student, as society renewed its commitment to equality of opportunity. With each of these changes, more and higher mathematical achievement was promised.

In the 1970s, when it became apparent that the promise of greater achievement had not fully materialised, another swing occurred in curriculum development. Emphasis was again placed on the skills needed for success in the real world. The minimal competency movement stressed the basics. As embodied in sets of objectives and in tests, the basics were considered to be primarily addition, subtraction, multiplication and division with whole numbers and fractions. Thus, the skills needed in colonial times were again being considered by many to be the sole necessities, even though children were now living in a world with calculators, computers and other features of a much more technological society.

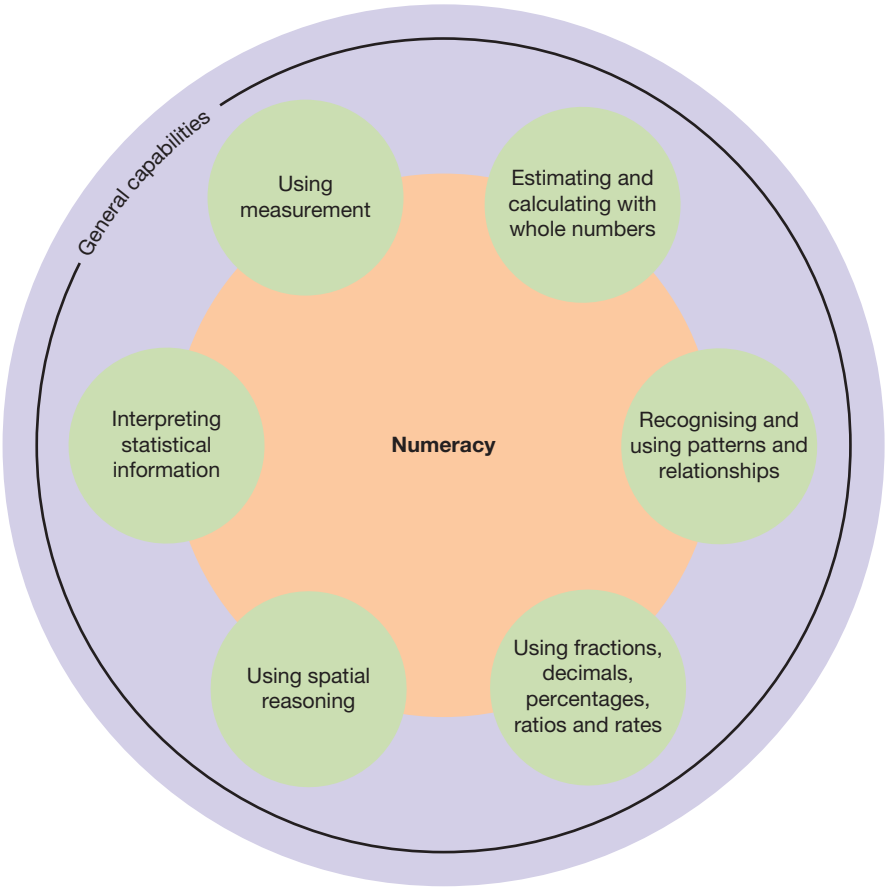
By the 1980s, it was acknowledged that no one knew exactly what skills were needed for the future but that everyone needed to be able to solve problems. The emphasis on problem solving matured through the last 20 years of the century to the point where problem solving was not seen as a separate topic but as a way to learn and to use mathematics. In the 1990s, the standards movement became a major focus. Along with standards came an emphasis on assessment, accountability and equity.

Historically, in Australia each state and territory has taken responsibility for developing a K–12 mathematics curriculum program. More recently, guided by the *Melbourne Declaration on Educational Goals for Young Australians* (Ministerial Council on Education, Employment, Training and Youth Affairs [MCEETYA] 2008), Australia has moved to implementation of a national curriculum from Foundation to Year 12 in specified learning areas.

