1

Introduction and Basic Principles

1.1 Introduction

Wind and water flows played an important role in the evolution of our civilization and provided inspiration in early agriculture, transportation, and even power generation. Ancient ship builders and architects of the land all respected the forces of nature and tried to utilize nature's potential. At the onset of the industrial revolution, as early as the nineteenth century, motorized vehicles appeared and considerations for improved efficiency drove the need to better understand the mechanics of fluid flow. Parallel to that progress the mathematical aspects and the governing equations, called the **Navier–Stokes** (NS) equations, were established (by the mid-1800s) but analytic solutions didn't follow immediately. The reason of course is the complexity of these nonlinear partial differential equations that have no closed form analytical solution (for an arbitrary case). Consequently, the science of fluid mechanics has focused on simplifying this complex mathematical model and on providing partial solutions for more restricted conditions. This explains why the term fluid mechanics (or dynamics) is used first and not aerodynamics. The reason is that by neglecting lower-order terms in the complex NS equations, simplified solutions can be obtained, which still preserve the dominant physical effects. Aerodynamics therefore is an excellent example for generating useful engineering solutions via "simple" models that were responsible for the huge progress in vehicle development both on the ground and in the air. By focusing on automobile aerodynamics, the problem is simplified even more and we can consider the air as incompressible, contrary to airplanes flying at supersonic speeds.

At this point one must remember the enormous development of computational power in the twenty-first century, which made numerical solution of the fluid mechanic equations a reality. However, in spite of these advances, elements of modeling are still used in those

Automotive Aerodynamics, First Edition. Joseph Katz.

solutions and the **understanding** of the “classical” but limited models is essential to successfully use those modern tools.

Prior to discussing the airflow over vehicles, some basic definitions, the engineering units to be used, and the properties of air and other fluids must be revisited. After this short introduction, the fluid dynamic equations will be discussed and the field of aerodynamic will be better defined.

1.2 Aerodynamics as a Subset of Fluid Dynamics

The science of fluid mechanics is neither really new nor biblical; although most of the progress in this field was made in the latest century. Therefore, it is appropriate to open this text with a brief history of the discipline with only a very few names mentioned.

As far as we could document history, fluid dynamics and related engineering was always an integral part of human evolution. Ancient civilizations built ships, sails, irrigation systems, or flood management structures, all requiring some basic understanding of fluid flow. Perhaps the best known early scientist in this field is Archimedes of Syracuse (287–212 BC), founder of the field now we call “fluid statics”, whose laws on buoyancy and flotation are used to this day.

Major progress in the understanding of fluid mechanics begun with the European Renaissance of the fourteenth to seventeenth centuries. The famous Italian painter sculptor, Leonardo da Vinci (1452–1519) was one of the first to document basic laws such as the conservation of mass. He sketched complex flow fields, suggested viable configuration for airplanes, parachutes, or even helicopters, and introduced the principle of streamlining to reduce drag.

During the next couple of hundred years, sciences gradually developed and then suddenly were accelerated by the rational mathematical approach of Englishman, Sir Isaac Newton (1642–1727) to physics. Apart from his basic laws of mechanics, and particularly the second law connecting acceleration with force, Newton developed the concept for drag and shear in a moving fluid, principles widely used today.

The foundations of fluid mechanics really crystallized in the eighteenth century. One of the more famous scientists, Daniel Bernoulli (1700–1782, Dutch-Swiss) pointed out the relation between velocity and pressure in a moving fluid, the equation of which bears his name in every textbook. However, his friend Leonhard Euler (1707–1783, Swiss born), a real giant in this field is the one actually formulating the Bernoulli equations in the form known today. In addition Euler, using Newton’s principles, developed the continuity and momentum equations for fluid flow. These differential equations, the Euler equations are the basis for modern fluid dynamics and perhaps the most significant contribution in the process of understanding fluid flows. Although Euler derived the mathematical formulation, he didn’t provide solution to his equations.

Science and experimentation in the field advanced but only in the next century were the governing equations finalized in the form known today. Frenchman, Claude-Louis-Marie-Henri Navier (1785–1836) understood that friction in a flowing fluid must be added to the force balance. He incorporated these terms into the Euler equations, and published the first version of the complete set of equation in 1822. These equations are known today as the Navier–Stokes equations. Communications and information transfer weren't well developed those days. For example, Sir George Gabriel Stokes (1819–1903) lived at the English side of the Channel but didn't communicate directly with Navier. Independently, he also added the viscosity term to the Euler equations, hence the shared glory by naming the equations after both scientists. Stokes can be also considered as the first to solve the equations for the motion of a sphere in a viscous flow, which is now called **Stokes flow**.

Although the theoretical basis for the governing equation was laid down by now, it was clear that the solution is far from reach and therefore scientists focused on "approximate models" using only portions of the equation, which can be solved. Experimental fluid mechanics also gained momentum, with important discoveries by Englishman Osborne Reynolds (1842–1912) about turbulence and transition from laminar to turbulent flow. This brings us to the twentieth century, when science and technology grew at an explosive rate, particularly, after the first powered flight of the Wright brothers in the US (Dec 1903). Fluid mechanics attracted not only the greatest talent but also investments from governments, as the potential of flying machines was recognized. If we mention one name per century then Ludwig Prandtl (1874–1953) of Gottingen Germany deserves the glory. He made tremendous progress in developing simple models for problems such as boundary layers and airplane wings.

The efforts of Prandtl lead to the initial definition of **aerodynamics**. His assumptions usually considered low-speed airflow as incompressible, an assumption leading to significant simplifications (as will be explained in Chapter 4). Also, in most cases the effects of viscosity were considered to be confined into a thin boundary layer, so that the viscous flow terms were neglected. These two major simplifications allowed the development of (aerodynamic) models that could be solved analytically and eventually compared well with experimental results!

This trend of solving models and not the complex Navier–Stokes equations continued well into the mid-1990s, until the tremendous growth in computer power finally allowed numerical solution of these equations. Physical modeling is still required but the numerical approach allows the solution of nonlinear partial differential equations, an impossible task from the pure analytical point of view. Nowadays, the flow over complex shapes and the resulting forces can be computed by commercial computer codes but without being exposed to simple models our ability to analyze the results would be incomplete.

1.3 Dimensions and Units

The magnitude (or dimensions) of physical variables is expressed using engineering units. In this text we shall follow the metric system, which was accepted by most professional societies in the mid-1970s. This international system of units (SI) is based on the decimal system and is much easier to use than other (e.g., British) systems of units. For example, the basic length is measured by meters (m) and 1000 m is called a kilometer (km) or 1/100 of a meter is a centimeter. Along the same line 1/1000 m is a millimeter.

Mass is measured in grams, which is the weight of one cubic centimeter of water. One thousand grams are one kilogram (kg) and 1000 kg is one metric ton. Time is still measured the old fashion way, by hours (h) and 1/60th of an hour is a minute (min), while 1/60 of a minute is a second (s).

For the present text velocity is one of the most important variables and its basic measure therefore is m/s. Vehicles speed are usually measured in km/h and clearly 1 km/h = 1000/3600 = 1/3.6 m/s Acceleration is the rate of change of velocity and therefore it is measured by m/s^2.

Newton's Second Law defines the units for the force F, when a mass m is accelerated at a rate of a

$$F = ma = \mathrm{kg} \cdot \frac{\mathrm{m}}{\mathrm{sec}^2}$$

Therefore, this unit is called Newton (N). Sometimes the unit **kilogram-force** is used (kgf) since the gravitational pull of 1 kg mass at sea level is 1 kgf. If we approximate the gravitational acceleration as g = 9.8 m/s^2, then

$$1\,\mathrm{kgf} = 9.8\,\mathrm{N}$$

The pressure, which is the force per unit area is measured using the previous units

$$p = \frac{F}{S} = \frac{\mathrm{kg} \cdot \dfrac{\mathrm{m}}{\mathrm{sec}^2}}{\mathrm{m}^2} = \frac{\mathrm{N}}{\mathrm{m}^2} = 1\ \mathrm{Pascal}$$

and this unit is called after the French scientist Blaise Pascal (1623–1662). Sometimes atmosphere (atm) is used to measure pressure and this unit is about 1 kgf/cm^2, or more accurately

$$1\,\mathrm{atm} = 1.013 \cdot 10^5\,\mathrm{N/m^2}.$$

There are a large number of engineering units and a list of the most common ones is provided in Appendix A.

The definition of engineering quantities, such as forces or pressures, requires the selection of a coordinate system. In this text, the preferred system is the Cartesian (named after the seventeenth century mathematician Rene Cartesius) shown in Fig. 1.1. The cylindrical

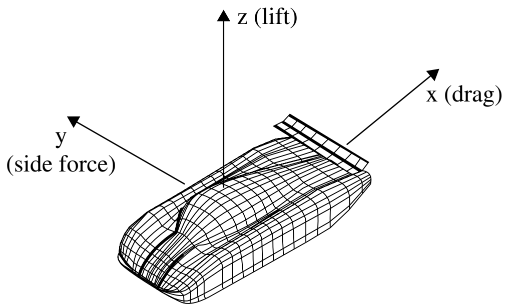


Figure 1.1 Cartesian coordinate system and its definition relative to an automobile

coordinates system (r, θ, x) will be used only when the problem formulation becomes significantly simpler.

In the next section, some examples are presented demonstrating the relevance of aerodynamics to vehicle design. The discussion that follows lists some of the more important properties of air and other fluids, along with the units used to quantify them.

1.4 Automobile/Vehicle Aerodynamics

Ask any fluid/aerodynamicist and he will tell you that "everything" is related to this science; weather, ocean flows, human organs such as the heart or lungs, or even the flow of concrete and metals. So if this science is so important there is nothing more rewarding than to study and explore its principle on an object close to all of us; the automobile. This will not deprive the discussion because all elements of fluid mechanics are included. Therefore, this prelude provides a comprehensive foundation for more advanced coursework the student may later take, focusing on more specific topics.

