BIG IDEA 1

Building with and Talking about Shapes

If you are reading this introduction, I am going to assume that you love beautiful, creative mathematics and want to introduce this mathematics to your students. After many years of working in mathematics education, researching and teaching different groups of students, I have come to realize that there are two completely different versions of mathematics. One version is all that many people know; it is a subject of rules and procedures. But there is another mathematics—a subject of beauty and creativity, of openness and exploration—and when people encounter this mathematics, they are forever changed. That is because this mathematics illuminates our world; and when people learn it, they start to see the world in a new light. Paul Lockhart is a mathematician who has worked to communicate the real nature of mathematics to readers. He writes:

> What makes a mathematician is not technical skill or encyclopedic knowledge but insatiable curiosity and a desire for simple beauty. Just be yourself and go where you want to go. Instead of being tentative and fearing failure or confusion, try to embrace the awe and mystery of it all and joyfully make a mess. (Lockhart, 2012, p. 10)

I love this advice. Your students are young mathematicians, and you can introduce them to the mathematics that will capture their interest and curiosity if you share with them tasks that invite their thinking and their creativity. This big idea is the first opportunity for that lovely work. As a teacher myself, I am always excited to introduce my students to this version of mathematics.

This big idea is all about composing and decomposing shapes—or, to say it another way, putting shapes together and taking them apart. This is a mathematically worthwhile activity, as it helps students learn the qualities and characteristics of shapes. It also gives students opportunities to build and to move shapes around, helping them get to know their world and the role of mathematics within it.

In our Visualize activity, we give students the opportunity to study two-dimensional images of buildings and notice the shapes that make up the buildings. Students work to decompose the two-dimensional representations of three-dimensional buildings. They then get to design and build their own large city, paying attention to the shapes that make up the buildings. Students will be able to stroll through the streets of their city as if they were giants!

Our Play activity introduces a historical puzzle called the tangram. A tangram is made up of seven two-dimensional pieces that can be cut from a square. Students will be working out how they can fit their seven shapes into a frame, which will engage their minds in important mathematical thinking. Students will turn and flip their shapes as they try to solve the puzzles. We then ask students to make some of their own puzzles. Students get to create a design, name it, and the trace the outline of the shape for another student to try to build. Tangrams, comprising these seven simple pieces, have brought lots of enjoyment to people all over the world.

Our Investigate activity asks students to use tangram pieces to make different squares. We ask students to make as many squares as possible using their pieces, which will be an exciting challenge. Students have the opportunity to discuss shapes by name as they create squares of different sizes using the tangram pieces. When their collection of squares is complete, they talk about the different sizes of the shapes they have made.

Jo Boaler

Reference

Lockhart, P. (2012). *Measurement*. Cambridge, MA: Harvard University Press.

Build(ings)

Snapshot

Students explore how three-dimensional shapes can be composed into more complex figures by first constructing buildings out of recycled materials and then designing a community as a class.

Connection to CCSS
1.G.2, 1.G.1

Agenda

Activity	Time	Description/Prompt	Materials
Launch	10–15 min	Show students the Noticing Buildings sheet and ask what students think makes these buildings different or the same, then discuss what shapes are used to make the buildings. Support students in noticing different shapes. Connect these two-dimensional shapes to a simple building you have created with blocks, and discuss what shapes are used to make your building. Tell students about the activity and introduce them to the materials available for constructing buildings.	• Noticing Buildings sheet, to display • Simple building made from two blocks • Materials for building construction, to show
Explore	30–45 min	Partners make a plan for the type of building they want to create and the shapes they will need to construct it. They construct their building, making modifications to the materials as needed. Confer with students about their plans and the shapes they are using to compose their buildings, supporting the use of geometric language.	Materials for building construction, such as recycled boxes, masking tape, scissors, and markers, or wooden blocks

Activity	Time	Description/Prompt	Materials
Discuss	30–45 min	Do a gallery walk of the buildings and ask students to figure out what each building is and how they know. As a class, discuss how the buildings are similar and different, focusing on their components. Develop a class plan for how to assemble the buildings into a community, and then construct a village, town, or city, including streets. Discuss the components that appear frequently in buildings and those that do not, and develop ideas about the reasons.	• Students' buildings • Masking tape

To the Teacher

Building—composing complex figures out of simpler components—is critical for developing spatial reasoning and relationships, which in turn support all mathematical thinking. Composing and decomposing with shapes is strongly connected to composing and decomposing with numbers, and we want students to learn that mathematics involves seeing both the simpler components and the complex wholes they can create. What may look like play is far from trivial, and we encourage you to use this Visualize activity as a springboard to more opportunities to build in your classroom.

You can use a variety of materials for constructing buildings in this activity, including any wooden blocks you may already have in your classroom, but we find that using recycled materials is the most engaging and flexible. Ahead of this activity, invite students to collect and bring in boxes of different shapes and sizes to contribute to the project, including empty cereal, tissue, oatmeal, cracker, and shipping boxes. Students could also collect other forms, such as toilet paper or paper towel tubes. Students love working with recycled materials because they can create large buildings and, ultimately, a town they can walk through like giants. The recycled materials can also be altered by cutting doors, marking windows, and cutting rectangular solids into new shapes. Masking tape works well to attach the boxes, and students can tear the tape themselves.

To make this activity more locally relevant, consider expanding the launch by taking a photo of your own neighborhood or shopping district to show students in addition to the Noticing Buildings sheet we've provided. Students can analyze the shapes that compose local or familiar buildings, including their school. You may even want to take a walking tour around the block to notice and analyze the shapes used to compose buildings. Alternatively, you could conduct this walking tour in conjunction with the discussion of how to construct a town from buildings, noticing what kinds of building typically go together and how streets are used to make a community.

For the launch, you'll need to make a simple building out of two blocks, such as a triangular prism on top of a rectangular solid, that you can show students. This model will help students make connections between two-dimensional and three-dimensional space and notice how they are different.

Note that the discussion can include several different structures, including a gallery walk, whole-class discussion, small-group planning, and class construction of a town. Instead of your holding one long whole-class discussion, we encourage you to think of it in parts that include discussions of different sizes. If you include the construction of the town and possibly have students walk through it, this discussion portion could be an entire day unto itself.

Activity

Launch

Launch the activity by showing students the Noticing Buildings sheet on the document camera. Ask, What do you notice about these buildings? What makes them different or the same? Give students a chance to turn and talk to a partner about the buildings. Take some student ideas and allow students to make all different kinds of observations. Students may notice, for instance, that the buildings get used for different purposes, that they are different heights, or that some look familiar while others do not.

Then ask, What shapes make these buildings? Again, give students a chance to turn and talk to a partner. Invite students to come up and point to the shapes they see composing these buildings. Mark up the image to show the shapes students notice. Point out that different kinds of buildings use different kinds of shapes and that buildings can be made in many different shapes.

