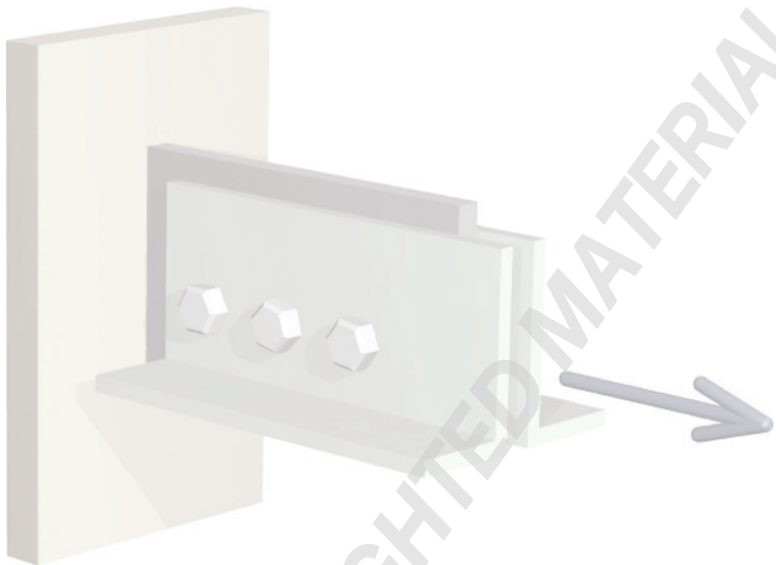
CHAPTER 1

Stress



1.1 Introduction

The three fundamental areas of engineering mechanics are statics, dynamics, and mechanics of materials. Statics and dynamics are devoted primarily to the study of *external* forces and motions associated with particles and rigid bodies (i.e., idealized objects in which any change of size or shape due to forces can be neglected). Mechanics of materials is the study of the *internal* effects caused by external loads acting on real bodies that deform (meaning objects that can stretch, bend, or twist). Why are the internal effects in an object important? The reason is that engineers are called upon to design and produce a variety of objects and structures, such as automobiles, airplanes, ships, pipelines, bridges, buildings, tunnels, retaining walls, motors, and machines—and these objects and structures are all subject to internal forces, moments, and torques that affect their properties and operation. Regardless of the application, a safe and successful design must address the following three mechanical concerns:

1. **Strength:** Is the object strong enough to withstand the loads that will be applied to it? Will it break or fracture? Will it continue to perform properly under repeated loadings?
2. **Stiffness:** Will the object deflect or deform so much that it cannot perform its intended function?
3. **Stability:** Will the object suddenly bend or buckle out of shape at some elevated load so that it can no longer continue to perform its function?

Addressing these concerns requires both an assessment of the intensity of the internal forces and deformations acting within the body and an understanding of the mechanical characteristics of the material used to make the object.

Mechanics of materials is a basic subject in many engineering fields. The course focuses on several types of components: bars subjected to axial loads, shafts in torsion, beams in bending, and columns in compression. Numerous formulas and rules for design found in engineering codes and specifications are based on mechanics-of-materials fundamentals associated with these types of components. With a strong foundation in mechanics-of-materials concepts and problem-solving skills, the student is well equipped to continue into more advanced engineering design courses.

1.2 Normal Stress Under Axial Loading

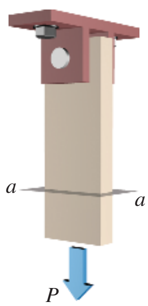


FIGURE 1.1*a* **Bar with axial load *P*.**

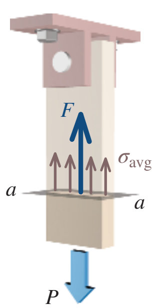


FIGURE 1.1*b* **Average stress.**

In every subject area, there are certain fundamental concepts that assume paramount importance for a satisfactory comprehension of the subject matter. In mechanics of materials, such a concept is that of **stress**. In the simplest qualitative terms, *stress is the intensity of internal force*. Force is a vector quantity and, as such, has both magnitude and direction. Intensity implies an area over which the force is distributed. Therefore, stress can be defined as

$$\text{Stress} = \frac{\text{Force}}{\text{Area}} \tag{1.1}$$

To introduce the concept of a **normal stress**, consider a rectangular bar subjected to an axial force (**Figure 1.1*a***). An **axial force** is a load that is directed along the longitudinal axis of the member. Axial forces that tend to elongate a member are termed **tension forces**, and forces that tend to shorten a member are termed **compression forces**. The axial force P in Figure 1.1a is a tension force. To investigate internal effects, the bar is cut by a transverse plane, such as plane a–a

> The technique of cutting an object to expose the internal forces acting on a plane surface is often referred to as the **method of sections**. The cutting plane is called the **section plane**. To investigate internal effects, one might simply say something like "Cut a section through the bar" to imply the use of the method of sections. This technique will be used throughout the study of mechanics of materials to investigate the internal effects caused by external forces acting on a solid body.

of Figure 1.1a, to expose a free-body diagram of the bottom half of the bar (**Figure 1.1*b***). Since this cutting plane is perpendicular to the longitudinal axis of the bar, the exposed surface is called a **cross section**.