Returning to automobiles, one must remember that aerodynamics relates to ventilation/AC, engine in-and-out flows, brake cooling, and resulting forces on the vehicle. To demonstrate the effect of aerodynamics on vehicles, let us start with a simple example; the drag (force resisting the motion), which also affects the shape and styling of modern vehicles. The forces that a moving vehicle must overcome increase with speed, and the tire rolling resistance and driveline friction effects are shown in Fig. 1.2, along with the total force resisting the motion (indicating the significance of aerodynamic drag).

From the early twentieth century, both fuel cost and vehicle speeds gradually increased and the importance of aerodynamic drag reduction, based on Fig. 1.2 is obvious. A careful

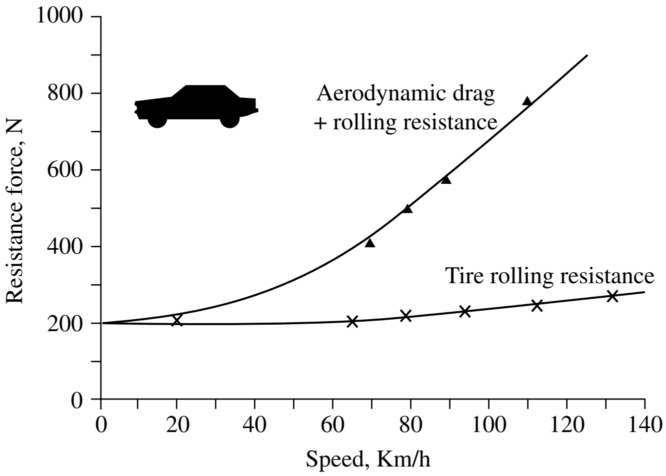


Figure 1.2 Increase of vehicle total drag and tires rolling resistance on a horizontal surface, versus speed (measured in a tow test of a 1970 Opel Record)

examination of the data in this figure reveals that the aerodynamic drag increases with the square of the velocity while all other components of drag change only marginally. Therefore, engineers devised a **nondimensional** number, called the drag coefficient (C_D), which quantifies the aerodynamic sleekness of the vehicle configuration. One of the major advantages of this approach is that scaling (e.g., changing the size) is quite simple. The definition of the drag coefficient is:

$$C_D = \frac{D}{0.5\rho U^2 S} \tag{1.1}$$

where D is the drag force, ρ is the density, U is vehicle speed, and S is the frontal area. Later, we shall see that the denominator represent a useful, widely used quantity. Now suppose that some manufacturer decides to reduce its vehicle dimensions by 10% and asks his engineers to estimate the fuel saving:

Example 1.1

A passenger car has a drag coefficient of 0.4 and management propose to reduce all dimensions by 10%. Apart from the weight saving, how much can be saved, based on the aerodynamic considerations?

Assuming that fuel consumption is related to the power (P), which is force (D) times velocity (U) we can write:

$$P = D \cdot U = C_D 0.5\rho U^2 S \cdot U$$

The scaling enters this formula via the frontal area S, which is now smaller by $0.9 \cdot 09$ (=0.81). So if vehicle shape is unchanged then the power for the 10% smaller vehicle will be:

$$P = D \cdot U = 0.81 \cdot C_D \left(0.5\rho U^3 S\right)$$

So at a specific speed, saving is estimated at 19%. Also note that power requirements increase with U^3.This simple example shows that by focusing on vehicle drag reduction, significant fuel savings can be achieved. Drag reduction trends over recent years are shown in Fig. 1.3, an overall trend that was probably driven by the increasing cost of fuel (and the environmental emission control of recent years).

Figure 1.3 also provides the range of practical drag coefficients, which could start as high as $C_D = 1.0$, but in recent years most manufacturers hope to cross the $C_D = 0.3$ "barrier". The trends of styling changes are hinted by the small sketches, and modern cars have smooth surfaces and utilize all available "practical" tricks to reduce drag (we can learn about this later). Also, two extreme examples were presented in this figure. First, the streamlined shape at the lower left part of the figure, which indicates that a $C_D \sim 0.15$ is possible. Furthermore, the placing of this shape indicates that engineers new early how to reduce drag but automobile designs were mostly driven by artistic considerations (not so in the twenty-first century). Just to prove this anecdotal point, Fig. 1.4 shows the 1924 Tropfenwagen (droplet-shaped

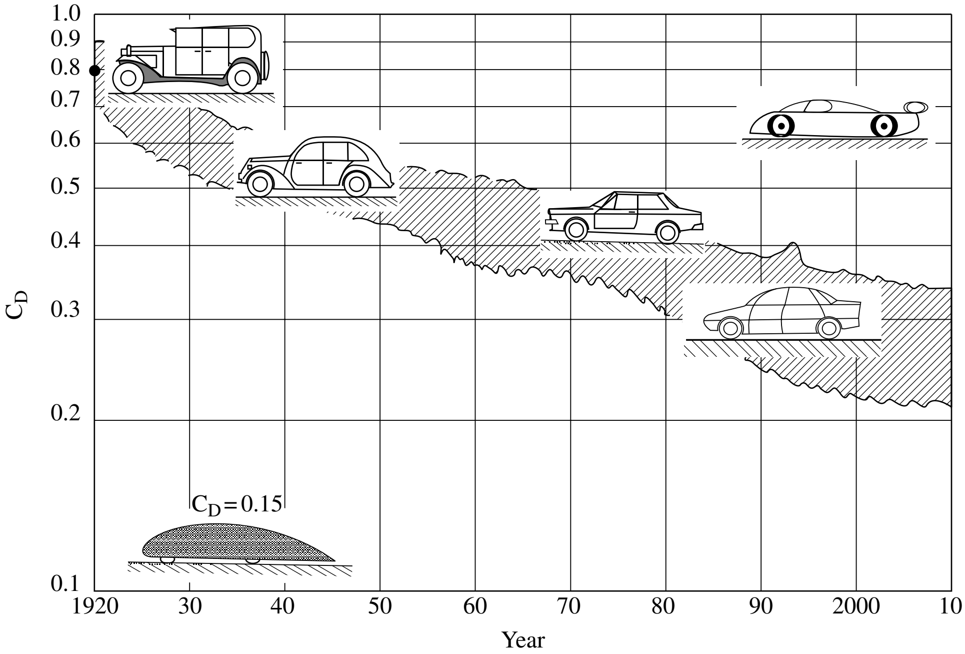


Figure 1.3 Schematic representation of the historic trends in the aerodynamic drag of Passenger cars

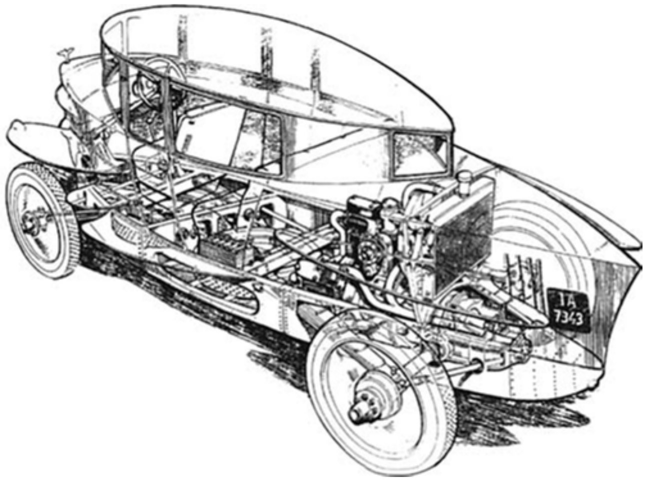


Figure 1.4 The 1924 Tropfenwagen, which had a better drag coefficient ($C_D = 0.28$) than most modern cars. Illustration by Brian Hatton

car, in German), designed by E. Rumpler. Both the vehicle body and its cabin had a teardrop shape, with the objective of reducing aerodynamic drag.

By the way, the original Tropfenwagen automobile residing in the German Museum in Munich, was tested in the VW AG wind tunnel in 1979. The measured drag coefficient surpassed most modern cars and was found at $C_D = 0.28$. This car also featured a mid-engine layout, which was reinvented in the 1960s by racecar engineers, but in the 1920s the design was too much for the traditional automobile buyer and resulted in commercial failure.

Let us now return to the second extreme example at the top right-hand side of Fig.1.3, representing the high drag of most modern racecars. This observation sounds contradictory to the purpose of racing fast and is the result of generating a force called "aerodynamic downforce", pushing the car to the ground. Because most races involve high-speed cornering and acceleration, increasing tire adhesion (using aerodynamic downforce) results in faster cornering, and in improved braking and acceleration. Of course top speed is compromised but overall vehicles utilizing downforce are not only faster on a closed track but also more stable.