Mathematics is also considered essential for a competitive society. As the acronym shows, mathematics is an essential element of Science, Technology, Engineering and Mathematics (STEM) and, as such, is inherent in the strong focus on the importance of STEM education and careers (Office of the Chief Scientist, 2014b). The report on Australia's STEM future by the Office of the Chief Scientist (2014a) recommended that opportunities be provided to enable the development of lifelong skills such as quantitative skills and critical thinking skills (p. 23). In a report that provides recommendations for both education sectors and industry to ensure the skills required so that Australia is positioned to be both productive and competitive, the Australian Industry Group [AIG] (2017) reiterate the importance of student participation in STEM and the development of STEM skills.

The Australian Curriculum also highlights the importance of mathematics when identifying numeracy as a general capability (see figure 1.2). A learning continuum for the general capability *numeracy* is provided through six interrelated elements that reflect the key ideas (ACARA, 2015d).

FIGURE 1.2 General capabilities required for Numeracy



Source: Adapted from © Australian Curriculum, Assessment and Reporting Authority 2010 to present.

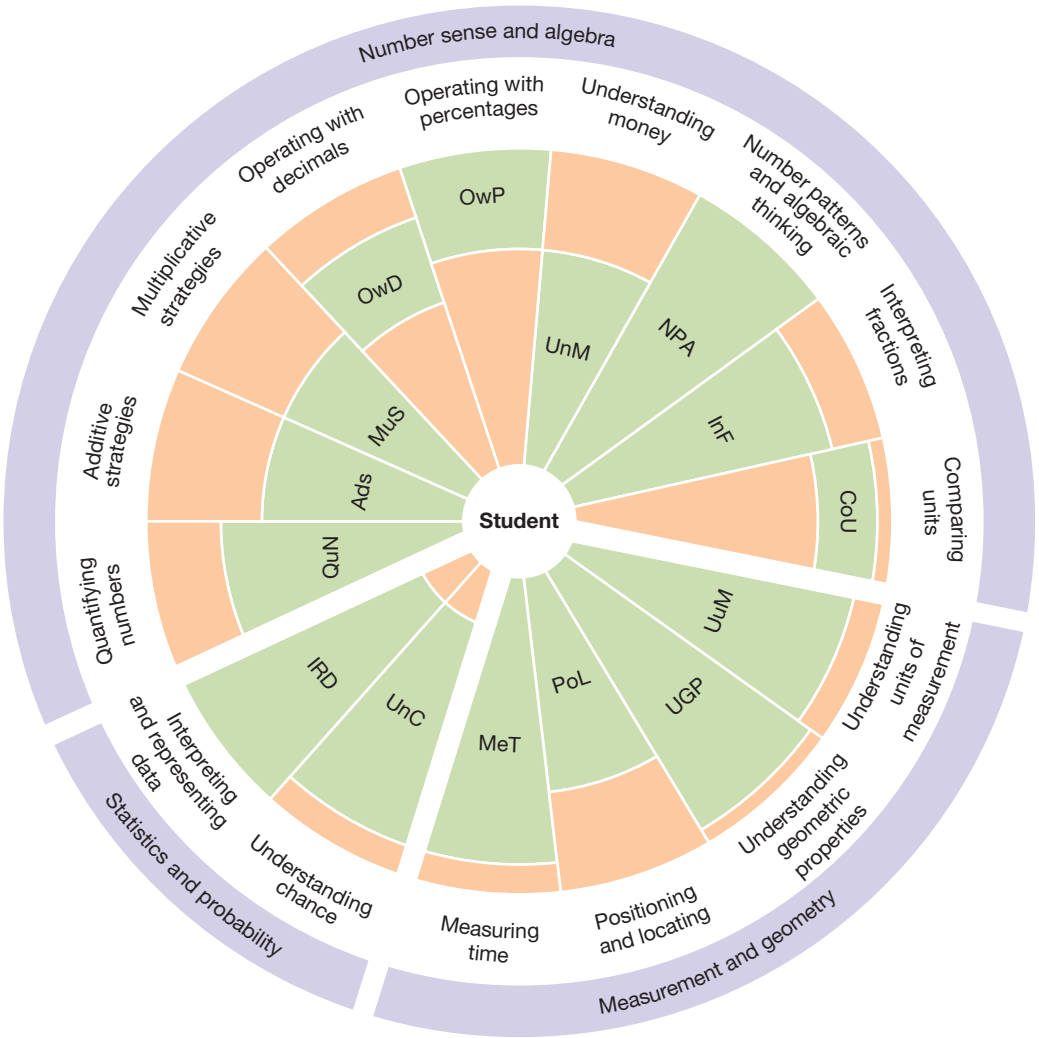
ACARA developed the National Numeracy Learning Progression (www.australiancurriculum.edu.au/resources/national-literacy-and-numeracy-learning-progressions/national-numeracy-learning-progression), and this is connected to the areas of the *Australian Curriculum: Mathematics* (see figure 1.3). Specifically, the three strands of the Mathematics curriculum are also referred to in the progression structure (see www.australiancurriculum.edu.au/resources/national-literacy-and-numeracy-learning-progressions/national-numeracy-learning-progression/how-is-the-numeracy-progression-structured).