Equilibrium of the lower portion of the bar is attained by a distribution of internal forces that develops on the exposed cross section. This distribution has a resultant internal force F that is normal to the exposed surface, is equal in magnitude to P, and has a line of action that is collinear with the line of action of P. The intensity of F acting in the material is referred to as stress.

In this instance, the stress acts on a surface that is *perpendicular* to the direction of the internal force F. A stress of this type is called a **normal stress**, and it is denoted by the Greek letter σ (sigma). To determine the magnitude of the normal stress in the bar, the average intensity of the internal force on the cross section can be computed as

$$\sigma_{\text{avg}} = \frac{F}{A} \tag{1.2}$$

where A is the cross-sectional area of the bar.

The sign convention for normal stresses is defined as follows:

- A positive sign indicates a *tensile normal stress*, and
- a negative sign denotes a *compressive normal stress*.

Consider now a small area ΔA on the exposed cross section of the bar, as shown in **Figure 1.1*c***, and let ΔF represent the resultant of the internal forces transmitted in this small area. Then the average intensity of the internal force being transmitted in area ΔA is obtained by dividing ΔF by ΔA. If the internal forces transmitted across the section are assumed to be uniformly distributed, the area ΔA can be made smaller and smaller, until, in the limit, it will approach a point on the exposed surface. The corresponding force ΔF also becomes smaller and smaller. The stress at the point on the cross section to which ΔA converges is defined as

$$\sigma = \lim_{\Delta A \to 0} \frac{\Delta F}{\Delta A} \tag{1.3}$$

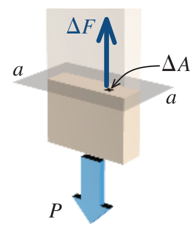


FIGURE 1.1*c* **Stress at a point.**

If the distribution of stress is to be uniform, as in Equation (1.2), the resultant force must act through the centroid of the cross-sectional area. For long, slender, axially loaded members, such as those found in trusses and similar structures, it is generally assumed that the normal stress is uniformly distributed except near the points where the external load is applied. Stress distributions in axially loaded members are not uniform near holes, grooves, fillets, and other features. These situations will be discussed in later sections on stress concentrations. *In this book, it is understood that axial forces are applied at the centroids of the cross sections unless specifically stated otherwise.*

Stress Units

Since the normal stress is computed by dividing the internal force by the cross-sectional area, stress has the dimensions of force per unit area. When U.S. customary units are used, stress is commonly expressed in pounds per square inch (psi) or kips per square inch (ksi) where 1 kip = 1,000 lb. When the International System of Units, universally abbreviated SI (from the French *Système International d'Unités*), is used, stress is expressed in pascals (Pa) and computed as force in newtons (N) divided by area in square meters (m^2). For typical engineering applications, the pascal is a very small unit and, therefore, stress is more commonly expressed in megapascals (MPa) where 1 MPa = 1,000,000 Pa. A convenient alternative when calculating stress in MPa is to express force in newtons and area in square millimeters (mm^2). Therefore,

$$\boxed{1\text{ MPa} = 1{,}000{,}000\text{ N/m}^2 = 1\text{ N/mm}^2} \tag{1.4}$$

Significant Digits

In this book, final numerical answers are usually presented with three significant digits when a number begins with the digits 2 through 9 and with four significant digits when the number begins with the digit 1. Intermediate values are generally recorded with additional digits to minimize the loss of numerical accuracy due to rounding.

In developing stress concepts through example problems and exercises, it is convenient to use the notion of a **rigid element**. Depending on how it is supported, a rigid element may move vertically or horizontally, or it may rotate about a support location. The rigid element is assumed to be infinitely strong.

EXAMPLE 1.1

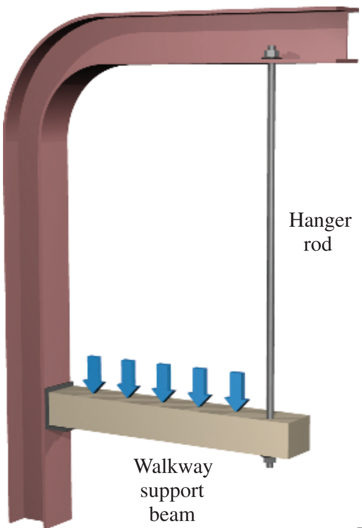
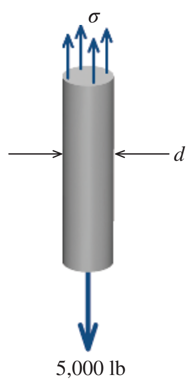


Figure 1

A solid 0.5 in. diameter steel hanger rod is used to hold up one end of a walkway support beam (**Figure 1**). The force carried by the rod is 5,000 lb. Determine the normal stress in the rod. (Disregard the weight of the rod.)