At this stage, students are likely to use two-dimensional language to describe what they see, such as *triangle* or *square*, and this makes sense with the image being shown. Tell students that while these images of buildings look flat, buildings in the world are made in three dimensions. Show students the building you built with blocks as an example of what it means to be three-dimensional. Point out that your building is solid, not flat, and so are buildings in the real world. Invite students to notice what shapes make your building, and provide language for these figures if students don't yet have words to describe them, such as *cube, rectangular solid,* or *triangular solid*. Tell students that they will be creating a building with a partner and that as a class you will be putting the buildings together to create a village, town, or city. Introduce students to the materials available for construction and the expectations for their use.

Explore

Provide partners with cardboard boxes or blocks to construct a building that can then be put together with others to compose a village, town, or city. If students are using recycled materials, they will also need access to scissors, masking tape, and markers. Partners discuss the following questions to create a plan:

- What do buildings look like?
- What kind of building might you build?
- What pieces will you need?
- How will you join the pieces together?

Encourage students to assemble and test different kinds of buildings, without attaching the pieces, before they settle on a plan for one building. As you circulate, conferring with students, ask questions to get them talking about the shapes they are using and the shapes' attributes or parts. Support students in using geometric language, such as *face* or *edge*, to describe what they are doing, joining, or planning.

Encourage students to add details to their buildings—for example, cutting doors, drawing windows, or labeling the buildings with signs—to make it clear what their building is.

Discuss

Hold a gallery walk of the buildings students have constructed. As students walk, ask them to think about the following questions:

- Can you figure out what each building is? What clues helped you?
- How are the buildings similar? How are they different?
- Which buildings would go near one another in a town or city? Why?

Gather as a class to discuss students' observations about the individual buildings and how they compare to one another. Ask, What buildings did you find most interesting or surprising? Why? As a class, discuss the question, How could we put our buildings together to make a village, town, or city? Develop a plan for assembling the buildings into a community. The class will need to consider where to put roads, what buildings go together and why, and how to make use of both sides of the street. You may want to allow students some small-group discussion time to develop ideas they can share and debate in the whole group.

Compose your city. You may want to mark off roads using masking tape. If you have built a large city with cardboard buildings, consider giving students a chance to walk through the city and make observations. Then compare buildings and their components by discussing the following questions:

- Which buildings have something in common? What is it?
- What differences do you see among the shapes we've made?
- What parts (3-D figures) were most commonly used? Why do you think that is?
- What parts (3-D figures) don't appear as parts of our buildings? Why do you think that is?

Look-Fors

- **How are students thinking about composing and decomposing with shapes to make buildings?** As you talk with students about their buildings, ask questions about the shapes they are using to compose larger buildings. What shapes might make up a building? How do different shapes create parts of a building? How do shapes work together to make a whole building? Buildings often have parts that we describe by their function rather than their shape, such as a lobby, apartments, or garage. Support students in connecting these parts of their buildings with the three-dimensional figures that they are using to represent them. If students are struggling to think about how to build a particular kind of building, it may be because they need a visual reference to decompose. You can support students in decomposing the building they hope to construct by showing them images of that type of building from the Noticing Buildings sheet, books, or the internet. When students can see the building they want to construct, ask, What shapes make this building? How could you make this building using the materials we have?

- **How are students grappling with different components and sizes when trying to make figures fit together?** If you are using recycled materials to construct buildings, you likely have a collection of boxes of various sizes that, unlike a set of wooden blocks, don't coordinate with one another. Making a tower, for instance, might require several boxes of the same size, but students may not have access to multiple identical shapes, those with the same width, or shapes of the same scale. As you circulate, pay attention to how students are reasoning about the ways that different solids fit together, or do not, and how students are attempting to solve problems of alignment or fit. If their boxes do not fit together in the ways they want, ask students, What could you do to get a better fit? Is there a way you could make this box smaller, or is there a different box you could choose? Encourage students to think about trimming or modifying boxes to get the lengths or angles they want. You may not have, for example, a triangular solid, which students may want for a roof. Ask, Is there a way to turn one of the boxes we have into the shape you want? Noticing that even the boxes themselves can be decomposed into other forms is a useful form of conceptual learning.
- **What language are students using to describe their buildings and their parts?** While students do not need the most formal geometric names for figures, having language for shapes makes it easier to discuss and compare them. Listen for the language students are already using, and support students in trying out new terms, particularly when they struggle to find a name for something they are trying to describe. Students will benefit from words to describe the forms they come in contact with most often—*prisms* or *solids*. With one of these core terms, students can then apply the two-dimensional names they have to describe the shape of the base of these objects to come up with a method for describing many of the solids they come across: rectangular solid or prism, triangular solid or prism, and so on. You may also have some additional shapes in your materials that have specialized names, such as cylinders or cubes. As you listen to students, ask them to describe the shapes they have used to construct their buildings and provide names for these parts when they use less formal language (such as *box*).

Reflect

What shapes are used most often to construct buildings? Why? What could it look like to construct a building that used unusual shapes?

Noticing Buildings

Tangram Puzzles

Snapshot

Students explore tangram puzzles as they develop ideas about composing and decomposing two-dimensional figures.