Assessment

People often think of assessment as testing to see what students have learned. However, it is much broader in scope. Helping all students learn mathematics requires that assessment be an integral part of the teaching program. Assessment should not be something that is done *to* students; rather, a mathematics program must include assessments that are done *for* students, to guide and enhance their learning (NCTM, 2000).

In order to develop a program that provides for the strengths and needs of individual students, teachers must continue to gather information about their students' progress. Useful and effective assessment is more than a test at the end of a unit. Asking students questions during lessons helps them describe their thinking and understanding; their responses help teachers make the decisions they must make to meet students' needs. Observing students as they solve problems or build ideas provides a different look at student understanding. Ultimately, 'Assessment should become a routine part of the ongoing classroom activity rather than an interruption' (NCTM, 2000, p. 23). (Assessment is considered in more detail in a later chapter.)

FIGURE 1.3 Elements and sub-elements of the National Numeracy Learning Progression



Source: Adapted from © Australian Curriculum, Assessment and Reporting Authority 2010 to present.

Today's society seems focused on assessments in the form of tests that are used to compare students' performance across schools, states and nations. In Australia in July 2006, MCEETYA declared that national testing in literacy and numeracy would proceed. Consequently in 2008, the National Assessment Program — Literacy and Numeracy (NAPLAN) commenced in Australian schools (National Assessment Program [NAP], n.d.). Every year, all students in Years 3, 5, 7 and 9 are assessed on the same days using national tests. NAPLAN results are reported using four scales, one for each of the following domains: Reading, Writing, Language Conventions (spelling, grammar and punctuation) and Numeracy. These reporting scales apply to each of the year levels tested and are accompanied by descriptions of the skills and understandings typical of students at various proficiency levels along each scale. Although NAPLAN has been conducted using paper-based tests, a move to computer-based testing is occurring, with trials in 2016 and the beginning of the transition in 2018 (NAP, 2017).

Accountability

An effort to hold schools accountable for student learning began with the new century. In Australia, the federal government recently adopted *My School* (www.myschool.edu.au) as a reporting and accountability resource for parents. *My School* is a web-based information service developed by ACARA, an independent authority with functions that include the publishing of nationally comparable data on all Australian schools. This responsibility is derived from the *Australian Curriculum, Assessment and Reporting Authority Act 2008* (the ACARA Act) and through the decisions of the Ministerial Council for Education, Early Childhood Development and Youth Affairs (MCEECDYA).