Free-body diagram of hanger rod.

Figure 2

Solution

A free-body diagram of the rod is shown (**Figure 2**). The solid rod has a circular cross section, and its area is computed as

$$A = \frac{\pi}{4}d^2 = \frac{\pi}{4}(0.5\text{ in.})^2 = 0.19635\text{ in.}^2$$

where d = rod diameter.

Since the force in the rod is 5,000 lb, the normal stress in the rod can be computed as

$$\sigma = \frac{F}{A} = \frac{5{,}000\text{ lb}}{0.19635\text{ in.}^2} = 25{,}464.73135\text{ psi}$$

Although this answer is numerically correct, it would not be proper to report a stress of 25,464.73135 psi as the final answer. A number with this many digits implies an accuracy that we have no right to claim. In this instance, both the rod diameter and the force are given with only one significant digit of accuracy; however, the stress value we have computed here has 10 significant digits.

In engineering, it is customary to round final answers to three significant digits (if the first digit is not 1) or four significant digits (if the first digit is 1). Using this guideline, the normal stress in the rod would be reported as

$$\sigma = 25{,}500 \text{ psi}$$ **Ans.**

In many instances, the illustrations in this book attempt to show objects in realistic three-dimensional perspective. Wherever possible, an effort has been made to show free-body diagrams within the actual context of the object or structure. In these illustrations, the free-body diagram is shown in full color while other portions of the object or structure are faded out.

EXAMPLE 1.2

Rigid bar *ABC* is supported by a pin at *A* and axial member (1), which has a cross-sectional area of 540 mm^2 (**Figure 1**). The weight of rigid bar *ABC* can be neglected. (**Note:** 1 kN = 1,000 N.)

a. Determine the normal stress in member (1) if a load of P = 8 kN is applied at *C*.

b. If the maximum normal stress in member (1) must be limited to 50 MPa, what is the maximum load magnitude P that may be applied to the rigid bar at *C*?

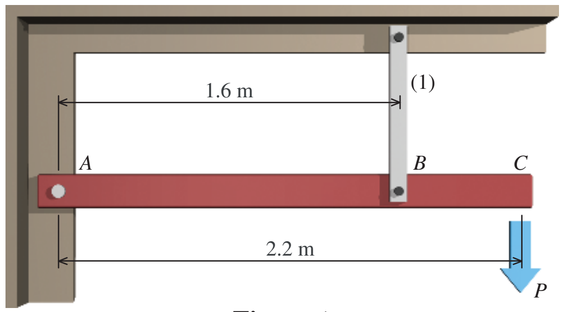
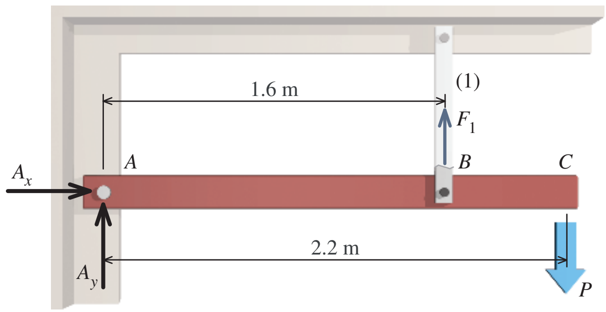


Figure 1



Free-body diagram of rigid bar *ABC*.

Figure 2

Plan the Solution

(Part a)

Before the normal stress in member (1) can be computed, its axial force must be determined. To compute this force, consider a free-body diagram of rigid bar *ABC* and write a moment equilibrium equation about pin *A* (**Figure 2**).

Solution

(Part a)

For rigid bar *ABC*, write the equilibrium equation for the sum of moments about pin *A*. Let F_1 = internal force in member (1) and assume that F_1 is a tension force. Positive moments in the equilibrium equation are defined by the right-hand rule. Then

$$\Sigma M_A = -(8 \text{ kN})(2.2 \text{ m}) + (1.6 \text{ m})F_1 = 0$$

$$\therefore F_1 = 11 \text{ kN}$$

The normal stress in member (1) can be computed as

$$\sigma_1 = \frac{F_1}{A_1} = \frac{(11 \text{ kN})(1{,}000 \text{ N/kN})}{540 \text{ mm}^2} = 20.370 \text{ N/mm}^2 = 20.4 \text{ MPa}$$ **Ans.**

(Note the use of the conversion factor 1 MPa = 1 N/mm^2.)

Plan the Solution

(Part b)

Using the stress given, compute the maximum force that member (1) may safely carry. Once this force is computed, use the moment equilibrium equation to determine the load P.