Connection to CCSS
1.G.2, 1.G.1

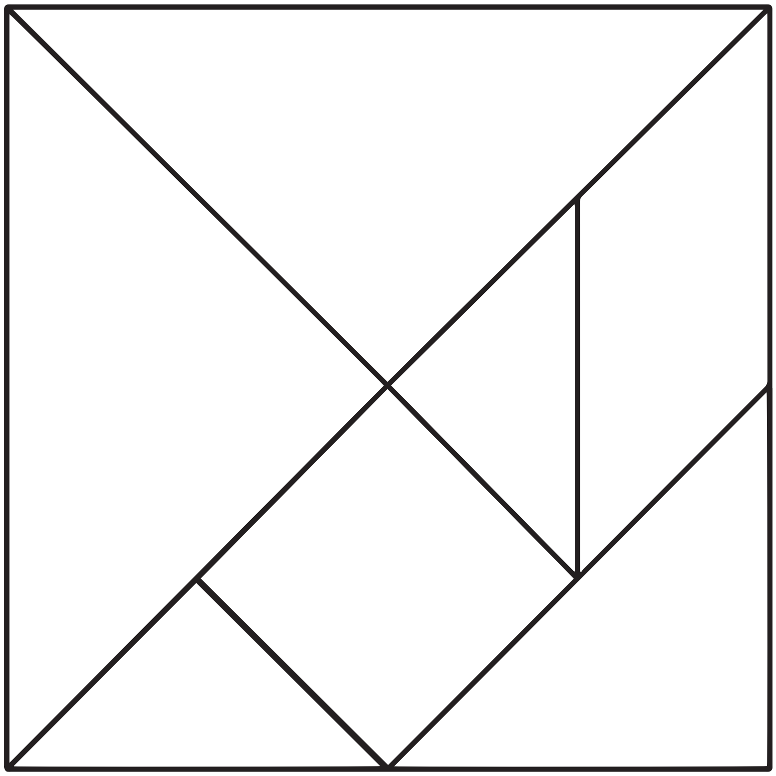
Agenda

Activity	Time	Description/Prompt	Materials
Launch	10 min	Show students the seven separate tangram pieces and ask students to make observations. Name each piece together. Tell students that this collection is called a tangram set, and provide a little history. Show students one of the tangram puzzles and explain that the goal is to use all seven pieces to compose the shape.	• Tangram set, to display • One of the tangram puzzles, to display
Play	25+ min	Partners choose a tangram puzzle to work on side by side, with each student having their own set of tangrams and copy of the puzzle sheet. Students explore how to compose the shape using the tangrams and outline the solutions they come up with.	• Tangram set, one per student • Make available: copies of all tangram puzzles, such that each student can complete multiple puzzles
Discuss	10–15 min	Discuss the strategies that students developed for solving the tangram puzzles, the clues in the figure outlines that helped them, and what made the puzzles challenging. Emphasize the language of composing and decomposing as you discuss students' thinking.	Tangram puzzles and tangram set, to display

Activity	Time	Description/Prompt	Materials
Extend	30+ min	Partnerships design their own tangram puzzle by creating a shape, outlining it, and giving it a title. Partnerships swap puzzles and try to solve. Discuss what made designing these puzzles challenging and how students had to think to compose a shape on their own.	• Tangram set, per student • Blank paper and marker • Optional: plastic sheet protectors and dry- or wet-erase markers

To the Teacher

In this Play activity, students explore ideas about composing and decomposing two-dimensional figures with tangrams, the classic dissection puzzle. Tangrams are considered a *dissection* because they are formed by dissecting, or decomposing a larger shape, in this case a square.

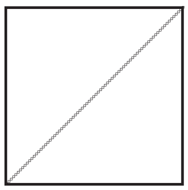


In this activity, we do not yet focus on the formation of the square—this will be at the heart of the Investigate activity. Rather, we familiarize students with tangrams through composing larger shapes in tangram puzzles. You can find literally thousands of these puzzles in books and online resources. We have provided six here, which are designed for students to overlay their pieces directly on the sheets. These puzzles fit tangrams that form a 4″ square. We encourage you to explore other puzzles, particularly if you'd like to find those that connect with your students' interests or other class activities, such as read-alouds or science explorations. You can also use additional puzzles to turn this activity into an ongoing center or station.

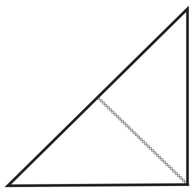
If you decide to incorporate other puzzles, bear in mind that the presentation of the puzzle makes a difference in its complexity for students. Puzzles that are the same size as the pieces and that can be copied for children to overlay their pieces, as ours are, allow students to focus their cognitive effort on composing the figure. Cognitive effort is increased by removing either of these supports. For instance, if you present the puzzle on a document camera, students will need to work to transfer between the image and their work surface. Similarly, if students have a printed image but it is too small to overlay, they will need to transfer their thinking between the image and their work. We suggest waiting until students have lots of experience with these puzzle before introducing these challenges.

If you don't have access to tangrams sets, students can construct their own out of square pieces of paper. Provide students with 4″ squares to match these puzzles, then lead them through the following directions to fold and cut pieces.

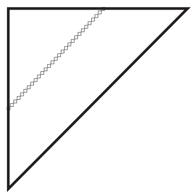
1. Start with a 4″ × 4″ square. Fold it in half along one diagonal. Cut along the diagonal making two large congruent triangles.



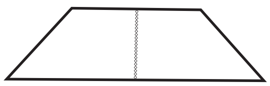
2. Cut one of the two large triangles in half.



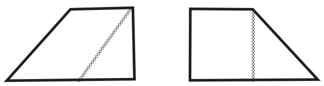
3. Take the other large triangle and fold the vertex of the right angle to the midpoint of the opposite side and cut along the line.



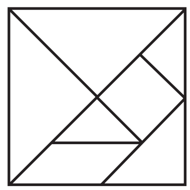
4. Put the two large and one medium triangles aside. Fold the remaining trapezoid in half and cut along the fold line.



5. Fold each trapezoid along the lines shown below and make a cut on the fold lines.



6. Now you have two large triangles, one medium triangle, two small triangles, one square and one parallelogram. The seven pieces make up the tangram set of shapes.



If you'd like to incorporate read-aloud into this lesson, there are several picture books that use tangrams. *Grandfather Tang's Story* by Ann Tompert is a classic, and you can use the tangram puzzles in the book as part of students' exploration.

Activity

Launch

Launch the activity by showing students the tangram pieces separately, rather than arranged in the square, on the document camera. Ask students, What do you notice about these shapes? Give students a chance to turn and talk about their observations. Discuss students' observations, descriptions, and names for these shapes. Be sure to come to agreement on a name for each shape.

Tell students that this is a famous collection of shapes called a *tangram set*, and tangrams are always composed of these seven shapes, sometimes called *tans*. Tangrams originated in China hundreds of years ago, though no one knows exactly when they were created or by whom. Tangrams are used for puzzles, which became very popular in Europe and America when they first arrived about 200 years ago. Show students one of the tangram puzzles they can work on today and tell them that in each puzzle, all seven pieces are used to make the shape, and the goal is to figure out how. Thousands of these puzzles have been created over hundreds of years, and today students will get to try out a few.

Play

Provide each student with a tangram set and access to tangram puzzles. Partners choose which puzzle to begin with, and each partner gets one copy of the puzzle. Partners explore the question, How can you use the set of tangram pieces to construct each shape? Students can work together on the same puzzle, but each must have their own space to explore, compose, and make adjustments. Encourage partners to sit side by side so that they can borrow each other's ideas and learn from each other's efforts. For each puzzle they solve, students outline their solutions on the puzzle sheet.

Discuss

Discuss the following questions as a class:

- What strategies did you use to solve these puzzles?
- What clues did you notice in the puzzles that helped you figure out where the pieces could go? (Show the puzzles that students want to discuss on the document camera, so that they can point out the features of the figure that supported them in finding a solution.)

- What made solving these challenging?
- What did you notice about the ways the pieces fit together (or didn't)? (Have the tangram pieces on the document camera so that students can show the ways they found the pieces fit together.)

As you discuss these puzzles, be sure to use the language of *composing* and *decomposing* shapes. Composing and decomposing, both with shapes and later with number, are big ideas that thread throughout first grade and beyond.

Extend

Invite students to create their own tangram puzzles. Partners design a figure, outline it on a blank piece of paper, and give it a title. Partnerships can swap their puzzles with those of other students and try to solve. If you'd like these puzzles to be used repeatedly, they can be placed in plastic sheet protectors so that the solution can be outlined with dry- or wet-erase markers and later erased. After students have designed their puzzles, discuss the following questions:

- What strategies did you use to design your puzzle?
- How did you have to think about the shapes to compose a larger figure?
- What made designing a figure challenging?

