

11TH EDITION

HELPING CHILDREN LEARN MATHEMATICS

ROBERT REYS
University of Missouri

MARY M. LINDQUIST
Columbus State University

DIANA V. LAMBDIN
Indiana University

NANCY L. SMITH
Emporia State University

WILEY



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About the Authors

Robert E. Reys is Curators' Professor Emeritus of Mathematics Education at the University of Missouri-Columbia, where he teaches courses for elementary, middle, and secondary mathematics teachers. He is a former mathematics teacher and district mathematics coordinator. His research interests are in the areas of calculators, mental computation, estimation, and number sense. Bob has authored over 220 articles in professional journals. He was General Editor for five yearbooks of the National Council of Teachers of Mathematics, and co-edited the 2010 NCTM Yearbook *Mathematics Curriculum: Issues, Trends, and Future Directions*. He was awarded the Lifetime Achievement Award for his service, leadership, and research by the Mathematics Education Trust of the National Council of Teachers of Mathematics.

Mary M. Lindquist is Fuller E. Callaway Professor of Mathematics Education, Emeritus at Columbus State University (Georgia). She taught undergraduate and graduate students in early childhood, middle grades, and secondary mathematics education. Mary was president of the National Council of Teachers of Mathematics and chair of the Commission on the Future of the Standards, the oversight committee for *Principles and Standards for School Mathematics* (NCTM, 2000). She has served or is serving on many national and international committees, including those involved with the National Assessment of Education Progress (NAEP) and the Trends in International Mathematics and Science Study (TIMSS). She was awarded the Lifetime Achievement Award for her service, leadership, and research by the Mathematics Education Trust.

Diana V. Lambdin is Armstrong Professor of Teacher Education Emerita and Professor of Mathematics Education Emerita at Indiana University in Bloomington, where she taught courses for prospective elementary teachers, supervised students' field experience, worked with masters and doctoral students in mathematics education, and served as Associate Dean for Teacher Education. Prior to entering the field of teacher education, she was a mathematics teacher in Massachusetts, Michigan, and Iowa. Diana has been active as an author, editor, project evaluator, and leader in the National Council of Teachers of Mathematics (NCTM). She was a member of the writing team for *Principles and Standards for School Mathematics* (NCTM, 2000) and served as a member of NCTM's Board of Directors from 2009 to 2012.

Nancy L. Smith has been an educator for over 30 years. She taught elementary school and middle school mathematics for ten years in Richmond, Missouri. She is currently Professor in the Department of Elementary Education/Early Childhood/Special Education at Emporia State University in Emporia, Kansas, where she teaches elementary mathematics education courses for preservice and inservice teachers, teaches general elementary education courses, and supervises student teachers in the Olathe Professional Development Schools.

Preface

Welcome to the 11th edition of *Helping Children Learn Mathematics*. Earlier editions have helped several generations of elementary teachers. We are grateful to the many instructors and students who have written us with ideas, stories, and suggestions about ways of helping children learn mathematics. This edition of *Helping Children Learn Mathematics* reflects the ever changing world of learning and teaching elementary school mathematics. You will experience many changes and have the opportunity to shape their direction.

Helping Children Learn Mathematics is built around three main themes:

- helping children make sense of mathematics
- incorporating practical experiences
- using research to guide teaching

Additionally, throughout the book you will also find connections and implications from the Common Core State Standards: Mathematics (CCSS-M).

Equity and diversity are key issues addressed throughout the book. Practical advice and suggestions about ways to help treat children equally and fairly are provided. Elementary teachers must strive to insure equity in their classrooms, and this becomes more challenging as the diversity within many classrooms grows.

This book is intended for those of you who are or who will be teachers of mathematics in elementary school. It is designed to help you help children develop understanding and proficiency with mathematics so they can solve problems. We hope our book will challenge your thinking and stimulate your interest in learning and teaching mathematics.

Helping Children Learn Mathematics consists of two main parts. The first part (Chapters 1–6) provides a basis for understanding the changing mathematics curriculum and how children learn mathematics. These chapters offer guidelines for planning, instructing, and evaluating that are aligned with the Mathematical Practices from the CCSS-M and include reference to the *Principles to Action: Ensuring Mathematical Success for All* (NCTM, 2014). Particular attention is given to problem solving and assessment, both of which have profound implications for mathematics teaching. Their importance is reflected by the integration of problem solving and assessment throughout the book.