Solution

(Part b)

Determine the maximum force allowed for member (1):

$$\sigma = \frac{F}{A}$$

$$\therefore F_1 = \sigma_1 A_1 = (50 \text{ MPa})(540 \text{ mm}^2) = (50 \text{ N/mm}^2)(540 \text{ mm}^2) = 27{,}000 \text{ N} = 27 \text{ kN}$$

Compute the maximum allowable load P from the moment equilibrium equation:

$$\Sigma M_A = -(2.2 \text{ m})P + (1.6 \text{ m})(27 \text{ kN}) = 0$$

$$\therefore P = 19.64 \text{ kN}$$ **Ans.**

EXAMPLE 1.3

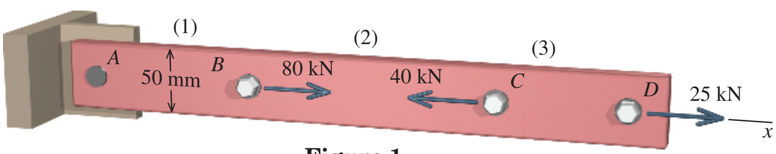


Figure 1

A 50 mm wide steel bar has axial loads applied at points B, C, and D (**Figure 1**). If the normal stress magnitude in the bar must not exceed 60 MPa, determine the minimum thickness that can be used for the bar.

Plan the Solution

Draw free-body diagrams that expose the internal force in each of the three segments. In each segment, determine the magnitude and direction of the internal axial force required to satisfy equilibrium. Use the largest-magnitude internal axial force and the allowable normal stress to compute the minimum cross-sectional area required for the bar. Divide the cross-sectional area by the 50 mm bar width to compute the minimum bar thickness.

Solution

Begin by drawing a free-body diagram (FBD) that exposes the internal force in segment (3) (**Figure 2**). Since the reaction force at A has not been calculated, it will be easier to cut through the bar in segment (3) and consider the portion of the bar starting at the cut surface and extending to the free end of the bar at D. An unknown internal axial force F_3 exists in segment (3), and it is helpful to establish a consistent convention for problems of this type.

Problem-Solving Tip:

When cutting an FBD through an axial member, assume that the internal force is tension and draw the force arrow directed *away from the cut surface*. If the computed value of the internal force turns out to be a positive number, then the assumption of tension is confirmed. If the computed value turns out to be a negative number, then the internal force is actually compressive.

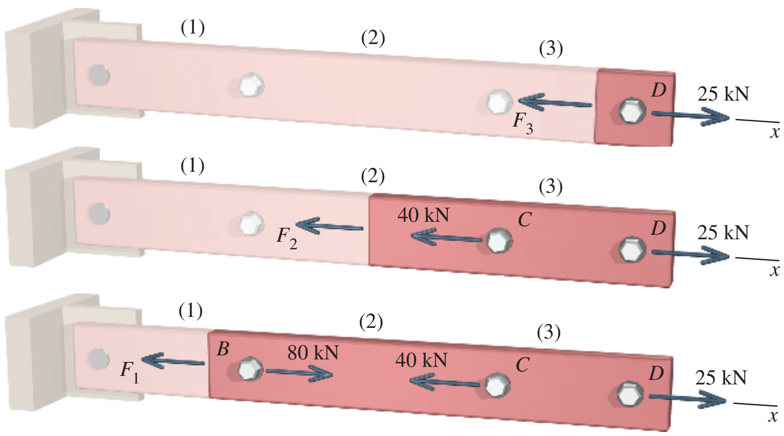


Figure 2

On the basis of an FBD cut through axial segment (3), the equilibrium equation is

$$\Sigma F_x = -F_3 + 25 \text{ kN} = 0$$
$$\therefore F_3 = 25 \text{ kN} = 25 \text{ kN (T)}$$

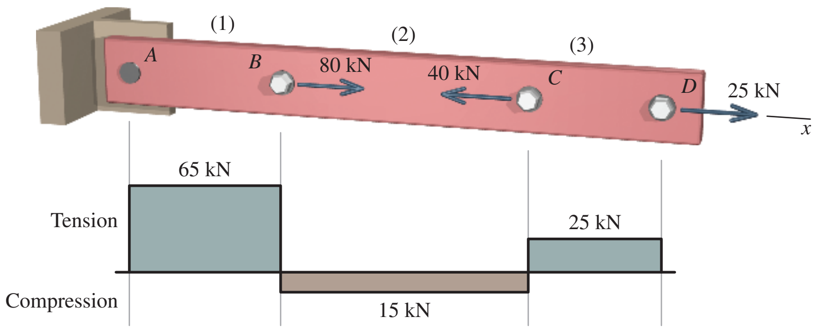
Repeat this procedure for an FBD exposing the internal force in segment (2):

$$\Sigma F_x = -F_2 - 40 \text{ kN} + 25 \text{ kN} = 0$$
$$\therefore F_2 = -15 \text{ kN} = 15 \text{ kN (C)}$$

Then repeat for an FBD exposing the internal force in segment (1):

$$\Sigma F_x = -F_1 + 80 \text{ kN} - 40 \text{ kN} + 25 \text{ kN} = 0$$
$$\therefore F_1 = 65 \text{ kN (T)}$$

It is always a good practice to construct a simple plot that graphically summarizes the internal axial forces along the bar. The axial-force diagram on the left shows internal tension forces above the axis and internal compression forces below the axis (**Figure 3**).