The evolution of the maximum lateral acceleration (during cornering) over the years is illustrated schematically in Fig. 1.5. The gray area shows the gradual improvement in sports

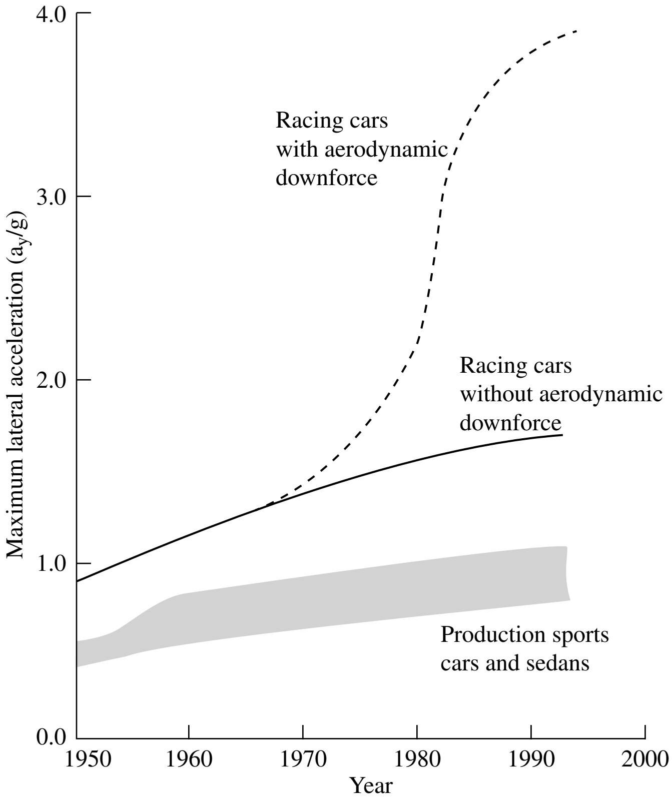


Figure 1.5 Trends of increased lateral acceleration over recent years for various racecars

Figure 1.6 Positive lift at high speed can make a racecar airborne (unintentionally), emphasizing the need for a reliable downforce mechanism. Courtesy of Mark Page, Swift Engineering

(and production) car handling, which is a direct result of improvements in tire (and suspension) construction. The solid line indicates a somewhat larger envelope of performance due to the softer and stickier tire compounds used for racing purposes. The gradual increase in racecars' maximum lateral acceleration, prior to 1966, is again a result of improvements in tire and chassis technology. However, the rapid increase that follows is due to the sudden utilization of aerodynamic downforce. The interesting question is: how, for the first 65 years of motor racing, was aerodynamics more like an art with a bit of drag reduction and why did no one notice the tremendous advantage of creating downforce on the tires without increasing the vehicle's mass? (We can always blame politics.)

Of course, the large values in Fig. 1.5 represent momentary limits and it is quite difficult to experience a lateral acceleration of three gs for more than a few seconds. For this reason, in many races where large lateral forces will be generated, the helmet of the driver is strapped to the vehicle's sides to avoid excess neck stress. If one must speculate about the future of racing, it seems that the 4 g shown in this diagram is a reasonable limit, and is based on human comfort (limits).

Most vehicles (e.g., passenger cars) have positive lift and not downforce and sport car manufacturers (like those with the red cars) make large efforts to generate even a small amount of downforce (which improves handling and safety). Also, the forces increase with the square of velocity (see Eq.1.1) and at high speed a vehicle can be lifted. Figure 1.6 proves that point, that even a racecar with significant level of downforce can become unintentionally airborne. We shall see later that this is a result of the large positive angle of the body relative to the surrounding air.

1.5 General Features of Fluid Flow

Fluid dynamics is the science dealing with the motion of fluids while aerodynamics is "restricted" to the flow of air. Fluids, contrary to solids cannot assume a fixed shape under load and will immediately deform. For example, if we place a brick in the backyard pool it will sink because the fluid below is not rigid enough to hold it.

Also, both gases and liquids behave similarly under load and both are considered fluids. A typical engineering question that we'll try to answer here is: what are the forces due to fluid motion? Examples could focus on estimating the aerodynamic forces acting on a car or loads needed to calculate the size and shape of a wing lifting an airplane. So let us start with the first question: what is a fluid?

As noted, in general, we refer to liquids and gases as fluids, but using the principles of fluid mechanics can treat the flow of grain in agricultural machines, a crowd of people leaving a large stadium, or the flow of cars on the highway. So one of the basic features is that we can look at the fluid as a continuum and not analyze each element or molecule (hence the analogy to grain or seeds). The second important feature of fluid is that it deforms easily, unlike solids. For example, a static fluid cannot resist a shear force without moving and, once the particles move, it is not a static fluid. So in order to generate shear force the fluid must be in motion. This will be clarified in the following paragraphs.

1.5.1 Continuum

Most of us are acquainted with Newtonian mechanics and therefore it would be natural to look at particle (or group of particles) motion and discuss their dynamics using the same approach used in courses such as dynamics. Although this approach has some followers, let us first look at some basics.

Consideration a: The number of molecules is very large and it would be difficult to apply the laws of dynamics, even when using a statistical approach. For example, the number of molecules in one gram-mole is called the Avogadro number (after the Italian scientist, Amadeo Avogadro 1776–1856). One gram-mole is the molecular weight multiplied by 1 gram. For example, for a hydrogen molecule (H_2) the molecular weight is two, therefore 2 g of hydrogen are 1 gram-mole. The Avogadro number N_A is:

$$N_A = 6.02 \cdot 10^{23} \text{ molecules/gmole} \tag{1.2}$$

Because the number of molecules is very large it is easier to assume a continuous fluid rather than discuss the dynamics of each molecule or even their dynamics, using a statistical approach.

Consideration b: In gases, which we can view as the least condensed fluid, the particles are far from each other, but as Brown (Robert Brown, botanist 1773–1858) observed in 1827, the molecules are constantly moving, and hence this phenomenon is called the Brownian motion. The particles move at various speeds and into arbitrary directions and the average distance between particle collisions is called the mean free path, λ, which for standard air is about $6 \cdot 10^{-6}$ cm. Now, suppose that a pressure disturbance (or a jump in the particles velocity) is introduced, this effect will be communicated to the rest of the fluid by these inter particle collisions. The speed that this disturbance spreads in the fluid is called the speed of sound and this gives us an estimate about the order of molecular speeds (the speed of sound is about 340 m/s in air at 288 K). Of course, many particles must move faster than this speed because of the three-dimensional nature of the collisions (see Section 1.6). It is only logical that the speed of sound depends on temperature since temperature is related to the internal energy of the fluid. If this molecular mean free path distance λ is much smaller than the characteristic length L in the flow of interest (e.g., $L \sim$ the length of a car) then, for example, we can consider the air (fluid) as a **continuum**! In fact, a nondimensional number, called the Knudsen number (after the Danish scientist Martin Knudsen: 1871–1949) exists based on this relation.

$$Kn = \frac{\lambda}{L} \tag{1.3}$$

Thus, if $Kn < 0.01$, meaning that the characteristic length is 100 times larger than the free mean path, then the continuum assumption may be used. Exceptions for this assumption of course would be when the gas is very rare $(Kn > 1)$, for example in a vacuum or at very high altitude in the atmosphere.

It appears that for most practical engineering problems, the aforementioned considerations (a) and (b) are easily met, justifying the continuum assumption. So if we agree to the concept of **continuum**, then we do not need to trace individual molecules (or groups of) in the fluid but rather observe the changes in the average properties. Apart from properties such as density or viscosity, the fluid flow may have certain features that must be clarified early on. Let us first briefly discuss frequently used terms such as laminar/turbulent and attached/separated flow, and then focus on the properties of the fluid material itself.

1.5.2 Laminar and Turbulent Flow

Now that via the continuum assumption we have eliminated the discussion about the arbitrary molecular motion, a somewhat similar but much larger scale phenomenon must be discussed. For the discussion let us assume a free-stream flow along the x-axis with uniform velocity U. If we follow the traces made by several particles in the fluid we would expect to see parallel lines as shown in the upper part of Fig. 1.7. If, indeed, these lines are parallel and follow in the direction of the average velocity, and the motion of the fluid seems to be "well organized", then this flow is called laminar. If we consider a velocity vector in a Cartesian system

$$\vec{q} = (u, v, w) \tag{1.4}$$

then for this steady state flow the velocity vector will be

$$\vec{q} = (U, 0, 0) \tag{1.4a}$$

and here U is the velocity into the x direction. Note that we are using $\vec{q}$ for the velocity vector!

On the other hand it is possible to have the same average speed in the flow, but in addition to this average speed the fluid particles will momentarily move into the other directions (lower part of Fig. 1.7). The fluid is then called turbulent (even though the average velocity

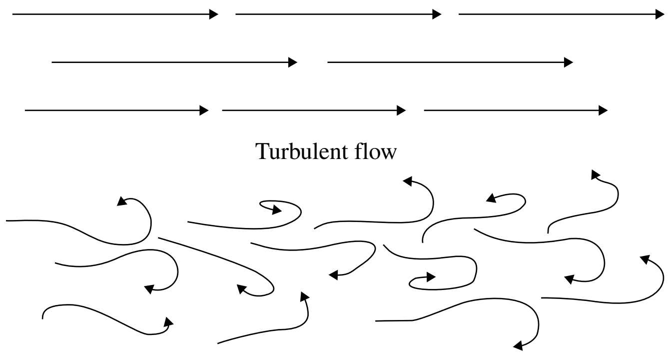


Figure 1.7 Schematic descriptions of laminar and turbulent flows with the same average velocity

U_{av} could be the same for both the laminar and turbulent flows). Again, note that at this point in the discussion the fluid is continuous and the turbulent fluid scale is much larger than the molecular scale. Also, in this two-dimensional case the flow is time dependent everywhere and the velocity vector then becomes

$$\vec{q} = (U_{av} + u', v', w') \tag{1.5}$$

and here u', v', w' are the perturbation into the x, y, and z directions. Also it is clear that the average velocities into the other directions are zero

$$V_{av} = W_{av} = 0$$

So if a simple one-dimensional laminar flow transitions into a turbulent flow, then it also becomes three-dimensional (not to mention time dependent). Knowing whether the flow is laminar or turbulent is very important for most engineering problems since features such as friction and momentum exchange can change significantly between these two types of flow. The fluid flow can become turbulent in numerous situations such as inside long pipes or near the surface of high-speed vehicles.