Look-Fors

- **Are students noticing shapes inside the tangram puzzles?** Some tangram puzzles provide clues about the ways pieces are used to construct them. Heads, feet, or other parts of the puzzles that stick out can point students toward particular pieces. By noticing these features, students can begin to decompose the whole into component parts, reducing the space that needs to be solved. If students are struggling to find an entry point for a puzzle, ask, Is there any part of this puzzle that gives you a clue about what piece might go there? Or, Can you see any shapes hidden inside this puzzle? Decomposing the tangram puzzles is a process, not a revelation, and needs to be tackled piece by piece.
- **Are students revising their solution ideas as they work?** As students work to solve these puzzles, it is useful if they recognize that each piece they put down may need to be moved. Some students may conceive of solving the puzzles as a series of choices (as in, "I'm going to put this piece here"), rather than as a process of testing and revising ideas toward a solution. Encourage students

to try out different pieces in different positions and to recognize that there may be multiple positions in which a particular piece *might* work. In this way, if students cannot find a solution with, for example, the largest triangles in a particular arrangement, then they would have something else to try next. Ask questions about possibility, such as, Where could this piece go? If this piece is here, where could these other pieces go? If this doesn't work, what could you change?

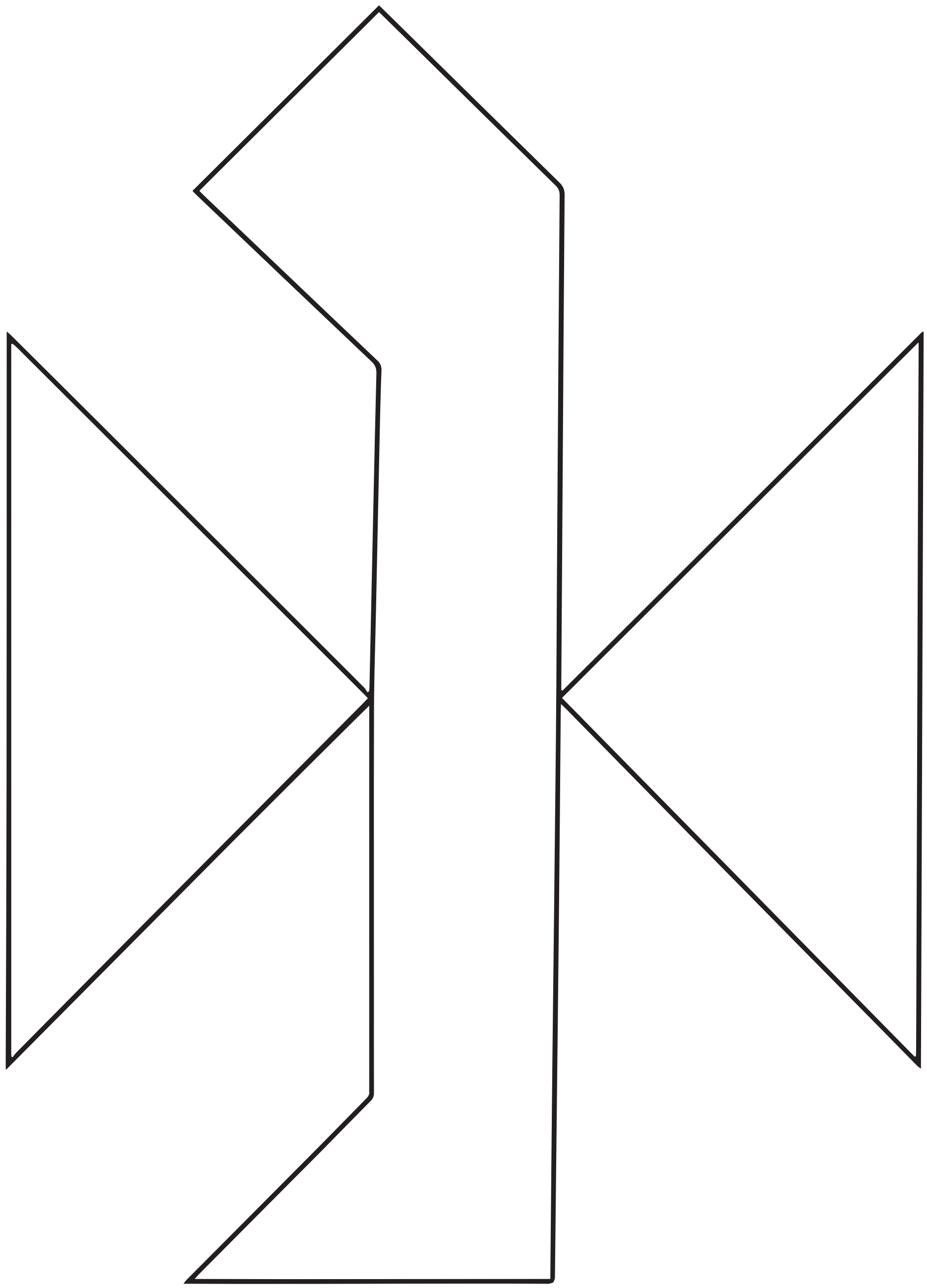
Reflect

What shapes in the tangram set fit well together? Show an example. Why do they fit together?