Axial-force diagram showing internal forces in each bar segment.
Figure 3

The required cross-sectional area will be computed on the basis of (the absolute value of) the largest-magnitude internal force. The normal stress in the bar must be limited to 60 MPa. *To facilitate the calculation, the conversion 1 MPa = 1 N/mm² is used; therefore, 60 MPa = 60 N/mm²*, and we have

$$\sigma = \frac{F}{A} \qquad \therefore A \geq \frac{F}{\sigma} = \frac{(65 \text{ kN})(1{,}000 \text{ N/kN})}{60 \text{ N/mm}^2} = 1{,}083.333 \text{ mm}^2$$

Since the flat steel bar is 50 mm wide, the minimum thickness that can be used for the bar is

$$t_{\text{min}} \geq \frac{1{,}083{,}333 \text{ mm}^2}{50 \text{ mm}} = 21.667 \text{ mm} = 21.7 \text{ mm}$$ **Ans.**

In practice, the bar thickness would be rounded up to the next-larger standard size.

Review

Recheck your calculations, paying particular attention to the units. Always show the units in your calculations because doing so is an easy and fast way to discover mistakes. Are the answers reasonable? If the bar thickness had been 0.0217 mm instead of 21.7 mm, would your solution have been reasonable, based on your common sense and intuition?

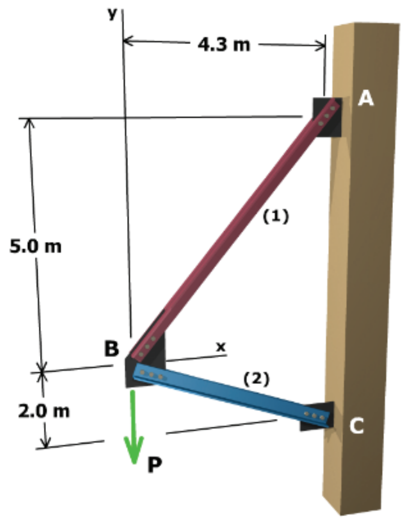
MecMovies

Example

M1.4 Two axial members are used to support a load P applied at joint B.

- Member (1) has a cross-sectional area of $A_1 = 3{,}080 \text{ mm}^2$ and an allowable normal stress of 80 MPa.
- Member (2) has a cross-sectional area of $A_2 = 4{,}650 \text{ mm}^2$ and an allowable normal stress of 75 MPa.

Determine the maximum load P that may be supported without exceeding either allowable normal stress.



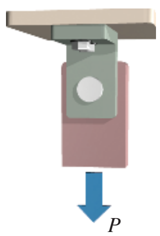


FIGURE 1.2*a* **Single-shear pin connection.**

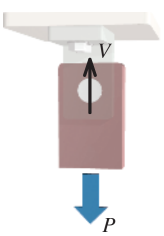


FIGURE 1.2*b* **Free-body diagram showing shear force transmitted by pin.**

1.3 Direct Shear Stress

Loads applied to a structure or a machine are generally transmitted to individual members through connections that use rivets, bolts, pins, nails, or welds. In all of these connections, one of the most significant stresses induced is a *shear stress*. In the previous section, normal stress was defined as the intensity of an internal force acting on a surface *perpendicular* to the direction of the internal force. Shear stress is also the intensity of an internal force, but shear stress acts on a surface that is *parallel* to the internal force.

To investigate shear stress, consider a simple connection in which the force carried by an axial member is transmitted to a support by means of a solid circular pin (**Figure 1.2*a***). The load is transmitted from the axial member to the support by a **shear force** (i.e., a force that tends to cut) distributed on a transverse cross section of the pin. A free-body diagram of the axial member with the pin is shown in Figure 1.2b. In this diagram, a resultant shear force V has replaced the distribution of shear forces on the transverse cross section of the pin. Equilibrium requires that the resultant shear force V equal the applied load P. Since only one cross section of the pin transmits load between the axial member and the support, the pin is said to be in **single shear**.

From the definition of stress given by Equation (1.1), an average shear stress on the transverse cross section of the pin can be computed as

$$\tau_{\text{avg}} = \frac{V}{A_V} \tag{1.5}$$

where A_V = area transmitting shear stress. The Greek letter τ (tau) is commonly used to denote shear stress. A sign convention for shear stress will be presented in a later section of the book.