1.5.3 Attached and Separated Flow

Tracing streamlines in the flow (by injecting smoke, for example) allows us to observe if the flow follows the shape of an object (e.g., vehicle's body) close to its surface. When the streamlines near the solid surface follow exactly the shape of the body (as in Fig. 1.8a) the flow is considered to be attached. If the flow does not follow the shape of the surface (as seen behind the vehicle in Fig. 1.8b) then the flow is considered detached or separated (in that region). Usually, such separated flows behind the vehicle will result in an unsteady wake flow, which can be felt up to large distances behind it. Also, in case

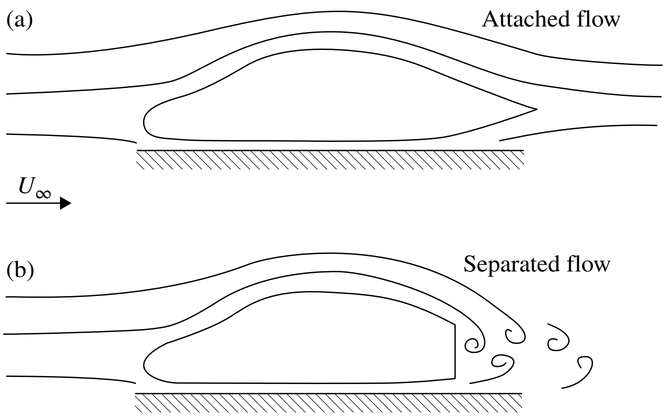


Figure 1.8 Attached flow over a streamlined car (a) and the locally separated flow behind a more realistic automobile shape (b)

of Fig. 1.8(b) the flow is attached on the upper surface and is separated only behind the vehicle. As we shall see later, having attached flow fields is extremely important because vehicles with larger areas of flow separation are likely to generate higher resistance (drag). Now, to complicate matters we may add that if the flow above this model is turbulent then, because of the momentum influx from the outer flow layers, the flow separation can be delayed.

1.6 Properties of Fluids

Fluids, in general, may have many properties related to thermodynamics, mechanics, or other fields of science. In the following paragraphs we shall mention only a few, which are used in introductory aero/fluid mechanics.

1.6.1 Density

The density by definition is mass (m) per unit volume. In case of fluids, we can define the density (with the aid of Fig. 1.9) as the limit of this ratio, when a measuring volume V shrinks to zero. We need to use this definition since density can change from one point to the other. Also in this picture we can relate to a volume element in space that we can call "control volume", which moves with the fluid or can be stationary (better if it is attached to an inertial frame of reference).

Therefore, the definition of density at a point is:

$$\rho = \lim_{V \to 0} \left(\frac{m}{V} \right) \tag{1.6}$$

Typical units are: kg/m^3 or g/cm^3

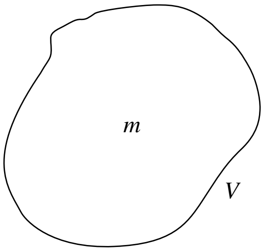


Figure 1.9 Mass m in a control volume V. Density is the ratio of m/V, as V shrinks

1.6.2 Pressure

We can describe the pressure p as the normal force F, per unit area, acting on a surface S(see Fig. 1.10). Again, we will use the limit process to define pressure at a point, since it may vary on a surface.

$$p = \lim_{S \to 0} \left(\frac{F}{S} \right) \tag{1.7}$$

Bernoulli pictured the pressure to be a result of molecules impinging on a surface (so this force per area is a result of the continuous bombardment of the molecules). The fluid pressure acting on a solid surface is normal to the surface as shown in the figure. Consequently, the direction is obtained by multiplying with the unit vector $\vec{n}$ normal to the surface. Thus, the pressure acts **normal** to a surface, and the resulting force, ΔF is:

$$\Delta F = -p\vec{n} \ dS \tag{1.8}$$

Here, the minus sign is a result of the normal unit-vector pointing outside the surface while the force due to pressure points inward. Also note that the pressure at a point inside a fluid is the same in all directions. This property of the pressure is called **isetropic**. The observation about the fluid pressure at a point, acting equally into any arbitrary directions, was documented first by Blaise Pascal (1623–1662).

The units used for pressure were introduced in Section 1.3. However, the Pascal is a small unit and more popular units are the kilopascal (kP), the atmosphere (atm), or the **bar**

$$1\,kP = 1000\,\frac{N}{m^2} \qquad 1\,atm = 101300\,\frac{N}{m^2} \qquad 1\,bar = 100000\,\frac{N}{m^2}$$

1.6.3 Temperature

The temperature is a measure of the internal energy at a point in the fluid. Over the years different methods evolved to measure temperature and, for example, the freezing point of water was considered as zero in the Celsius system while water boiling temperature under

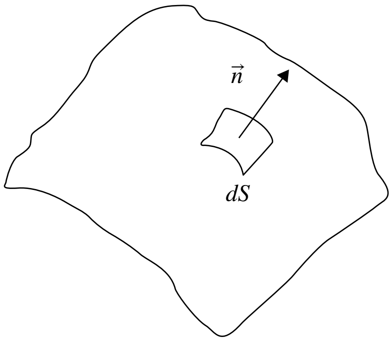


Figure 1.10 Pressure acts normal to the surface dS ($\vec{n}$ is the unit vector normal to the surface)

standard condition is 100°C. Kelvin units are similar to Celsius, however, they measure temperature from absolute zero, the temperature found in space that represents when molecular motion will stop. The relation between the two temperature measuring systems is:

$$K = 273.16 + C^\circ \tag{1.9}$$

The Celsius system is widely used in European countries while in the US the Fahrenheit scale is still used. In this case the 100°F was set to be close to the human body's temperature. The conversion between these temperature systems is

$$C^\circ = 5/9(F^\circ - 32) \tag{1.10}$$

Which indicates that $0°C = 32°F$. The absolute temperature in these units is called the Rankine scale and this is higher by 459.69°.

$$R^\circ = 459.69 + F^\circ \tag{1.11}$$

Now that we have introduced density, pressure, and temperature it is important to recall the ideal gas relation, where these properties are linked together by the gas constant, R.

$$p/\rho = RT \tag{1.12}$$

If we define v as the volume per unit mass then $v = 1/\rho$, and we can write

$$pv = RT \tag{1.13}$$

However, R is different for various gases or for their mixtures, but it can be easily calculated by using the universal gas constant, $\mathcal{R}$ ($\mathcal{R}$ = 8314.3 J/mol K). Then R can be found by dividing this universal $\mathcal{R}$ by the average molecular weight M of the mixture of gases.

Example 1.2 The ideal gas formula
As an example, for air we can assume that the molecular weight is M = 29, and therefore

$$R = \mathcal{R}/M = 8314.3/29 = 286.7 \text{ m}^2/\left(\sec^2 K\right) \quad \text{for air} \tag{1.14}$$

Suppose we want to calculate the density of air when the temperature is 300 K, and the pressure is 1 kgf/cm^2.

$$\rho = p/RT = 1 \cdot 9.8 \cdot 10^4/(286.7 \cdot 300) = 1.139 \text{ kg/m}^3$$

Here we used 1 kgf/cm^2 = $9.8 \cdot 10^4$ N/m^2, and g = 9.8 m/s^2.

Another interesting use of the universal gas constant is when we calculate the volume (V) of one gram-mole of gas in the following conditions: $T = 300°$K and p = 1 atm = 101,300 N/m^2. For air we can take 29 g (since M = 29) and then $\mathcal{R}$ is multiplied by 10^{-3} because we considering one gram-mole and not one kg-mole. Based on Eq. 1.3 and 1.4, and using the fact that the volume per unit mass is *V/M* we can write:

$$pV/M = (\mathcal{R}/M)T$$

or

$$V = \mathcal{R}\ T/p = 8314.3 \cdot 10^{-3} \cdot 300/101300 = 24.62 \cdot 10^{-3} \text{m}^3 = 24.62 \text{ liter}$$

Note that the molecular weight was cancelled, and 1 gram-mole of any gas will occupy the same volume because we have the same number of molecules (as postulated by Avogadro). Also 1 liter is equal to 0.001 m^3.

1.6.4 Viscosity

The viscosity is a very important property of fluids, particularly when fluid motion is discussed. In fact the schematic diagram of Fig. 1.11 is often used to demonstrate the difference between solids and fluids. A fluid must be in motion in order to generate a shear force, while a solid can support shear forces in a stationary condition.

In this figure the upper plate moves at a velocity of U_∞ while the lower surface is at rest. A fluid is placed between these parallel plates and when pulling the upper plate, a force F is needed. At this point we can introduce another important observation. The fluid particles in immediate contact with the plates will not move relative to the plate (as if they were glued to it). This is called the **no-slip boundary condition** and we will use this in later chapters. Consequently, we can expect the upper particles to move at the upper plate's speed while the lowest fluid particles attached to the lower plate will be at rest. Newton's Law of Friction states that:

$$\tau = \mu \frac{dU}{dz} \tag{1.15}$$

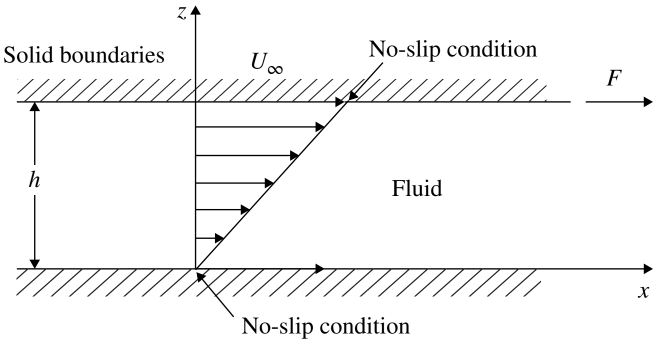


Figure 1.11 The flow between two parallel plates. The lower is stationary while the upper moves at a velocity of U_∞

here τ is the shear force per unit area and μ is the fluid viscosity. In this case the resulting velocity distribution is linear and the shear will be constant inside the fluid (for $h > z > 0$). For this particular case we can write:

$$\tau = \mu \frac{U_\infty}{h} \tag{1.16}$$

A fluid that behaves like this is called a **Newtonian fluid**, indicating a linear relation between the stress and the strain. As noted earlier, this is an important property of fluids since without motion there is no shear force.

The units used for τ are force per unit area and the units for the viscosity μ are defined by Eq. 1.15. Some frequently used properties of some common fluids are provided in Table 1.1.

Also note that the viscosity of most fluids depends on the temperature and this is shown for several common fluids in Fig. 1.12.