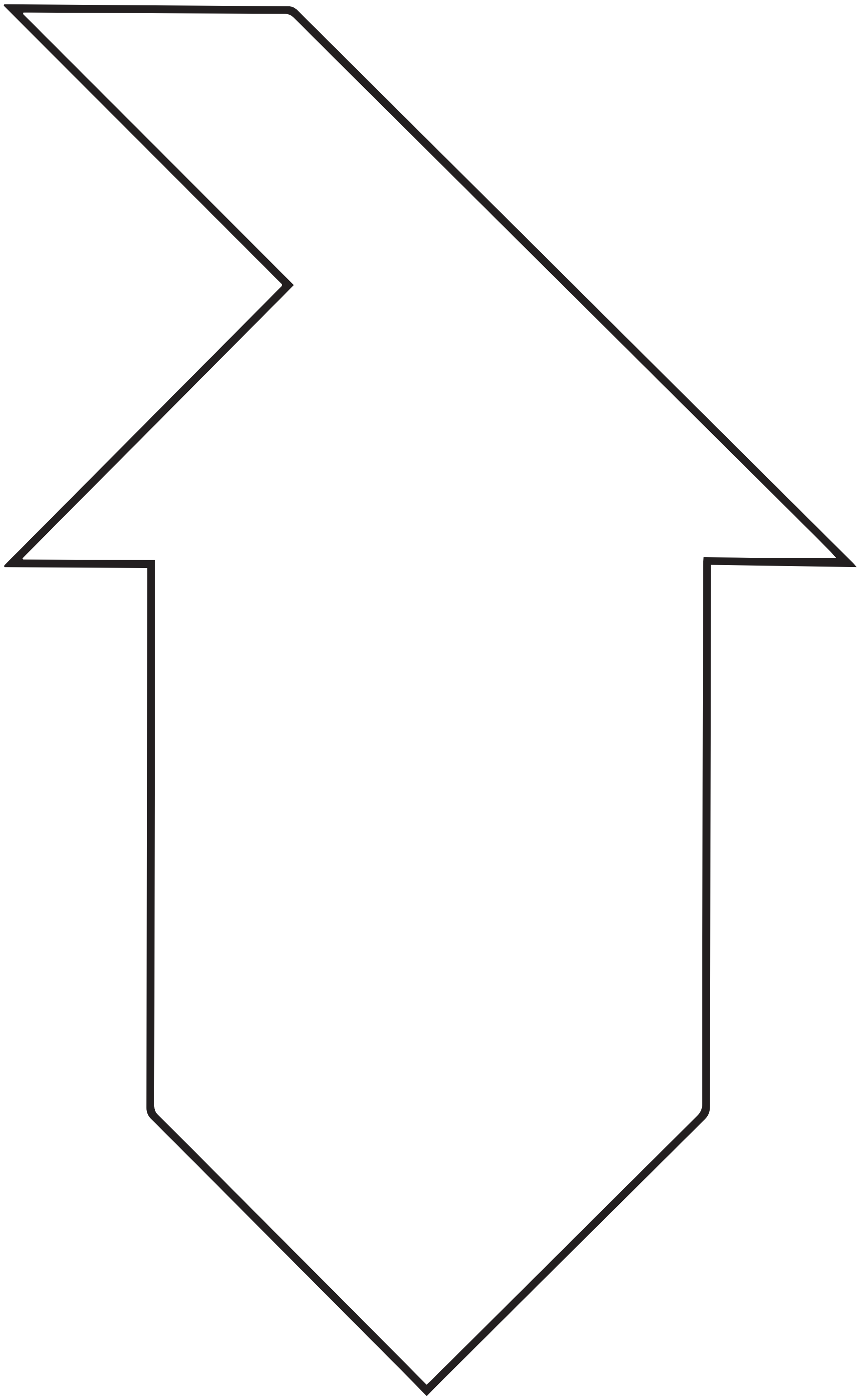
Reference

Tompert, A. (1997). *Grandfather Tang's story*. New York, NY: Dragonfly Books.