The stress at a point on the transverse cross section of the pin can be obtained by using the same type of limit process that was used to obtain Equation (1.3) for the normal stress at a point. Thus,

$$\tau = \lim_{\Delta A_V \to 0} \frac{\Delta V}{\Delta A_V} \tag{1.6}$$

It will be shown later in this text that the shear stresses cannot be uniformly distributed over the transverse cross section of a pin or bolt and that the *maximum shear stress* on the transverse cross section may be much larger than the average shear stress obtained by using Equation (1.5). The design of simple connections, however, is usually based on average-stress considerations, and this procedure will be followed in this book.

MecMovies 1.7 and 1.8 present animated illustrations of single- and double-shear bolted connections.

MecMovies 1.9 presents an animated illustration of a shear key connection between a gear and a shaft.

The key to determining shear stress in connections is to visualize the failure surface or surfaces that will be created if the connectors (i.e., pins, bolts, nails, or welds) actually break (i.e., fracture). The shear area A_V that transmits shear force is the area exposed when the connector fractures. Two common types of shear failure surfaces for pinned connections are shown in **Figures 1.3** and **1.4**. Laboratory specimens that have failed on a single shear plane are shown in **Figure 1.3**. Similarly, a pin that has failed on two parallel shear planes is shown in **Figure 1.4**.

Jeffery S. Thomas

FIGURE 1.3 **Single-shear failure in pin specimens.**

Jeffery S. Thomas

FIGURE 1.4 **Double-shear failure in a pin specimen.**

EXAMPLE 1.4

Chain members (1) and (2) are connected by a shackle and pin (**Figure 1**). If the axial force in the chains is $P = 28$ kN and the allowable shear stress in the pin is $\tau_{allow} = 90$ MPa, determine the minimum acceptable diameter d for the pin.

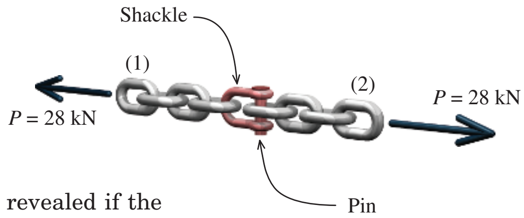
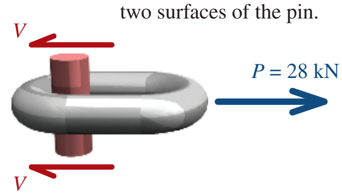


Figure 1

Plan the Solution

To solve the problem, first visualize the surfaces that would be revealed if the pin fractured because of the applied load P. Shear stress will be developed in the pin on these surfaces, at the two interfaces (i.e., common boundaries) between the pin and the shackle. The shear area needed to resist the shear force acting on each of the surfaces must be found, and from this area the minimum pin diameter can be calculated.

Shear forces V act on two surfaces of the pin.



Free-body diagram of pin.
Figure 2

Solution

Draw a free-body diagram (FBD) of the pin, which connects chain (2) to the shackle (**Figure 2**). Two shear forces V will resist the applied load $P = 28$ kN. The shear force V acting on each surface must equal one-half of the applied load P; therefore, $V = 14$ kN.

Next, the area of each surface is simply the cross-sectional area of the pin. The average shear stress acting on each of the pin failure surfaces is, therefore, the shear force V divided by the cross-sectional area of the pin. Since the average shear stress must be limited to 90 MPa, the minimum cross-sectional area required to satisfy the allowable shear stress requirement can be computed as

$$\tau = \frac{V}{A_{pin}} \qquad \therefore A_{pin} \geq \frac{V}{\tau_{allow}} = \frac{(14 \text{ kN})(1{,}000 \text{ N/kN})}{90 \text{ N/mm}^2} = 155.556 \text{ mm}^2$$

The minimum pin diameter required for use in the shackle can be determined from the required cross-sectional area:

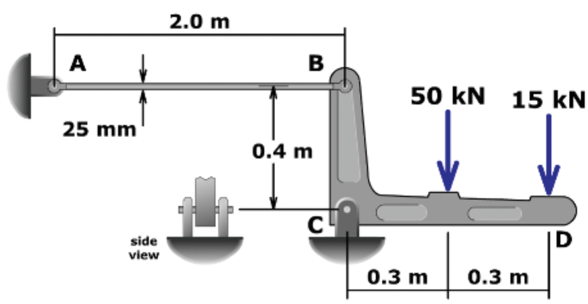
$$A_{pin} \geq \frac{\pi}{4} d_{pin}^2 = 155.556 \text{ mm}^2 \qquad \therefore d_{pin} \geq 14.07 \text{ mm} \quad \text{say, } d_{pin} = 15 \text{ mm} \qquad \textbf{Ans.}$$

In this connection, two cross sections of the pin are subjected to shear forces V; consequently, the pin is said to be in **double shear**.