There is also a focus on you as an educator and what the community expects. To demonstrate that pre-service teachers are in the top 30 per cent of the adult Australian population for personal literacy and numeracy, requirements were put in place from 1 July 2016. All of those studying to become a teacher need to complete the Literacy and Numeracy Test for Initial Teacher Education, often referred to as *LANTITE* (Australian Government Department of Education and Training [DET], 2017). In terms of numeracy, DET describes LANTITE as being designed 'to assess the specific personal literacy and numeracy skills required by graduate teachers working in Australian schools' (para. 3). As a pre-service teacher, you will need to ensure you have investigated the format of the tests and the content that is covered (see https://teacheredtest.acer.edu.au). In addition, the Australian Institute for Teaching and School Leadership [AITSL] provides Australian Professional Standards for Teachers (see www.aitsl.edu.au/teach/understand-the-teacher-standards). There are seven standards grouped into three domains of teaching (AITSL, 2011, p. 3).

- *Professional knowledge*
  1. Know students and how they learn.
  2. Know the content and how to teach it.
- *Professional practice*
  3. Plan for and implement effective teaching and learning.
  4. Create and maintain supportive and safe learning environments.
- *Professional engagement*
  5. Assess, provide feedback and report on student learning.
  6. Engage in professional learning.
  7. Engage professionally with colleagues, parents/carers and the community.

Equity

Equity relates to ensuring that all students learn mathematics. The Australian Curriculum (ACARA, 2015, p. 8) emphasises the need to cater for the diversity of learners, recognising that:

> Australian students have multiple, diverse, and changing needs that are shaped by individual learning histories and abilities as well as personal, cultural language backgrounds and socio-economic factors.

This vision can be realised only if each person involved in education firmly believes that all children can learn mathematics and that each child should be expected to do so. Every child must be given the opportunity to learn worthwhile mathematics. This means designing instructional programs that can encompass all the different interests, strengths, needs, cultures and mathematical backgrounds of students. Educational experiences must be culturally relevant for all learners; otherwise there is a risk that not all children will develop mathematical understandings, leading to inequalities in mathematics education (Averill, 2018). Plenty of evidence supports the idea that all students can learn mathematics. High-quality instructional programs are needed that let well-prepared teachers and other school personnel respond to students' varied strengths and needs.

Our schools are characterised by diversity — students from many different cultures and languages; from many different economic and home backgrounds; with many different strengths, ways of learning mathematics and past experiences with mathematics. Equity *does* mean that all children must learn worthwhile mathematics, but it *does not* mean that all should have the same instruction. In fact, it means that children can reach the high expectations set for them only if we meet the individual needs of each child. Your repertoire of ways to reach children will grow as you teach and learn. At this point, you can begin by challenging the popular belief that only some children can learn mathematics. This is an important first step in becoming a teacher who can help every child learn.

Your role

Before reading the previous section, you might have responded with a blank look at the question, 'What determines the mathematics being taught?' Hopefully, this discussion has helped you see that answering this question leads us to consider the needs of the subject, the child and society. These needs have always been considered, but the relative importance assigned to them has changed over time. There is also a greater focus on what you will bring to the role, as evidenced by LANTITE (DET, 2017) and the teaching standards (AITSL, 2011).

Your actions within your role as an educator are influenced by your disposition towards mathematics. Cooke (2015) proposed that disposition towards mathematics incorporated four measurable components — attitudes towards mathematics, mathematics anxiety, confidence with mathematics, and conceptualisation of mathematics. In addition, your mathematical knowledge and understandings need to be sufficiently developed and usable when engaging with children. Your disposition towards and competency with mathematics will impact on what you will do as a teacher in a mathematics lesson.

If your goal is to help each child become mathematically proficient, you will have to consider many factors, some of which you can control and some of which you cannot. Certainly, you can hold high expectations for each child by believing that every child can learn mathematics. Equally certainly, you cannot control whether your students will have to take mandated tests. This text focuses on the things you can control — the things that you can do in the classroom to provide a worthwhile mathematics program that enables each child to develop their mathematical understandings and disposition.