Example 1.3 The units of shear
To demonstrate the units of shear let us calculate the force required to pull a plate floating on a 2-cm thick layer of SAE 30 oil at U_∞ = 3 m/s.

Taking the value of the viscosity from Table 1.1: μ = 0.29 kg/m · s at 20°C

Thus:

$$\tau = 0.29\ 3/0.02 = 43\ \text{kg/m} \cdot \text{s}^2 = 43\ \text{N/m}^2$$

So if the plate area is 2 m^2 then a force of 86 N will be required to pull it at 3 m/s.

Sometimes the ratio between viscosity and the density is denoted as ν, the "kinematic viscosity". Its definition is:

$$\nu = \frac{\mu}{\rho} \tag{1.17}$$

Table 1.1 Approximate properties of some common fluids at 20°C (ρ = density, μ = viscosity)

	ρ (kg/m^3)	μ (N s/m^2)
Air	1.22	$1.8 \cdot 10^{-5}$
Helium	0.179	$1.9 \cdot 10^{-5}$
Gasoline	680	$3.1 \cdot 10^{-4}$
Kerosene	814	$1.9 \cdot 10^{-3}$
Water	1000	$1.0 \cdot 10^{-3}$
Sea water	1030	$1.2 \cdot 10^{-3}$
Motor oil (SAE 30)	919	0.29
Glycerin	1254	0.62
Mercury	13,600	$1.6 \cdot 10^{-3}$

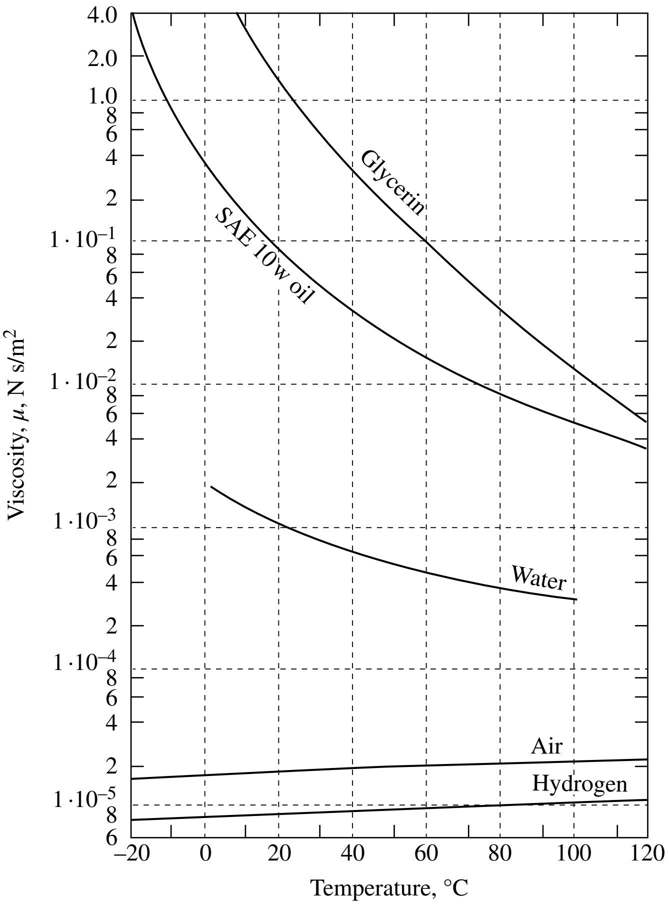


Figure 1.12 Variation of viscosity versus temperature for several fluids

1.6.5 Specific Heat

Fluids have several thermodynamic properties and we shall mention only two related to heat exchange. For example, if heat Q is added in a constant pressure process to a mass m, then the relation between temperature change and heat is stated by the simple formula

$$Q = mc_p \Delta T \tag{1.18}$$

Here, c_p is the specific heat coefficient used in a constant pressure process. However, if the fluid is not changing its volume during the process then c_v is used for the specific heat in this process.

$$Q = mc_v \Delta T \tag{1.19}$$

The ratio between these two specific heat coefficients is denoted by γ

$$\gamma = \frac{c_P}{c_V} \tag{1.20}$$

The heat (energy) required to raise the temperature of 1 g of water by 1°C is called a calorie (cal). Therefore, the units for c_p or c_v are $\frac{\text{cal}}{\text{kg} \cdot \text{C}^\circ}$ and 1 cal = 4.2 J (J = Joule). Work in mechanics is force times distance and therefore units of 1 Joule are

$$1\,\text{J} = \text{kg}\frac{\text{m}}{\text{sec}^2}\text{m} = \text{kg}\frac{\text{m}^2}{\text{sec}^2}$$

Also, for an ideal gas undergoing an adiabatic process, the two heat capacities relate to the gas constant, R (see Ref. 1.1 p. 90) by:

$$c_p - c_v = R \tag{1.21}$$

1.6.6 Heat Transfer Coefficient, k

Heat transfer can take several forms such as conduction, convection, or radiation (see Chapter 11). As an example at this introductory stage, we can mention only one basic mode of heat transfer, called conduction. The elementary one-dimensional model is depicted in Fig. 1.13 where the temperature at one side of the wall is higher than on the other side. The basic heat transfer equation for this case, called the **Fourier equation**, states that the heat flux Q is proportional to the area A, the temperature gradient, and to the coefficient k, which depends on the material through which the heat is conducted.

$$Q = -kA\frac{dT}{dx} \tag{1.22}$$

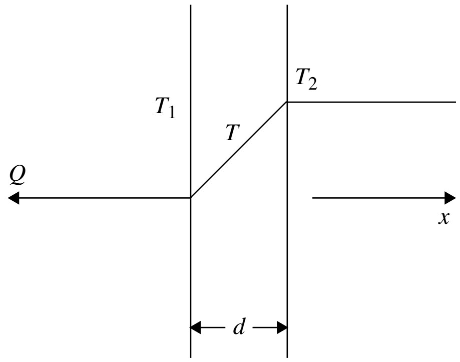


Figure 1.13 Conductive heat transfer through a wall of thickness d

For the case in the figure we could state that the heat flux is

$$Q = -kA\frac{T_2 - T_1}{d} \tag{1.22a}$$

And here T_2 is larger than T_1, and the minus sign indicates that the heat flux is in the left direction. The units for k are defined by Eq. 1.22 as W/(m°C) or cal/(m°C s). Note that the temperature distribution in the wall in Fig. 1.13 is linear. This is proved later in Chapter 11.

1.6.7 Modulus of Elasticity, E

The modulus of elasticity E is a measure of compressibility. It can be defined as

$$E = dp/(dV/V) \quad \text{or} \quad dp = E(dV/V) \tag{1.23}$$

And the second form indicates how much pressure is needed to compress a material having a modulus of E. Also, the change in volume is directly related to the change in density, and we can write:

$$d\rho/\rho = dV/V \tag{1.24}$$

And by substituting dV/V instead of $d\rho/\rho$ in Eq. 1.23 we get:

$$E = dp/(d\rho/\rho) \tag{1.25}$$

Most liquid are not very compressible, but gases are easily compressed and for an ideal gas we already introduced this relation (in Eq. 1.12):

$$dp/d\rho = RT \tag{1.26}$$

Therefore, substituting Eq. 1.26 into Eq. 1.25 results (for an ideal gas):

$$E = \rho RT \tag{1.27}$$

The units for E, based on Eq. 1.23 are $\dfrac{\mathrm{N/m^2}}{\mathrm{m^3/m^3}} = \mathrm{N/m^2}$

Example 1.4 Compressibility of a liquid

For this example, let us consider the compressibility of sea water. The modulus of elasticity is $E = 2.34 \cdot 10^9\ \mathrm{N/m^2}$ and let us evaluate the change in volume at a depth of 1 Km. The change in pressure at 1000 m depth is

$$dp = \rho gh = 1000 \cdot 9.8 \cdot 1000\ \mathrm{N/m^2}$$

and

$$dV/V = dp/E = 1000 \cdot 9.8 \cdot 1000/2.34 \cdot 10^9 = 4.188 \cdot 10^{-3}\ (0.42\%)$$

which is less than half a percent. This shows that water is really incompressible.

It is interesting to point out that compressibility relates to the speed of sound in a fluid. If we use the letter a to denote the speed of sound, then later we shall see that

$$a^2 = dp/d\rho \tag{1.28}$$

For liquids, we can use Eq. 1.25 to show that

$$a^2 = dp/d\rho = E/\rho \tag{1.29}$$

For ideal gases undergoing an adiabatic process (thermally isolated), the relation between pressure and density (Ref. 1.1) is:

$$\frac{p}{\rho^\gamma} = C \tag{1.30}$$

where C is a constant. To find the speed of sound the derivative ${}^{dp}\!/_{d\rho}$ is evaluated using Eq. 1.30 and the ideal gas definition:

$$\frac{dp}{d\rho} = C \cdot \gamma \cdot \rho^{\gamma-1} = C \cdot \gamma \cdot \frac{\rho^\gamma}{\rho} = \frac{p}{\rho^\gamma} \cdot \gamma \cdot \frac{\rho^\gamma}{\rho} = \gamma \frac{p}{\rho} = \gamma RT \tag{1.31}$$

Therefore

$$a = \sqrt{\gamma RT} \tag{1.32}$$

indicating that the speed of sound is a function of the temperature.

Example 1.5 The speed of sound
Let us calculate the speed of sound in air at 300°K. Taking the value of R from Eq. 1.14 and assuming $\gamma = 1.4$:

$$a = \sqrt{1.4 \cdot 286.6 \cdot 300} = 346.9\,\text{m/s for air}$$

Now, to calculate the speed of sound in water we must use Eq. 1.29. Based on the modulus of elasticity of sea water:

$$a = \sqrt{\frac{E}{\rho}} = \sqrt{\frac{2.34 \cdot 10^9}{1000}} = 1529\,\text{m/s}$$

and the resulting speed of sound is significantly higher.