MecMovies

Example

M1.5 A pin at C and a round aluminum rod at B support the rigid bar BCD. If the allowable pin shear stress is 50 MPa, what is the minimum diameter required for the pin at C?



EXAMPLE 1.5

A belt pulley used to drive a device is attached to a 30 mm diameter shaft with a square shear key (**Figure 1**). The belt tensions are 1,500 N and 600 N, as shown. The shear key dimensions are 6 mm by 6 mm by 25 mm long. Determine the shear stress produced in the shear key.

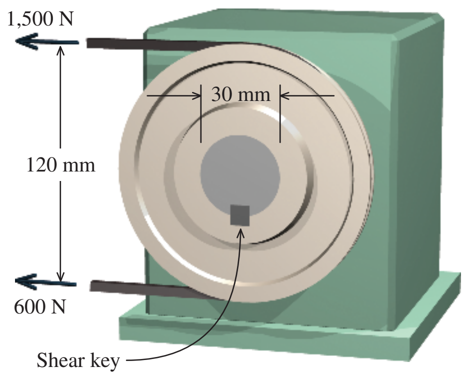


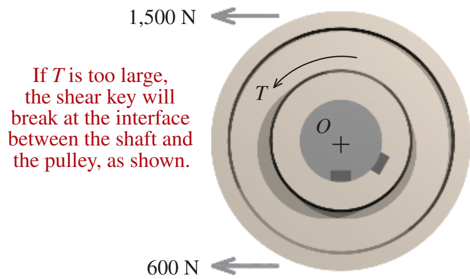
Figure 1

Plan the Solution

A shear key is a common component used to connect pulleys, chain sprockets, and gears to solid circular shafts. A rectangular slot is cut in the shaft, and a matching notch of the same width is cut in the pulley. After the slot and the notch are aligned, a square metal piece is inserted in the opening. This metal piece is called a shear key; it forces the shaft and the pulley to rotate together.

Before beginning the calculations, try to visualize the failure surface in the shear key (**Figure 2**). Since the belt tensions are unequal, a moment is created about the center of the shaft. This type of moment, called a **torque**, causes the shaft and pulley to rotate. If the torque T created by the unequal belt tensions is too large, the shear key will break at the interface between the shaft and the pulley, allowing the pulley to spin freely on the shaft. This failure surface is the plane at which shear stress is created in the shear key.

From the belt tensions and the pulley diameter, determine the torque T exerted on the shaft by the pulley. From a free-body diagram (FBD) of the pulley, determine the force that must be supplied by the shear key to satisfy equilibrium. Once the force in the shear key is known, the shear stress in the key can be computed by using the shear key dimensions.

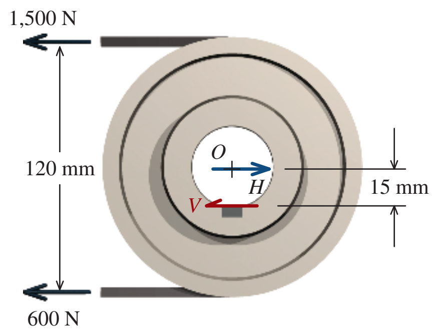


Visualize failure surface in shear key.

Figure 2

Solution

Consider an FBD of the pulley (**Figure 3**). This FBD includes the belt tensions, but it specifically excludes the shaft. The FBD cuts through the shear key at the interface between the pulley and the shaft. We will assume that there could be an internal force acting on the exposed surface of the shear key. This force will be denoted as shear force V. The distance from V to the center O of the shaft is equal to the radius of the shaft. Since the shaft diameter is 30 mm, the distance from O to shear force V is 15 mm. The magnitude of shear force V can be found from a moment equilibrium equation about point O, which is the center of rotation for both the pulley and the shaft. In this equation, positive moments are defined by the right-hand rule:



Free-body diagram of pulley.

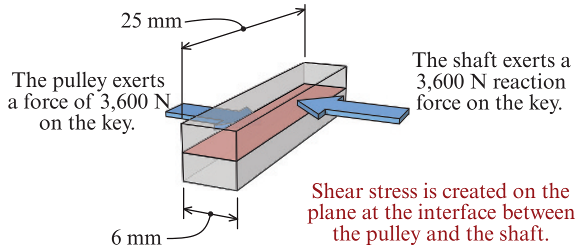
Figure 3

$$\Sigma M_O = (1{,}500\text{ N})(60\text{ mm}) - (600\text{ N})(60\text{ mm}) - (15\text{ mm})V = 0$$

$$\therefore V = 3{,}600\text{ N}$$

For the pulley to be in equilibrium, a shear force of $V = 3{,}600$ N must be supplied by the shear key.