1.3 Where can you turn?

LEARNING OUTCOME 1.3 Evaluating resources to determine those of use in your continuing development of mathematics, mathematical learning and teaching mathematics.

There are many places you can turn to for help with the curriculum, with helping students learn and with continuing to develop your knowledge of mathematics and of mathematics learning and teaching. There are many resources available to support you. In this section, we discuss a few of the resources that we reference throughout this text.

National guidelines for school mathematics

In this chapter, we have briefly outlined the content and proficiency strands of the *Australian Curriculum: Mathematics* (ACARA, 2015a, 2015c). This curriculum document, along with the accompanying website www.australiancurriculum.edu.au, will be a primary source of information for your broad curriculum planning. Its scope and sequence charts will provide details of specific content and sequencing within the content strands and sub-strands.

Professional organisations also have recommendations for school mathematics. Of particular value are the standards and policy statements of the AAMT. These complement both the Australian Curriculum documents and previous and current curriculum documents of the states and territories.

State and local guidelines

Although the Australian education ministers endorsed the *Australian Curriculum: Mathematics* in 2015, the curriculum documents of the Australian states and territories cannot be ignored.

The Australian Curriculum website provides implementation advice for states and territories (www.australiancurriculum.edu.au/f-10-curriculum/implementation-of-the-australian-curriculum). The advice provides information regarding the education authority within each state that is responsible for implementing the curriculum.

Research

Research is referenced throughout this text, not only to acquaint you with research in mathematics education, but also to illustrate or support discussions in the text. There is a substantial body of research in mathematics education, both about children's learning and about teaching. The Mathematics Education Research Group of Australasia (MERGA) (www.merga.net.au) is a leading research organisation in Australia. MERGA publishes two international journals, *Mathematics Education Research Journal* (MERJ) and *Mathematics Teacher Education and Development* (MTED), both of which are excellent

sources of current research in mathematics learning, teaching and assessment, as well as professional learning for maths educators. In addition, MERGA publishes four-year reviews of mathematics education research in Australasia, *Research in Mathematics Education in Australasia* [RiMEA] (see www.merga.net.au/Public/Publications/RiMEA/Public/Publications/RiMEA.aspx?hkey=e35e45af-8f4d-47b6-9ab9-cec66c1a122c). Another relevant source of research information is the *Second Handbook for Research on Mathematics Teaching and Learning* (Lester, 2007).

Research reports and related articles appear in many teacher professional journals, making these journals very valuable resources for effective research-derived mathematics learning and teaching activities. For use in primary school mathematics in Australia, the AAMT publishes *Australian Primary Mathematics Classroom* and *Australian Mathematics Teacher*. In England, the NCETM produces publications addressing mathematics in the early years and primary school (*Primary Magazine*), and secondary- and tertiary-level mathematics (*Secondary Magazine*). In the United States, the NCTM publishes *Teaching Children Mathematics* and *Mathematics Teaching in the Middle School*.

Cultural and international resources

Improving the outcomes in mathematics for Indigenous learners is supported by the *Make it Count* website (https://mic.aamt.edu.au). The website provides papers presented at the *AAMT Special Interest Conference on Numeracy, Mathematics and Indigenous Learners* and other conferences (see https://mic.aamt.edu.au/Resources/Professional-reading). There are also resources, such as *Significant Episodes* (https://mic.aamt.edu.au/Resources/Significant-episodes) and *Units of Learning* that are grouped by early years, primary and middle years (https://mic.aamt.edu.au/Resources/Units-of-learning), that can help inform your work with Indigenous students.