The second part (Chapters 7–18) discusses teaching strategies and techniques, as well as learning activities related to specific mathematical topics. The emphasis is on making mathematics make sense for our diverse population of students.

FEATURES OF THIS TEXT

Helping Children Learn Mathematics reflects current recommendations from professional associations, as well as recent research findings relevant to teaching mathematics. We have maintained the characteristics and features that have made this book a popular choice of instructors for many years. A useable text for instructors, it is also readable and understandable by students who are being introduced to teaching elementary mathematics. Yet, its depth also makes it appropriate for teachers to use as they continue to learn about teaching.

NEW Snapshots of a Lesson reflect video clips. These videos are new and drawn from the Teaching Channel. Each snapshot is aligned with the CCSS-M and is easily accessed online. Each chapter-opening snapshot is an excerpt from an actual mathematics lesson or mathematical situation. These videos demonstrate many effective classroom practices and provide a smooth, practical segue into the body of the chapter. You'll see the snapshots called out with this camera icon: 

NEW Reflecting on the Snapshot Lesson provides questions to stimulate discussion about the Snapshot of a Lesson. Answering these questions will be a good preparation for licensure tests.

NEW Common Errors in Chapters 7–18 illustrate errors that children or teachers often make. Both research and experiences in working with children reveal frequent errors and misconceptions. We have identified errors that are often predictable and highlighted them in Common Error boxes. This feature is designed to stimulate your thinking as you strive to help children make sense of mathematics.

NEW Tech Connect. The Tech Connect boxes highlight a variety of technology resources, including apps for tablets and smartphones, and URLs that link to information aligned with the content being discussed.

NEW Learning Outcomes are identified early in each chapter and address major issues within each chapter. While these learning outcomes are not intended to be exhaustive, they do provide some key questions to guide reading and promote discussions.

Chapter on Fractions. Research has shown the importance of children having a conceptual understanding of fractions as well as fluency with computation. A chapter in this edition is

devoted entirely to common fractions. Decimals have been moved to the place value chapter.

In the Classroom activities are found throughout the text in special boxes. These activities provide a wealth of ideas and strategies for helping children explore, learn, and in some cases practice skills related to specific mathematical topics. Lesson plans for most of these activities can be easily found in the companion book, *Teaching Elementary Mathematics: A Resource for Field Experiences*.

Children's Literature. Literature can be a powerful ally in helping children learn. We have made a concerted effort to identify books that you can use effectively to stimulate children's interest in mathematics and to promote mathematics learning. We cite and discuss specific books at various points within the text, and we provide an updated annotated list of useful books in the **Book Nook** section at the end of each chapter (except Chapter 1).

Updated and Streamlined Research. Updated references to important research are found throughout the text. As a teacher, you are often called on to provide a rationale for curricular or instructional decisions, and we think you will find these references to relevant research useful; much of the research we cite has important implications for learning and teaching. Complete reference information for each citation is available on the book companion site at www.wiley.com/college/reys.

Things to Do sections found near the end of each chapter embody our active learning and teaching approach to mathematics. Divided into two parts, the Things to Do sections are designed to engage you in inquiring and thinking about mathematics—to offer you experiences and introduce you to investigations that will help you achieve the understanding and insight you need to be a successful teacher. The first part, “From What You’ve Read,” relates directly to this book. The second part, “Going Beyond This Book,” generally offers four types of activities: In the field, In your journal, With additional resources, and With technology. These include several activities linked to *Teaching Elementary Mathematics: A Resource for Field Experiences*.

WILEY E-TEXT

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The Wiley E-Text: Powered by VitalSource gives students anytime, anywhere, access to the best math methods teaching content when and where they study: on their desktop, laptop, tablet, or smartphone. Students can search across content, highlight, and take notes that they can share with teachers and classmates.