1.6.8 Vapor Pressure

Vapor pressure is a property related to the phase change of fluids. One way to describe it is to observe the interface between the liquid and the gas phase of a particular fluid and the vapor pressure indicates that there is equilibrium between the molecules leaving and joining the liquid phase. The best example is to examine the vapor pressure of water as shown in Fig. 1.14. Because molecular energy is a function of the temperature, it is clear

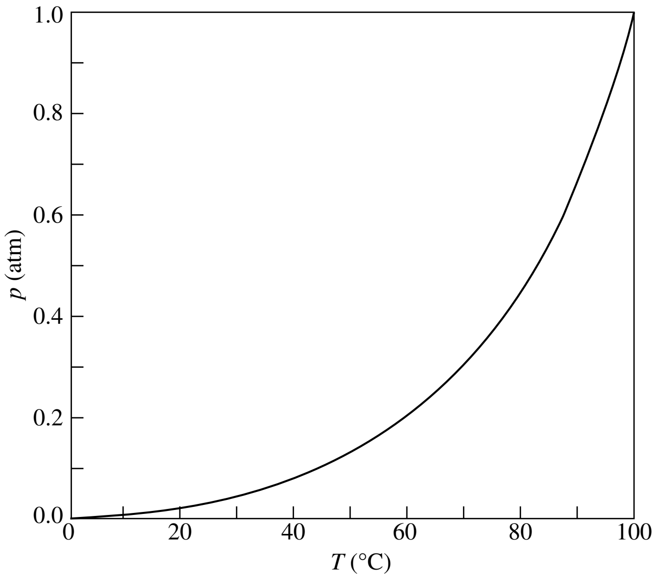


Figure 1.14 Vapor pressure of air versus temperature

that vapor pressure will increase with temperature. The vapor pressure is zero at 0°C, and of course is equal to one atmosphere at 100°C, which is the standard boiling point of water.

In later chapters we shall see that the pressure can change in a moving fluid. So even if there is no temperature change, there could be a situation when the pressure in the fluid (liquid) falls below the vapor pressure. The result is formation of bubbles, because at this condition the liquid will evaporate locally. This phenomenon is called **cavitation** and can happen in an overheating engine coolant when the radiator pressure cap cannot hold the designed pressure. If pump cavitation occurs, this will reduce coolant flow rate and cooling system performance, and will affect the pump efficiency. One possible solution is to increase the pressure in an engine cooling system to delay cavitation, resulting in better cooling performance.

Fluids have many more properties such as enthalpy, entropy, internal energy, and so on, but they are not used in this text.

1.7 Advanced Topics: Fluid Properties and the Kinetic Theory of Gases

Gases were defined earlier as fluids where the molecules can move freely and are far from each other, occasionally colliding with each other. This model led Daniel Bernoulli in 1738 to explain pressure in gases based on this type of molecular motion. Bernoulli considered a cylindrical container, filled with gas, as shown in the figure. As the molecules move inside the container, they also impinge on the walls as shown in Fig. 1.15. Now we may neglect the intermolecular collisions and assume that when a molecule hits the wall it will bounce back without losses (elastic collision). This assumption also includes pure elastic collisions with the sides of the cylinder.

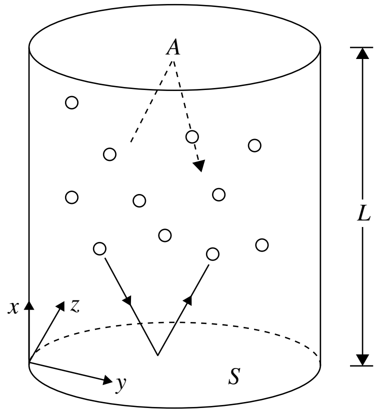


Figure 1.15 Gas molecules moving randomly inside a container

Therefore the total forces due to these collisions must produce the pressure on the container's walls. For example, the particle in Fig. 1.15 hitting the top has a velocity

$$q = (u, v, w) \tag{1.33}$$

and when it hits the top, the change in its linear momentum in the x direction is

$$2\,mu$$

and the 2 is a result of the elastic collision, while m is the mass of the molecule, and u is the velocity component into the x direction. Because the particle is contained inside the cylinder and it is continuously bouncing back and forth, we can estimate the time Δt between these collisions on the upper wall by

$$\Delta t = 2\,L/u$$

where L is the length of the cylinder. The force due to the collisions of this particle, based on Newton's Momentum Theory is

$$F = \frac{\Delta(mu)}{\Delta t} = \frac{2\,mu}{2\,L/u} = mu^2/L \tag{1.34}$$

Now recall that the particles are likely to move at the same speed into any direction and

$$q^2 = u^2 + v^2 + w^2$$

and if all directions are of the same order of magnitude we can assume

$$q^2 \approx 3u^2$$

Now suppose that there are N particles in the container and therefore the force due to the inner gas is

$$F = N\frac{mq^2/3}{L} \tag{1.35}$$

and the pressure is simply the force per unit area

$$p = \frac{F}{S} = N\frac{mq^2/3}{LS} = \frac{N}{3\,V}mq^2 \tag{1.36}$$

and here the volume $V = LS$, and S is the cylinder top (or bottom) area. This is a surprisingly simple approach that connects the pressure to the molecular kinetic energy. Now if we recall the ideal gas equation

$$pV = nRT = \frac{N}{N_A}RT \tag{1.37}$$

Where $n = N/N_A$, is the number of moles in the cylinder (recall that N_A is the Avogadro number in Eq. 1.2). By equating these two equations (1.36 and 1.37) we solve for the temperature:

$$T = \frac{N_A}{3R}mq^2 \tag{1.38}$$

This simple model shows that for an ideal gas the molecular kinetic energy is proportional to the absolute temperature. This means that at the absolute zero the molecular motion will stop; a concept that wasn't received well in Bernoulli's era. About 100 years later, the Scottish physicist, James Maxwell (1831–1879) revived this theory and introduced a statistical approach. He suggested a universal velocity distribution (Fig. 1.16) that shows the velocity range of the molecular motion. Our interest at this point is to demonstrate the magnitude of the molecular velocity, which mainly depends on temperature and molecular weight. The probability is depicted on the ordinate and the probable velocity is on the abscissa. Of course the total area under the curve is always one, because all particles in the container are included. Note that the average velocity is a bit over the top to the right (468 m/s for air) which is somewhat higher than the speed of sound, mentioned earlier.

Another interesting aspect of this molecular model is that for flows over bodies, it can intuitively explain the effect of curvature on the pressure distribution. For example, Fig. 1.17 shows a generic automobile, which is moving forward at a constant speed.

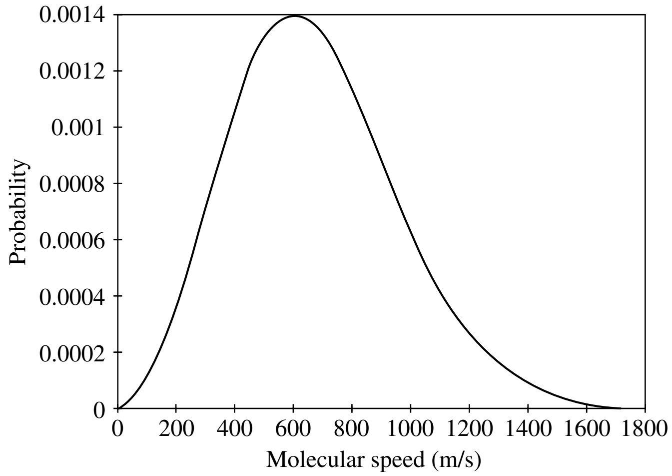


Figure 1.16 Maxwell's universal velocity distribution for the molecules of air (M = 29), at 300 K

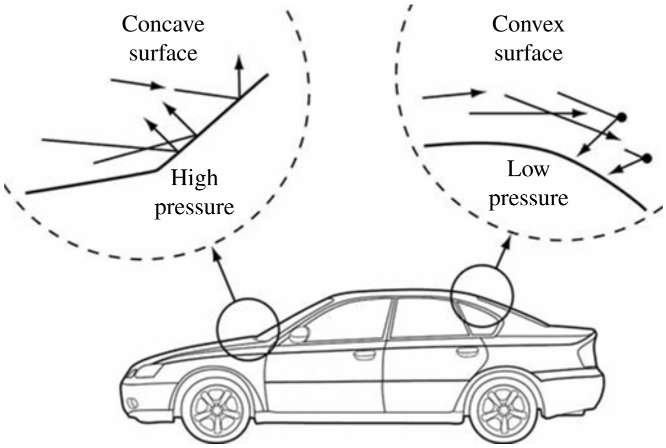


Figure 1.17 Using the kinetic theory of gases we can explain the high pressure near a concave curvature and the lower pressure near a convex curvature

The air molecules are moving towards the car at an average velocity, in addition to their Brownian motion (see Fig. 1.17). At the base of the windshield the number of collisions will increase because the incoming molecules will hit head on and some may even bounce back again due to intermolecular collisions. On the other hand, when observing the flow over a convex surface as shown in the figure (e.g., behind the roof top). The particles will not hit the rear window head on. They will fill the void mainly due to intermolecular collisions. Hence a lower pressure is expected there. We can also guess that the velocity at the base of the windshield (concave surface) slows down while the undisturbed particles at the back (convex surface) will accelerate to cover the additional distance created by the void. This generic discussion suggests that the pressure is lower if the velocity is increased in such flows. We shall see later that this observation led to the formulation of the well-known Bernoulli equation.

1.8 Summary and Concluding Remarks

In this introductory chapter, the properties of fluids were discussed. The reader must have seen those during earlier studies and the only ones worth mentioning again are related to the forces in fluids. The first is of course the pressure, which acts **normal** to a surface, and the second is the shear force. The shear stress in a fluid exists only when the fluid is in motion, contrary to solids that can resist shear under static conditions. This situation is created by the "no slip boundary condition", which postulates that the fluid particles in contact with a surface will have zero relative velocity at the contact area.