The Programme for International Student Assessment (PISA) is conducted by the Organisation for Economic Cooperation and Development (OECD). The focus is on the skills and knowledge of children aged 15 years of age; specifically, reading, science and mathematics (OECD, n.d.). The testing of the application of skills reflects how mathematics relates to what children who have finished compulsory schooling will take with them into the community (OECD, n.d.). PISA has been conducted every three years from 2000. In the 2015 results, the OECD reported that Australia's mathematics result was higher than the OECD average but had declined since 2006 (www.compareyourcountry.org/pisa/country/AUS?lg=en).

The Trends in Mathematics and Science Study (TIMSS) is an international study that focuses on students in Years 4 and 8. TIMSS is conducted every four years since 1995 by the International Study Centre at the International Association for the Evaluation of Educational Achievement located at Boston University (https://timssandpirls.bc.edu/about.html). In the 2015 study, Australia had the same average achievement as the 2011 study for both Year 4 and Year 8 (Mullis, Martin, Foy, & Hooper, 2016). However, in 2015, whereas Year 4 showed a higher average achievement than in the first study in 1995, Year 8 showed the same average achievement as the first study (Mullis et al., 2016).

The TIMSS also collects information about curriculum, teaching and teachers. In their study of the latest TIMSS Video Study from 1999, Stigler and Hiebert (2004) compared seven countries, including Australia. They found that countries that had higher Year 8 average achievement in mathematics incorporated time in their lessons for students to make mathematical connections by studying the mathematical concepts addressed in the problems they were solving rather than giving the students a procedure to solve the problem.

Many of the other chapters in this text suggest resources that will help you understand how mathematics is taught and learned in other cultures and that show you ways to use culturally oriented activities as you strive to teach each child.

History

Before 1900, arithmetic was taught in later school years, and it then became a primary school subject. You can gain some historical perspective on mathematics education, and have an enjoyable experience at the same time, by looking at mathematics textbooks from the 1800s and 1900s. Read the prefaces of the books, note their lengths and look at the exercises and the activities. Are the activities relevant for today's children in learning mathematics? If you are a student of history, you may want to examine the trends in mathematics education over the decades. How has the emphasis changed and how has it remained the same? Did students spend more time on learning procedures or on problem solving? Were students provided with manipulatives to help them grasp the ideas?

Textbooks and other materials

Many teachers use a textbook when teaching primary and middle school mathematics. Today's textbooks provide supplementary materials such as assessments, problems to solve and extra practice. Teachers' manuals also provide a wealth of materials and teaching suggestions.

There are many different types of textbook. Some provide a lot of drill and practice but offer little help in developing understanding and using mathematics. Others do the opposite: providing a great deal of help in developing students' understanding but falling short on practice of necessary skills. As you gain experience, you will be better able to judge the quality of a textbook and better able to depart from it as appropriate. If you have not had much experience, you may want to stick with the textbook until you become more comfortable with teaching. Be sure to always ask why you need to teach a given lesson. Does it help children develop the mathematics they need? Does it achieve this goal in a way that makes sense to children? Does it help children make sense of the mathematics? Also consider how you implement the question — refer to Stigler and Hiebert's (2004) findings — do the children have the opportunity to explore the underlying concepts or are they given the procedure to solve the problem?

Electronic materials

The kind and quantity of materials readily available are increasing every day. The web provides immediate access to lesson plans, help with the mathematics itself, assessment items and information that can be used in teaching mathematics. Additionally, many sites have videos of mathematics classes. As an educator, you need to learn to make informed and knowledgeable decisions about the quality of the plethora of online resources. Check the source. Usually, '.edu' within the URL would indicate an educational institution. Also, check if the resource has been provided by a reputable source, such as ACARA, AAMT, NCETM or NCTM). Links to a number of useful websites have been included in the resources at the end of each chapter.

Professional organisations

Being a part of a profession opens up many opportunities and imposes many responsibilities. Professional association with others and the support you can find from being a part of a professional organisation will enhance your teaching career. Journals, conferences and other materials of a professional organisation are often available through schools and even to individuals who are not members of the organisation.