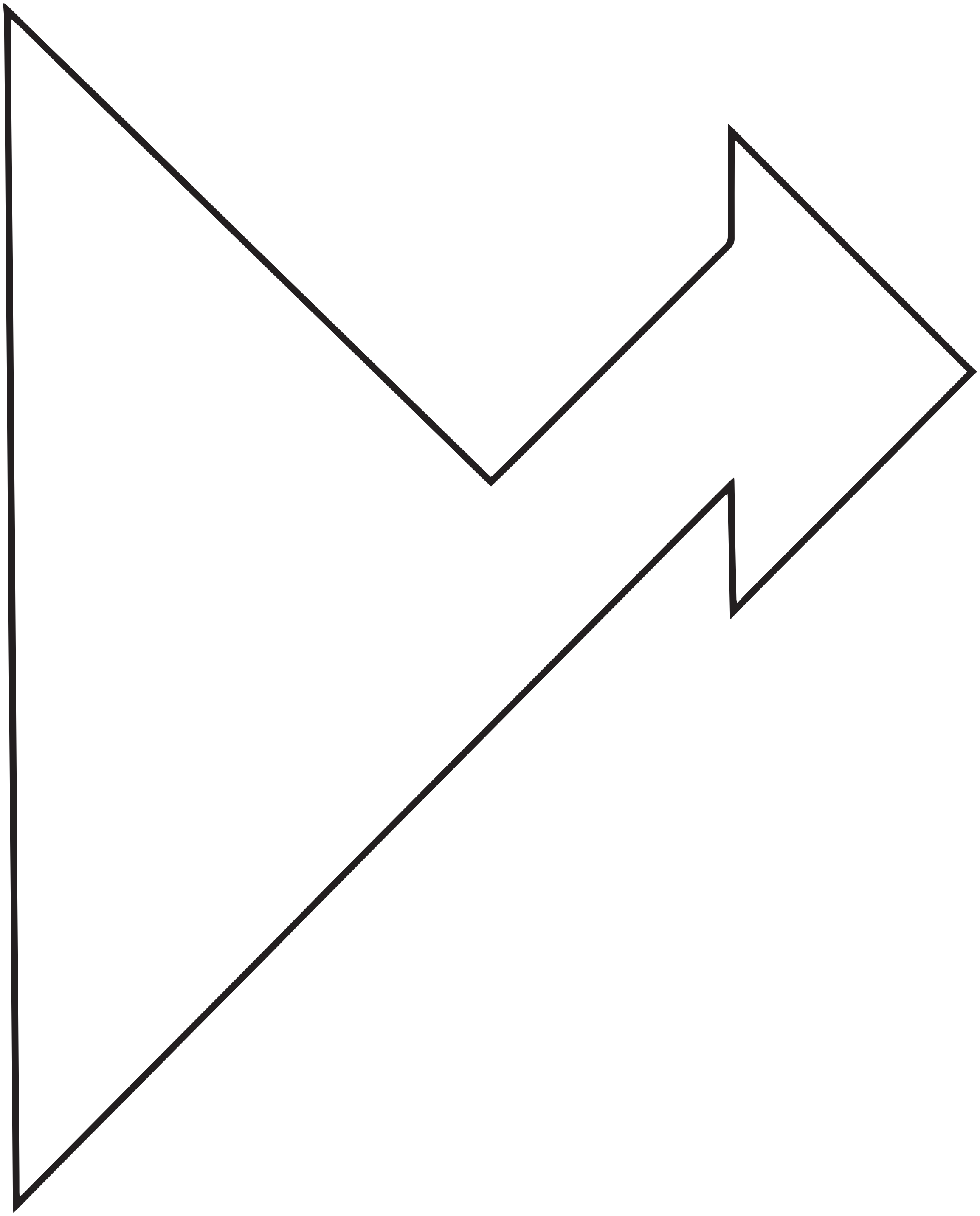
Tangram Puzzle: Dragon



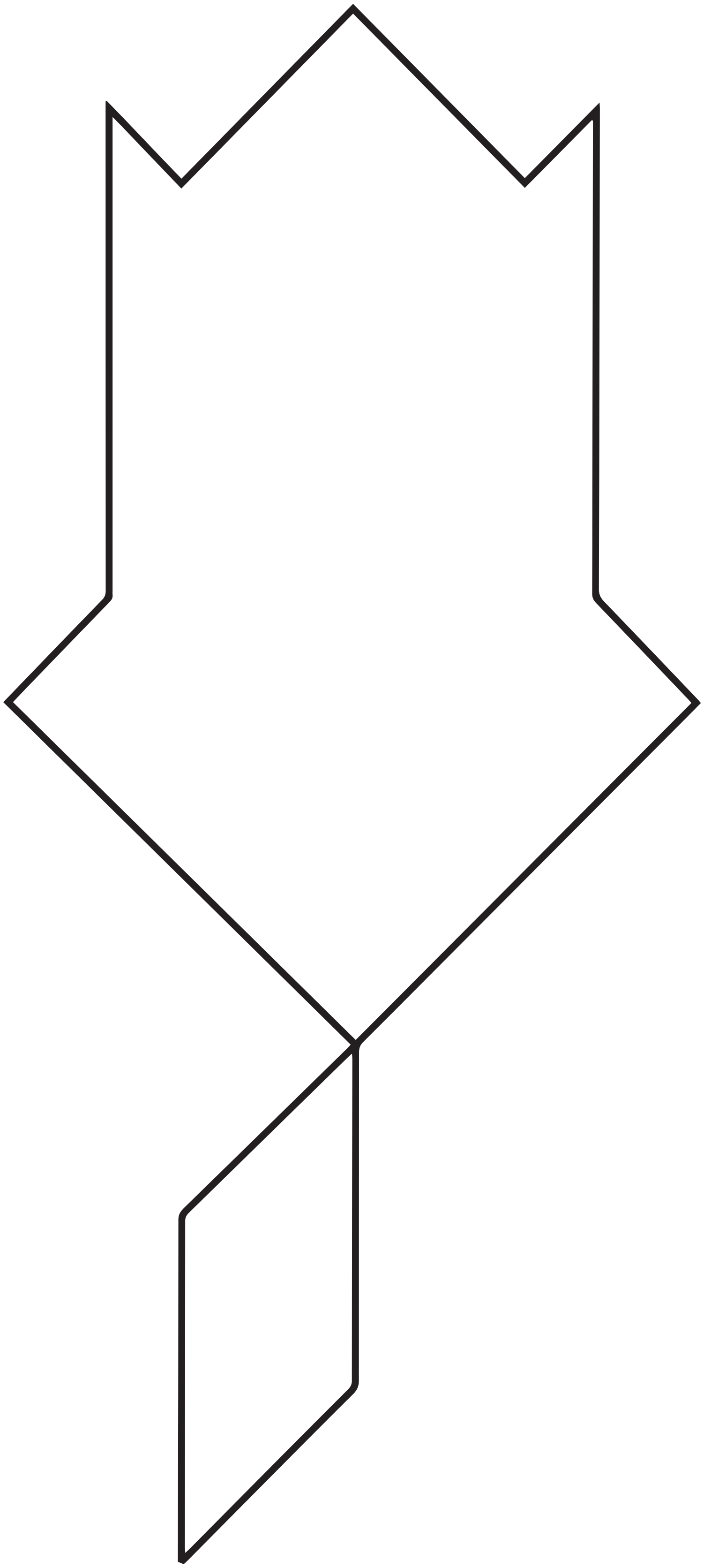
Tangram Puzzle: Acorn



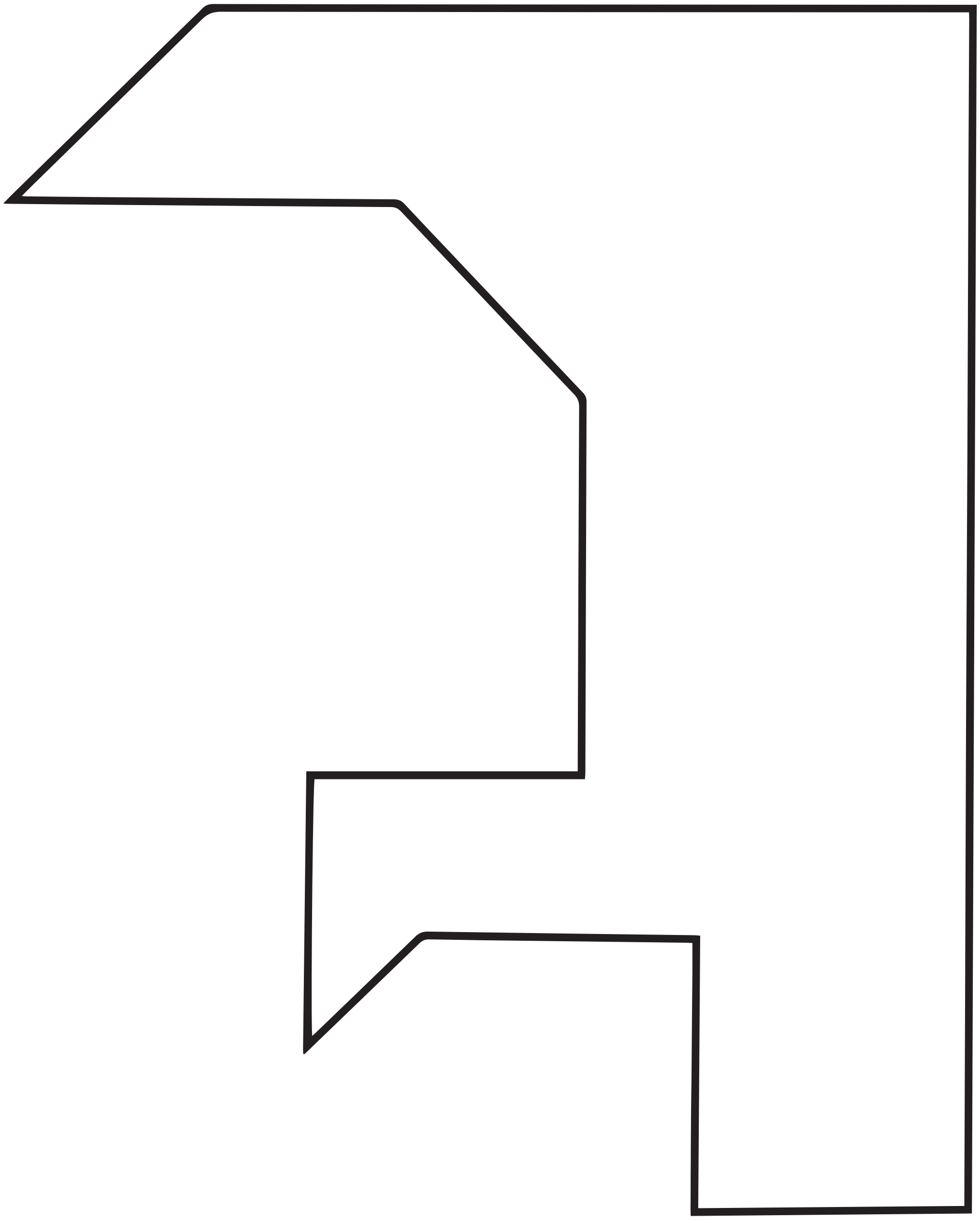
Tangram Puzzle: Storm



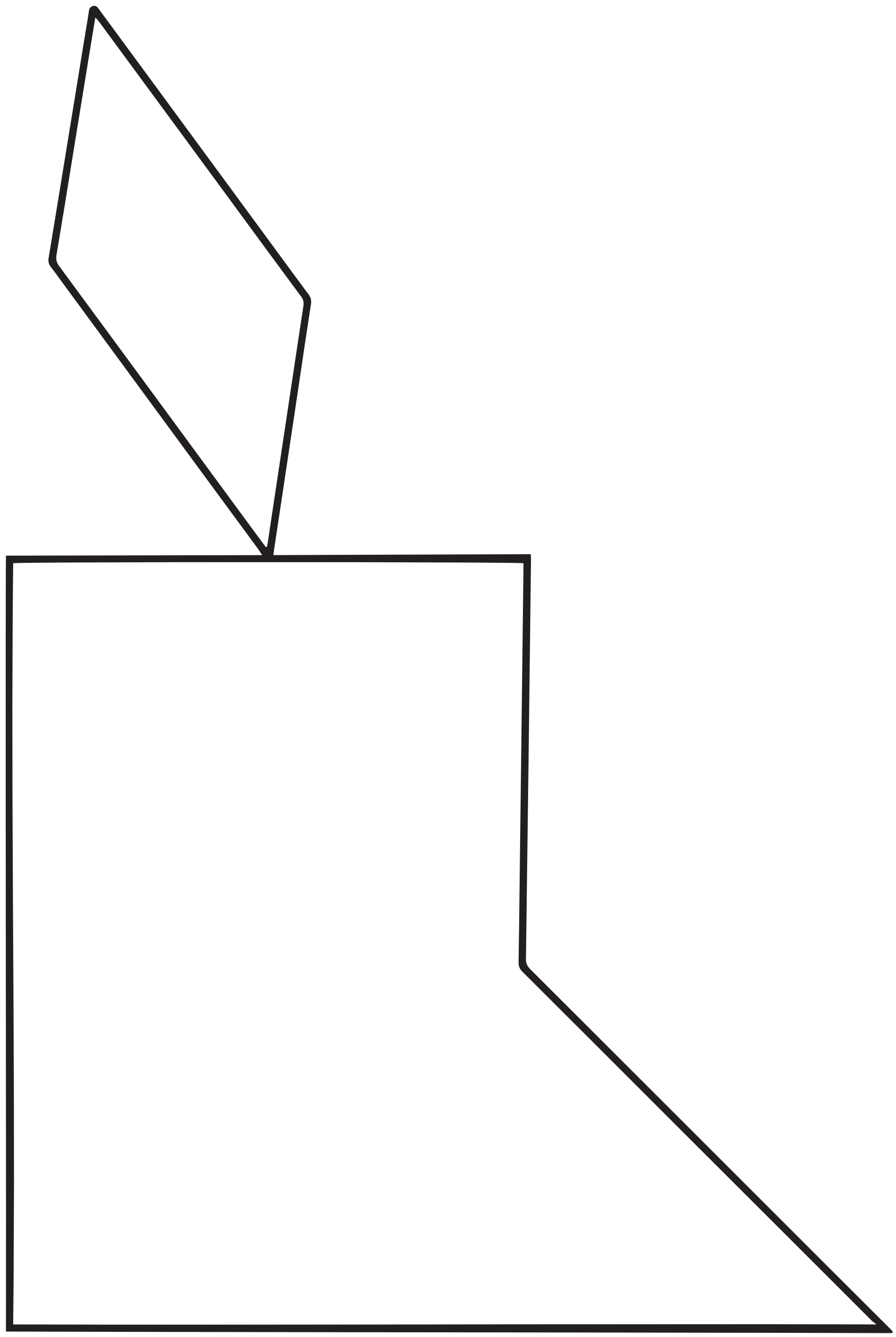
Tangram Puzzle: Flying Squirrel



Tangram Puzzle: Factory



Tangram Puzzle: Candle



Tangram Squares

Snapshot

Students use tangrams to explore composing squares and investigate what makes two shapes or solutions the same or different.

Connection to CCSS
1.G.2, 1.G.1

Agenda

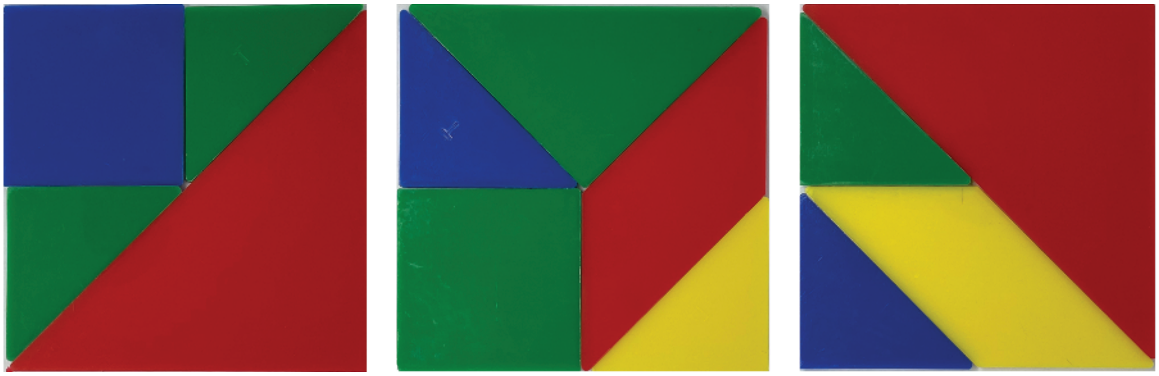
Activity	Time	Description/Prompt	Materials
Launch	10–15 min	Ask students what they know about squares, and after a turn and talk, record students' ideas on a chart. Support students in distinguishing between defining attributes and those that are not (such as color and size). Show students a set of separate tangram pieces and ask, What squares could you make with these pieces? Give students some time to think and visualize.	• Chart and markers • Tangram set, to display
Explore	20–30 min	Partners work together to generate as many squares using tangram pieces as they can, recording each on paper. Students investigate the question, What is the smallest square you can make? What is the largest square you can make? How do you know?	• Tangram set, per partnership • Tools for recording, such as half sheets of blank paper and rulers
Discuss	15 min	Invite students to share some of the squares they designed, then as a class figure out how to order the squares everyone made from smallest to largest in a display space. Looking at all the squares together, discuss whether there are different ways of constructing squares of the same size and whether the class has found all the possible squares.	• Display space and tools for posting students' squares

Activity	Time	Description/Prompt	Materials
Extend	30+ min	Provide partners with two sets of tangrams. Students investigate what squares they can make with 14 pieces and how this changes the size of the squares and number of solutions.	• Two sets of tangrams, per partnership • Tools for recording, such as half sheets of blank paper and rulers (consider using a different color of paper for the second set of squares)

To the Teacher

This activity, adapted from a task developed by NRich (nrich.maths.org), invites students to explore how to use the shapes in a set of tangrams to compose squares of different sizes. Key to this exploration is a shared understanding of what a square is. Students are likely to have informal knowledge of squares, which they encounter regularly and which are often named by adults, but they may not have ever articulated clear criteria for what makes a square. We encourage you to spend time in the launch discussing what features a square always has so that students can move forward with the investigation with a shared definition.