Reference

1.1. Karlekar, B. V., *Thermodynamics for Engineers*, Prentice-Hall, Englewood Cliffs, NJ,1983.

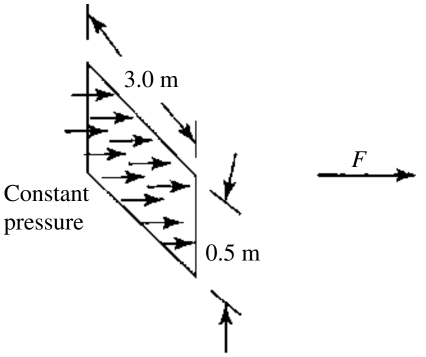
Problems

1.1. The recommended pressure on the tires of a sedan is 30 psi. Convert this to units of N/m^2 and atmospheres.

1.2. The frontal area of a pickup truck is 2 m^2 and the air resistance (drag) at 100 km/h is 500 N. Calculate the truck's drag coefficient (assume air density is 1.2 kg/m^3).

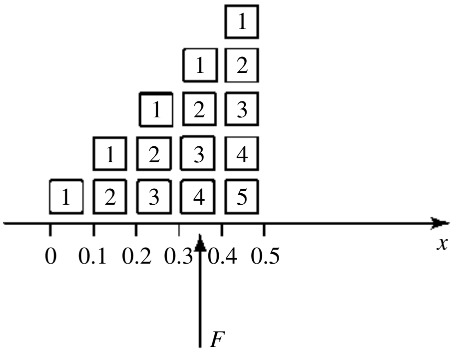
1.3. How many kg of air at standard conditions are in a container with a volume of 2 m^3?

1.4. A uniform pressure is acting on a plate 0.5 m tall and 3.0 m wide. Assuming the pressure difference between the two sides of the plate is 0.05 atm, calculate the resultant force.



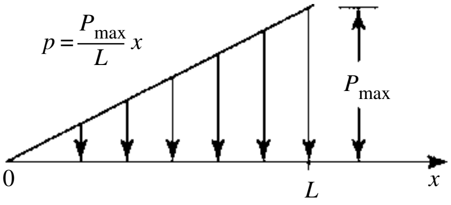
Problem Figure 1.4

1.5. Identical bricks 0.1 m wide, and weighing 2 kg are placed on a plate (assume the plate has negligible weight). Calculate the total weight (and force F required to balance the plate). How far from $x = 0$ should F be placed so that the plate will not tip over?



Problem Figure 1.5

1.6. A linearly varying pressure $[P(x) = P_{max} \cdot x/L]$ is acting on a plate. Calculate the total force (resultant) and how far it acts from the origin. (Later we shall call this the **center of pressure**.)



Problem Figure 1.6

1.7. Suppose that a 1 m^3 metal container holds air at standard conditions (p = 1 atm and T = 300 K).
   a. Calculate the pressure inside the container if it is heated up to 400 K?
   b. Calculate the density ρ inside the container.

$$\left[R = 286.6\,\mathrm{m}^2/\left(\sec^2\mathrm{K}\right)\right]$$

1.8. A two-dimensional velocity field is given by the following formulation

$$u = \frac{x}{x^2 + z^2} \qquad w = \frac{z}{x^2 + z^2}$$

Calculate the value of the velocity vector $\vec{q}$ at a point (1, 3).

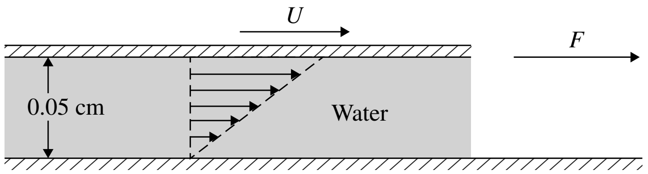
1.9. On a warm day the thermometer reads 30°C. Calculate the absolute temperature in Kelvin and also the temperature in degrees Fahrenheit.

1.10. 1 m^3 of air at 1 atm, and at 300 K is sealed in a container. Calculate the pressure inside the container if:
   a. The volume is reduced to 0.5 m^3 but the temperature cooled off to 300 K, and
   b. when the temperature was 350 K.

1.11. A 1 m^3 balloon is filled with helium at an ambient temperature of 30°C. The pressure inside the balloon is 1.1 atm while outside it is 1.0 atm. The molecular weight of helium is about 4 and the surrounding air is about 29. Calculate the weight of the helium inside the balloon. What is the weight of outside air that has the same volume as the balloon? What is the meaning of this weight difference?

1.12. Usually, we check the tire pressure in our car early morning when the temperature is cold. Suppose that the temperature is 288 K (about 15°C), the volume of air inside is 0.025 m^3, inside air density is 2.4 kg/m^3, and the tire pressure gauge indicates a pressure of 2 atm ($2 \cdot 1.1013 \cdot 10^5$ N/m^2).
   a. What is the tire pressure when the car is left in the summer sunshine and the tire temperature reaches 333 K?
   b. Suppose the tire is inflated with helium (M ~ 4) instead of air (M ~ 29); how much weight is saved?

1.13. A 200 cm^3 container is filled with air at standard conditions. Estimate the number of air molecules in the container.
1.14. The temperature inside the container in the previous question was raised to 350 K. Calculate the pressure, density, and the number of air molecules inside the container.
1.15. A 3 m^3 tank is filled with helium at standard conditions. If the molecular weight of helium is 4.0, calculate the mass of the gas inside the tank.
1.16. The tire pressure in a car was measured in the morning, at 280 K and was found to be 2.5 atm. After a long trip on a warm afternoon the pressure rose to 3.1 atm. Assuming there is no change in the tire volume, calculate the air temperature due to the pressure rise.
1.17. A flat plate floating above a 0.05 cm thick film of oil being pulled to the right at a speed of 1 m/s (see sketch in Fig. 1.18). If the fluid viscosity is 0.4 N s/m^2, calculate the shear force τ on the lower and upper interfaces (e.g., on the floor and below the plate) and at the center of the liquid film.
1.18. A flat plate floating on a 0.05 cm thick water film is pulled by the force F. Calculate F for an area of 1 m^2 and for U = 1 m/s (note that for water, μ = 0.001 kg/(m/s).

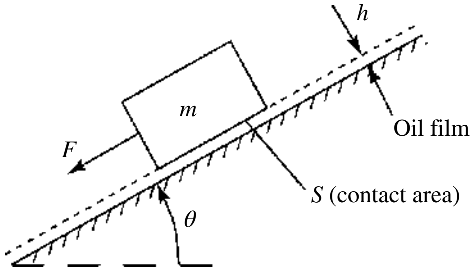


Problem Figure 1.18

1.19. A flat plate is pulled to the right above a 0.1 cm thick layer of viscous liquid (see sketch in Fig. 1.18) at a speed of 1 m/s. If the force required to pull the plate is 200 N per 1 m^2, then calculate the viscosity of the liquid.
1.20. Consider a stationary vertical line in the figure of the previous problem (fixed to the lower surface). Calculate how much water per 1 m width flows during 1 s to the right across that line?
1.21. The velocity distribution above a solid surface represented by the x coordinate is

$$u = 3z - 3z^3$$
$$w = 0$$

Calculate the shear stress on the surface (at z = 0) and at z = 0.5.



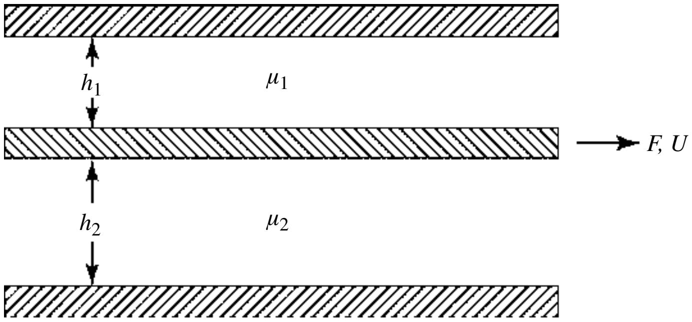
Problem Figure 1.21

1.22. A thin oil film covers the surface of an inclined plane, as shown. Develop an expression for the terminal velocity of a block of weight W, sliding down the slope. Assume that the oil film thickness and viscosity are known, as well as the incline angle and the contact surface area.

1.23. Calculate the terminal velocity of a 0.2 m wide, 0.3 m long, and 5 kg block sliding down an incline of 30°, as shown in sketch for Problem 1.22. Assume the oil film thickness is 1 mm and the oil viscosity (from Table 1.1) is 0.29 N s/m^2.

1.24. The block in the sketch for Problem 1.22 slides at a velocity of 2 m/s due to the force F. In this case, however, the slope $\theta = 0$. Calculate the magnitude of the force if the oil film thickness is 1 mm and the oil viscosity (from Table 1.1), is 0.29 N s/m^2.

1.25. A thin plate is pulled to the right, between two parallel plates at a velocity U, as shown. It is separated by two viscous fluids with viscosity μ_1, μ_2, and the spacing is h_1, h_2, accordingly. Assuming that the plates are very large, calculate the force per unit area required to pull the central plate.



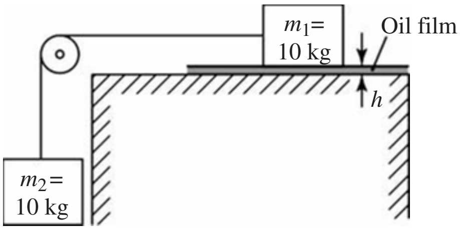
Problem Figure 1.25

1.26. A small bird with a characteristic length of $L = 0.2$ m flies near the ocean at a speed of 14 m/s. The mean free path of air molecules at sea level is about $\lambda = 6.8 \cdot 10^{-8}$ m, and the average molecular sped can be estimated as $c = 468$ m/s. Calculate the Knudsen number. Can you assume that the fluid is continuous?

1.27. At standard atmospheric condition the average speed of the air molecules is estimated at $c = 468$ m/s (see Fig. 1.16). Calculate the speed of sound for this condition (at $T =$ 300 K). Can you explain the large difference?