The professional journals of the AAMT, MERGA, the NCETM and the NCTM have already been listed for you. These professional organisations also sponsor conferences. The state-based affiliated organisations of the AAMT (see table 1.3) provide local conferences, professional development activities and teaching resources.

TABLE 1.3 **Affiliated associations of the Australian Association of Mathematics Teachers**

State/territory	Mathematics association
New South Wales	Mathematical Association of New South Wales (MANSW), www.mansw.nsw.edu.au
Victoria	Mathematical Association of Victoria (MAV), www.mav.vic.edu.au
South Australia	Mathematical Association of South Australia (MASA), www.masaonline.org.au
Australian Capital Territory	Canberra Mathematical Association (CMA), www.canberramaths.org.au
Western Australia	Mathematical Association of Western Australia (MAWA), https://mawainc.org.au
Northern Territory	Mathematics Teachers Association of the Northern Territory (MTANT), http://mtant.weebly.com
Queensland	Queensland Association of Mathematics Teachers (QAMT), http://qamt.qld.edu.au
Tasmania	Mathematical Association of Tasmania (MAT), https://mat.aamt.edu.au

Professional development

Start taking advantage of professional development opportunities. Some of these will be formal, such as workshops, university courses and conferences; others will include informal study groups. Your school, region or state will provide some opportunities for you. Others will be commercially sponsored or sponsored by a professional organisation. An increasing number of opportunities on the web are designed so you can participate as your schedule permits.

Often, schools or regions have funds set aside for professional development that are available on request. Some districts have teachers design their own professional development plans and support them in carrying out those plans. The importance of professional development is recognised by many organisations. Sometimes these are provided as video resources, such as ACARA (for example, see www.australiancurriculum.edu.au/resources/mathematics-proficiencies) and AITSL (such as www.aitsl.edu.au/tools-resources/resource/2d-shapes-and-their-features-illustration-of-practice) or they are in the form of reports, such as the AIG (2017). NCETM provides self-evaluation tools (available from www.ncetm.org.uk/self-evaluation) to help you investigate your understanding of mathematics and how mathematics is taught.

Other teachers

Teachers learn from each other. You will learn from your school experiences, but do not let learning stop there. Look for schools where sharing ideas about helping students learn mathematics and sharing teaching tips and materials are the norm. Look for teachers in other schools, either near you or far away but connected electronically, who are willing to discuss and to share. A good teacher who is willing to work with you is an invaluable resource.

MAKING CONNECTIONS

As you prepare for teaching mathematics, be sure to think about the broader context of your work and carefully consider these three challenges.

1. *Examine your own disposition towards mathematics and your beliefs about who can learn mathematics*. Be ready to question your beliefs, to evaluate proposed changes and to make a difference in helping children learn mathematics.
2. *Take seriously the title of this text.* Teaching means helping students learn, not merely giving out information. As you begin working with children, stop and listen to them, individually and collectively. Reflect on what you are hearing, and learn with and from the children.
3. *Realise that doing mathematics and teaching mathematics are different*. Teaching mathematics requires a depth of understanding about mathematics (Ma, 1999), about students, about schools, about curriculum and about pedagogy. If you come to this realisation and actively seek knowledge and experiences that integrate these areas, you are well on your way to becoming a good teacher.

A GLANCE AT WHERE WE'VE BEEN

Teaching mathematics in a changing world means that the curriculum and instruction must change to reflect the needs of the subject, the child and society. In this chapter, you have been challenged to consider your view of mathematics as a subject. You have been given a glimpse of the changes in mathematics education through the past as well as recommendations for teaching mathematics in the twenty-first century. The aim and rationale of the *Australian Curriculum: Mathematics* along with the principles put forth by the NCTM underpin many of the recommendations for today. Resources have been identified to support your study throughout the rest of this text and, more importantly, as you teach. The challenge is to keep an open mind and continue your own learning about teaching children mathematics. Prepare to help your students make sense of mathematics.