As students explore, the question of what constitutes "the same" is going to come up, and we see the discussion as a space for the class to work together to define what *same* means. For instance, are two squares that are the same size always the same, even if they have been constructed with different pieces? Are two squares that use the same pieces always the same, or does the orientation of those pieces matter? We think there is reason to include all versions of any formation of a square in the class's collection of squares. But students may compose a square out of the same pieces only rotated or flipped. Discuss whether two such squares are the same or different, and in doing so, you will be foreshadowing complex mathematical ideas about congruence and geometric transformations, regardless of the decision the class comes to.



The class needs to decide, Are any of these squares "the same"?

Activity

Launch

Launch the activity by asking, What do we know about squares? Give students a chance to turn and talk to a partner. Discuss what students know about squares, such as that they have four sides, they have square corners (right angles), and their sides are the same length. On a chart, record these ideas and draw a square to point out each feature. To help students articulate some of the features that matter, you might say, "I want to draw a square. How do I do that?" As you discuss squares, be sure to support students in distinguishing between attributes that define squares and attributes that do not (such as color or size). Students may also need to specifically discuss rectangles and note that a square is a kind of a rectangle that has sides that are all the same. Be sure that the class has a shared understanding of what makes a square.

Show students the set of tangrams as separate pieces on the document camera. Ask, What squares can you make with these pieces? How could you make a square? Allow students a minute to imagine how they might use the pieces to construct a square, but do not discuss solutions at this point.

Explore

Provide partners with a set of tangrams and space to record their squares. Half sheets of paper will enable students to record each solution separately, which will make ordering them by size easier later. Students explore the following questions:

- What squares can you make using any of the tangram shapes?
- How can you record the square you made?
- How many different squares can you make?

- What is the smallest square you can make? How do you know?
- What is the largest square you can make? How do you know?

For each square students make, they record both the outline and the shapes used to make it. Students may want access to rulers to support them in recording their thinking.

Discuss

Invite students to share some of the squares they made. For each one that students share on the document camera, you might ask, Did anyone else make this same square in the same way? Discuss the following questions:

- Which shapes were most useful in making squares? Why?
- Which ones were hardest to use to make a square? Why?

As a class, try to order by size the squares that students made. Ask, How can we put these in order from smallest to largest? Work together to create a display of the squares students made. Looking at the collection, discuss the following questions:

- Are there different ways of constructing squares of the same size? What makes two squares "different" squares?
- Do you think these are all the possible squares that could be made with these pieces? Why or why not?

Be sure to dig into discussions of what makes two squares the same or different, and into conversations about systematicity in looking for other squares. These get at larger mathematical ideas around structure—both looking for structure and using it—that will continue to come up throughout students' mathematical lives.

Extend

Extend the investigation by expanding the set of shapes that students can use to make squares. Provide each partnership with two sets of tangrams, a total of 14 pieces. Students explore the same questions:

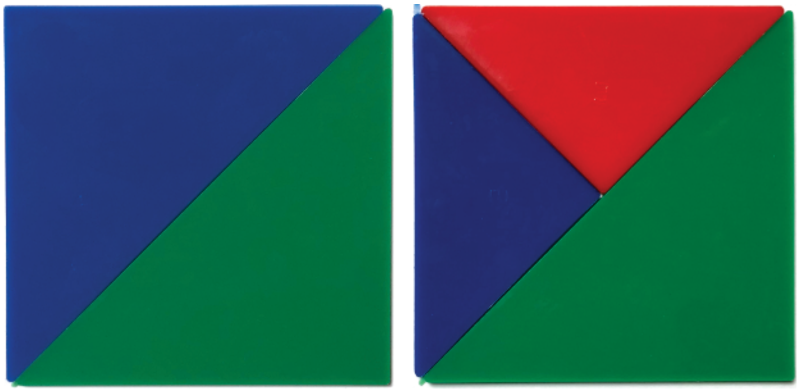
- What squares can you make now?
- What squares can you make with two sets of tangrams that you could not make with one set?

- How many different squares can you make?
- What is the smallest square you can make? How do you know?
- What is the largest square you can make? How do you know?

Add the new squares to your display and discuss how having more pieces changes the number and possible sizes of squares that can be made. You may want to provide students with different colored paper or markers for this extension so that when you add the new squares to your display, you can distinguish between those that were made with one and two sets of tangrams.

Look-Fors

- **Do students have a clear working definition of a square?** Before sending students off to work on this investigation, it is critical that they have a clear and shared working definition of a square, developed during the launch. Students may use informal language to describe a square, such as simply providing examples in the classroom. Use these examples to press the class to develop criteria for what makes a square. You can revise the question to say, "I want to draw a square. How do I do it?" and use students' directions as places to press for reasoning. For instance, a student might say, "Draw a line," and you can ask, "How do I know when to stop?" which can ultimately get students to articulate that the sides need to be the same length. You might deliberately draw lines in ways you know will not create a square—for example, making an acute angle or sides of two different lengths—and when students protest, again press for why. Another frequent confusion is that squares are not rectangles. If you hear students voice this, gently reframe their language. For instance, if you drew two sides with different lengths, students may say, "You can't do that. That will make a rectangle," then you can say, "A square is a kind of rectangle. What makes it different from this one?"
- **Are students using equivalence to generate new solutions?** This task is a great place for students to think systematically about how to generate new solutions using equivalence. Several of the pieces in a tangram set can be swapped out for other pieces that cover an equivalent area, making a new solution to the task. For instance, students might make a square using the two largest triangles. If they notice that one of the triangles is equivalent to two medium triangles, they can exchange one large for two medium triangles and create a new solution.



Do students use equivalence to substitute the large triangle for two medium triangles?

Students are unlikely to have language to describe the relationships they are working with, but the underlying idea of equivalence is a critical one in first grade, whether we are thinking of shapes or number. If you notice students generating solutions this way, ask them questions about their strategy and invite them to share in the discussion. To encourage students to think about equivalence, you can ask students questions such as, How could you change the square you made to make a new square?

- **How are students conceiving of size when comparing squares?** We have challenged students to create the smallest and largest squares possible and, ultimately, to order these squares by size. Our intent is to think about a square's size by its side lengths, such that squares with longer sides are larger. However, some students may be conceiving of square size based on the number of pieces used to construct it. In this way, the two squares discussed in the previous question would be considered different sizes, because one is made with two pieces and the other is made with three. As you circulate, ask questions about which squares are bigger. Remind students that the question is what makes a square larger, and to focus attention on the outline of the figure rather than its components when considering size. During the discussion, press students to think about what makes a square bigger.

Reflect

What questions are you wondering now about tangrams? How could you explore your questions?