1.28. An important parameter for grouping different flow regimes (called the Reynolds number) represents the ratio between the actual and the molecular scaling of length times velocity. It can be approximated by the following formulation $\mathrm{Re} = 2\frac{V}{c}\frac{L}{\lambda}$ (see Chapter 4, Section 4.2). Calculate this ratio for the small bird of the previous problem flying at a speed of 14 m/s. (recall that $c = 468$ m/s, and $\lambda = 6.8 \cdot 10^{-8}$ m).

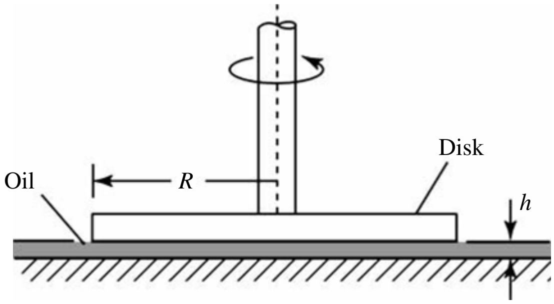
1.29. A 0.3 m wide, 0.5 m long, and 10 kg block (m_1) slides on a 1 mm thin oil film, pulled by the mass m_2, as shown. Calculate the terminal velocity using the oil viscosity from Table 1.1 (0.29 N s/m^2).



Problem Figure 1.29

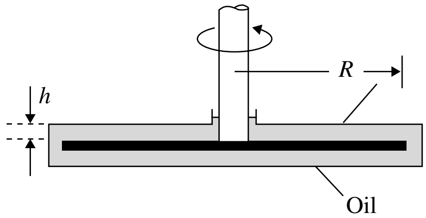
1.30. Calculate the terminal velocity in Problem 1.29, but now m_1 = 20 kg.

1.31. The disk shown in the figure rotates at a speed of 50 RPM above a stationary plane separated by an oil film. If the oil viscosity is μ = 0.01 kg/(m/s), the spacing between the disc and the stationary surface is 2 mm, and R = 5 cm, calculate the torque required to rotate the disk (assume linear velocity distribution in the gap).



Problem Figure 1.31

1.32. A rotary damper consists of a disk immersed in a container as shown in the figure. Assuming that the gap h is the same on both sides and the viscosity μ and the disk radius R are known, calculate the torque required to rotate the disk at a particular RPM.

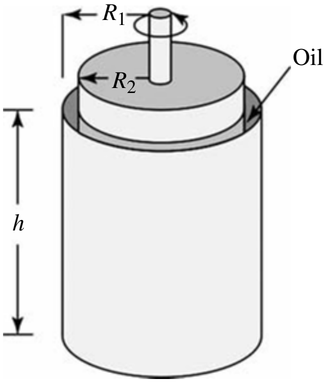


Problem Figure 1.32

1.33. The diameter of the rotary damper, shown in the figure is $2R$ = 20 cm. The oil viscosity is μ = 0.29 kg/(m/s), and the gap is h = 1 mm. Calculate the torque on the shaft at 1000 RPM.

1.34. Suppose the gap is increased to h = 2 mm in both sides, by how much would the torque change?

1.35. Two concentric cylinders with radius R_1 and R_2 are separated by an oil film with viscosity μ as shown in the figure. Next, the inner cylinder is rotated and a linear velocity distribution is assumed in the gap between the cylinders (the lower surface is not active). Develop a formula for the torque on the shaft, as a function of rotation speed.



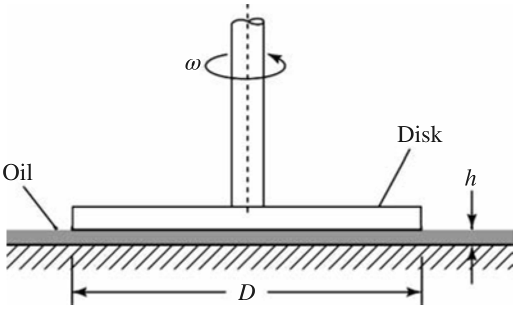
Problem Figure 1.35

1.36. The two concentric cylinders shown in the sketch for Problem 1.35 are separated by an oil film with viscosity μ = 0.023 kg/(m/s). If the shaft rotates at 200 RPM, calculate the torque on the shaft (R_1 = 15.12 cm, R_2 = 15 cm, and h = 70 cm).

1.37. The device, based on the two concentric cylinders (shown in the sketch for Problem 1.35) can be used to measure the viscosity of a fluid. Assuming that the shaft rotates at 200 RPM and the torque measured is 6 Nm, calculate the viscosity of the fluid. (Use the dimensions from the previous problem (R_1 = 15.12 cm, R_2 = 15 cm, and h = 70 cm).

1.38. Some desalination processes are based on evaporating the sea water. Energy can be saved by reducing the boiling temperature of the water. Based on Fig. 1.14, determine the water boiling temperature if the pressure is lowered to 0.5 atm.

1.39. The disk shown in the sketch rotates at a speed of ω = 50 rad/s above a stationary plane separated by an oil film. If the oil viscosity is μ = 0.01 kg/(m/s), the spacing between the disc and the stationary surface is 2 mm, and D = 10 cm, calculate the torque required to rotate the disk.

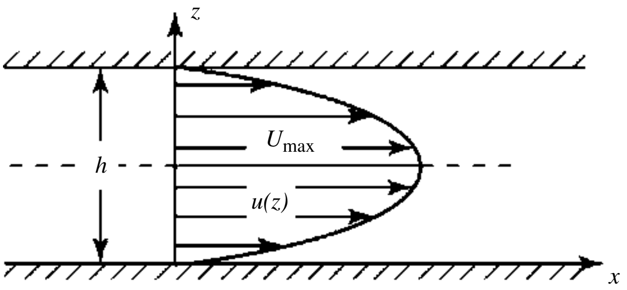


Problem Figure 1.39

1.40. Oil with a viscosity of μ flows between two parallel plates, as shown. Suppose the velocity distribution is given as:

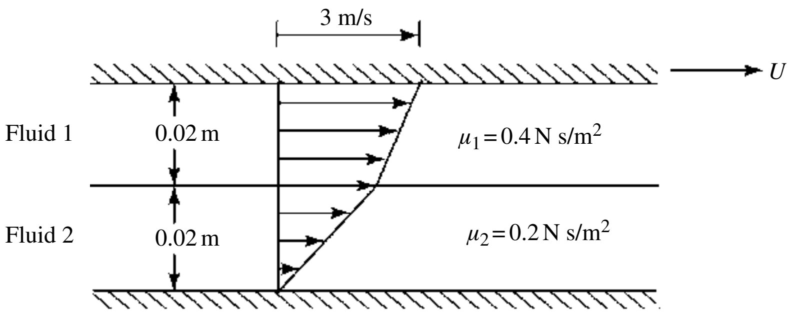
$$u(z) = -k\left(\frac{z}{h} - \frac{z^2}{h^2}\right)$$

Then plot and calculate the shear stress as a function of z. Where (in terms of z) is the highest and where is the lowest shear stress? What is the relation between k in the equation and U_{max} in the figure?



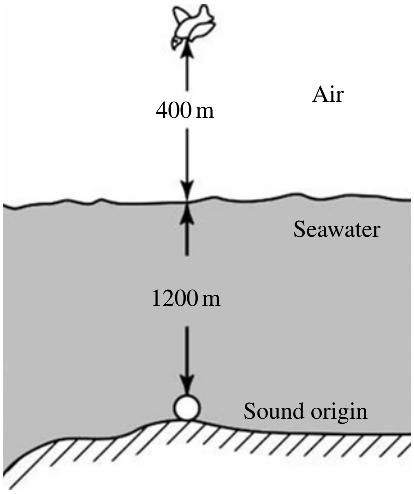
Problem Figure 1.40

1.41. Two layers of fluid are dragged along by the motion of an upper plate as shown (without mixing). The bottom plate is stationary. The top fluid applies a shear stress on the upper plate and the lower fluid applies a shear stress on the bottom plate. Calculate the velocity of the boundary between the two fluids.



Problem Figure 1.41

1.42. A noise created by small earthquake at a depth of 1200 m in the ocean propagates upward and eventually reaches a bird flying above, at an altitude of 400 m. Calculate how long it takes for the noise to reach the bird.
For the seawater use $E = 2.34 \cdot 10^9$ N/m^2, $\rho = 1030$ kg/m^3, and for air, $\gamma = 1.4$ and $T = 270$ K).



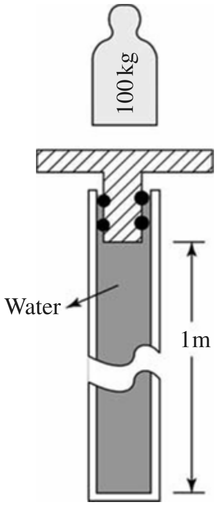
Problem Figure 1.42

1.43. A small explosion in the ocean is 3000 m from two swimmers. The first has his ears under water while the second swimmer's head is above the water. How soon will each hear the noise of the explosion? For the seawater use $E = 2.34 \cdot 10^9$ N/m^2, $\rho = 1030$ kg/m^3, and for air, $\gamma = 1.4$ and $T = 300$ K).

1.44. The speed of an airplane is frequently stated in terms of the Mach number, which is simply the ratio between the actual speed and the speed of sound $M = U/a$. Suppose an airplane flies at sea level at $M = 0.8$ where the temperature is 27°C, calculate the actual speed of the airplane.

Next, calculate the speed at the same Mach number but at an altitude of 13 km, where the temperature is −57°C (for air, $\gamma = 1.4$).

1.45. A piston floats over a 1 m high column of water enclosed in a 2 cm diameter, pressure-tight cylinder. Calculate how deep the 100 kg weight will push the cylinder down. Assume the water modulus of elasticity is $E = 2.34 \cdot 10^9$ N/m^2.



Problem Figure 1.45