THINGS TO DO: FROM WHAT YOU'VE READ

1. What are the three general goals mentioned in the introduction? Which do you think is the most important? Explain why.
2. Give an example (different from those in this chapter) of how mathematics is a study of patterns and relationships, a way of thinking, an art and a language.
3. Which of the resources discussed in this chapter have you already used? Which ones do you think will be most helpful to you?
4. Explain in your own words the aims and rationale of the *Australian Curriculum: Mathematics*.
5. In the Australian Curriculum, four proficiency strands are highlighted. Describe the four strands, using specific examples from your personal experiences. Explain how these strands are interwoven and interdependent.
6. Consider the 'big ideas' approach explored by Charles (2005), Siemen et al. (2012) or Hurst and Hurrell (2014). How could you apply big ideas to engage students with mathematics, develop their mathematical thinking and help with their mathematical sense-making?

THINGS TO DO: GOING BEYOND THIS TEXT

IN THE FIELD

1. *Links between mathematics at home and mathematics at school.* What does mathematics look like at school? Consider how the mathematics might link back to the experiences children bring with them to the classroom. What might the teacher do to make connections between the mathematical knowledge and understandings children have from home and the mathematical experiences created at school?
2. *Equity*. What does equity look like in the classroom? How might the ideas raised by the *Australian Curriculum: Mathematics* be enacted? How does focusing on equity in a mathematical experience look different to focusing on equality?

IN YOUR JOURNAL

3 The Australian Curriculum recommends that technology should support effective mathematics teaching. Write a statement of your experience in using calculators and describe your philosophy regarding calculators in learning primary mathematics.

4 Several educators have noted that the curriculum is in a continuous process of change in order to maintain balance as the needs of the subject, the child and society pull it first one way and then another. Discuss this comment.

5 React to the three challenges presented in the close of the chapter.

WITH ADDITIONAL RESOURCES

6 Describe the four strands of mathematical proficiency and how they are intertwined and interdependent. What practices are needed if all children are to become mathematically proficient?

7 Find a recent issue of *Australian Primary Mathematics Classroom* (*APMC*) or *Australian Mathematics Teacher* (*AMT*). Select an article that describes a classroom application. Describe one or more of the principles that are considered in the recommendations for this classroom application.

8 Access the resources available from the *Australian Curriculum: Mathematics* (www.australiancurriculum.edu.au/resources/student-diversity). Consider how can you ensure children with diverse needs have the opportunity to learn in your classroom and outline what this might look like in your classroom.

WITH TECHNOLOGY

9 Access the *My School* website (www.myschool.edu.au). After looking through the website, identify how your perspective as a pre-service teacher may have an impact on what you reviewed. Now, consider what might be focused on if you are a teacher or a parent/guardian — how do these perspectives change what is viewed and how it is viewed? Does focusing on mathematics change the similarities and differences of these perspectives?

10 There are many websites that provide mathematical activities, but you need to be selective in using them. Have a look at three — www.connectwith.earlyyears.aamt.edu.au from Australia, www.illuminations.nctm.org from the US, and www.nrich.maths.org/frontpage from the UK. Explore the websites and then compare what each offers. Consider how you might use the resources from these websites when you teach.

11 Go to the National Literacy and Numeracy Week in NSW website (www.nlnw.nsw.edu.au/numvideos.htm) and select a video that corresponds to a mathematical area you might have struggled with at some time (for example, decimals or fractions). How might the points raised in the video connect to your difficulties?

12 Consider what your mathematical understandings are for teaching mathematics. In literature, these are often referred to using specific terminology, such as mathematical content knowledge. The NCETM provides self-evaluation tools on their website (www.ncetm.org.uk/self-evaluation). Experiment with the various ones available.

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ACKNOWLEDGEMENTS

Adapting author for this Australian edition: Audrey Cooke
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