This exciting new learning model brings textbook pages to life—no longer just a static e-book, the E-Text enriches the study experience with dynamic features:

- Pre- and Post-chapter quizzes
- *Teaching Channel* video introductions to each chapter

- Integrated *Teaching Elementary Mathematics: A Resource for Field Experiences* activities
- Access to student study tools, including Lecture PowerPoints and Blackline Masters

TEACHING AND LEARNING RESOURCES

Teaching Elementary Mathematics: A Resource for Field Experiences, 4th Edition

Featuring a wealth of materials to use in schools, this resource is designed for students to use when they participate in field experiences such as practicums, observations, and professional development school (PDS) experiences in K–8 classrooms. It is aligned with *Helping Children Learn Mathematics* and provides:

- **In the School** activities focus on observing and gathering information about the school and its resources. Resources for teacher and student interviews are included.
- **Helping Children Learn** activities include mini lessons for small groups or whole class, that include games, technology, and In the Activities in this *Resource* provide opportunities in collecting information about the school and school resources, observing and interviewing teachers and children, and doing mathematics with children in the form of mini-lessons, some involving games and technology. Pages from this *Resource* may be downloaded and printed for classroom use.

NEW Appendix and References

The Appendix and References are available on the book companion site at www.wiley.com/college/reys.

SUPPLEMENTS

The eleventh edition of *Helping Children Learn Mathematics* is accompanied by the following instructor and student supplements:

NEW Instructor's Manual

The Instructor's Manual is a useful resource for both veteran and new elementary methods instructors. It is available on the Wiley Web site at www.wiley.com/college/reys. Each chapter includes:

- Chapter overview, student objectives, and key vocabulary
- Supplemental lecture ideas, textbook extension ideas, and class and field activity suggestions
- Extensive resource list (both print and media)

Pre- & Post-Chapter Test

Objective questions are available online. These short quizzes may be helpful for focusing learning and reflecting on the content of each chapter. (More in-depth questions are available at the end of each chapter.)

Test Bank

Password protected in the instructor's section of www.wiley.com/college/reys, a test bank of over 500 items features both objective and open-ended questions with varying levels of complexity.

PowerPoint Slides

Our PowerPoint™ presentations contain a combination of key concepts allowing you to illustrate important topics.

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We thank Marilyn Suydam for her hard work and insights during the early years of this text's development. Marilyn is now retired, but her legacy of significant contributions remains a vital part of this book.

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 MARTHA EGGERS, McKendree University
 DIANNE ERICKSON, Oregon State University
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 CLAIRE GRAHAM, Framingham State College
 ANDREA GUILLAUME, California State University
 JANET HANDLER, Mount Mercy College
 KIM HARRIS, University of North Carolina-Charlotte
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 ALICE MILLS, Quincy University
 JEAN MITCHELL, California State University-Monterey Bay
 EULA EWING MONROE, Brigham Young University
 SARAH MURRAY, Centre College
 MARGARET (MAGGIE) NIESS, Oregon State University
 JAMAR PICKREIGN, Rhode Island College
 DON PLOGER, Florida Atlantic University
 SARA POWELL, University of Charleston (SC)

FRANK POWERS, University of Idaho
JACELYN MARIE REES, McNeese State University
GAY RAGAN, Southwest Missouri State University
DENISE M. REBOLI, King's College (PA)
ANDY REEVES, University of South Florida-Tampa
BARBARA RIDENER, Florida Atlantic University
CANDICE L. RIDLON, Towson University
TOM ROMBERG, University of Wisconsin
THOMAS E. ROWAN, University of Maryland, College Park
MARY ELLEN SCHMIDT, Ohio State University-Mansfield
LINDA SHEERAN, Oklahoma State University
JASON SILVERMAN, Drexel University
TINA SLOAN, Athens State University
MARIAN SMITH, Florida A&M University
Marilyn Soucie,
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FRANCES STERN, New York University

DAVID L. STOUT, University of West Florida
JANE STRAWHECKER, University of Nebraska at Kearney
GERTRUDE R. TOHER, Hofstra University
FREDERICK L. UY, California State University, Los Angeles
JUAN VAZQUEZ, Missouri Southern State College
ELSA VILLA, The University of Texas at El Paso
BETH VINSON, Athens State
KAY WALL, University of Central Oklahoma
PAT WALL, Northern Arizona University
JUDY WELLS, Indiana State University
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