An enlarged view of the shear key is shown (**Figure 4**). The torque created by the belt tensions exerts a force of 3,600 N on the shear key. For equilibrium, a force equal in magnitude, but opposite in direction, must be exerted on the key by the shaft. This pair of forces tends to cut the key, producing a shear stress. The shear stress acts on the plane highlighted in red.



Enlarged view of shear key.

Figure 4

An internal force of V = 3,600 N must exist on an internal plane of the shear key if the pulley is to be in equilibrium. The area of this plane surface is the product of the shear key width and length:

$$A_V = (6 \text{ mm})(25 \text{ mm}) = 150 \text{ mm}^2$$

The shear stress produced in the shear key can now be computed:

$$\tau = \frac{V}{A_V} = \frac{3{,}600 \text{ N}}{150 \text{ mm}^2} = 24.0 \text{ N/mm}^2 = 24.0 \text{ MPa} \quad \textbf{Ans.}$$

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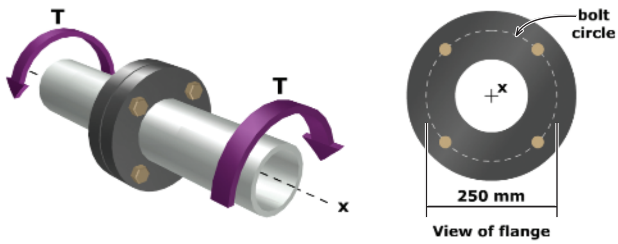
Example

M1.6 A torque of T = 10 kN·m is transmitted between two flanged shafts by means of four 22 mm diameter bolts. Determine the average shear stress in each bolt if the diameter of the bolt circle is 250 mm. (Disregard friction between the flanges.)



Another common type of shear loading is termed **punching shear**. Examples of this type of loading include the action of a punch in forming rivet holes in a metal plate, the tendency of building columns to punch through footings, and the tendency of a tensile axial load on a bolt to pull the shank of the bolt through the head. Under a punching shear load, the significant stress is the average shear stress on the surface defined by the *perimeter* of the punching member and the *thickness* of the punched member. Punching shear is illustrated by the three composite wood specimens shown in **Figure 1.5**. The central hole in each specimen is a pilot hole used to guide the punch. The specimen on the left shows the surface initiated at the outset of the shear failure. The center specimen reveals the failure surface after the punch is driven partially through the block. The specimen on the right shows the block after the punch has been driven completely through the block.

MecMovies 1.10 presents an animated illustration of punching shear.

Jeffery S. Thomas

FIGURE 1.5 **Punching shear failure in composite wood block specimens.**

EXAMPLE 1.6

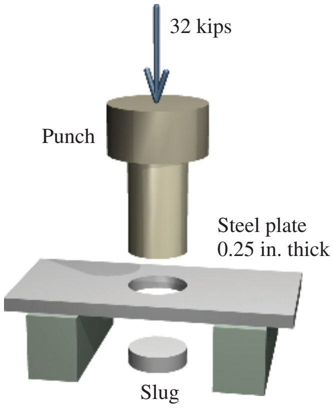


Figure 1

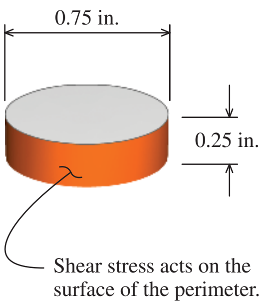


Figure 2

A punch for making holes in steel plates is shown (**Figure 1**). A downward punching force of 32 kips is required to punch a 0.75 in. diameter hole in a steel plate that is 0.25 in. thick. Determine the average shear stress in the steel plate at the instant the circular slug (the portion of the steel plate removed to create the hole) is torn away from the plate.

Plan the Solution

Visualize the surface that is revealed when the slug is removed from the plate (**Figure 2**). Compute the shear stress from the applied punching force and the area of the exposed surface.

Solution

The area subjected to shear stress occurs around the perimeter of the slug. Use the slug diameter d and the plate thickness t to compute the shear area A_V:

$$A_V = \pi\, dt = \pi(0.75 \text{ in.})(0.25 \text{ in.}) = 0.58905 \text{ in.}^2$$

The average shear stress τ is computed from the punching force P = 32 kips and the shear area:

$$\tau = \frac{P}{A_V} = \frac{32 \text{ kips}}{0.58905 \text{ in.}^2} = 54.3 \text{ ksi}$$ **Ans.**

1.4 Bearing Stress

A third type of stress, **bearing stress**, is actually a special category of normal stress. Bearing stresses are compressive normal stresses that occur on the surface of contact *between two separate interacting members*. This type of normal stress is defined in the same manner as normal and shear stresses (i.e., force per unit area); therefore, the average bearing stress σ_b is expressed as

$$\boxed{\sigma_b = \frac{F}{A_b}} \tag{1.7}$$

where A_b = area of contact between the two components.

EXAMPLE 1.7

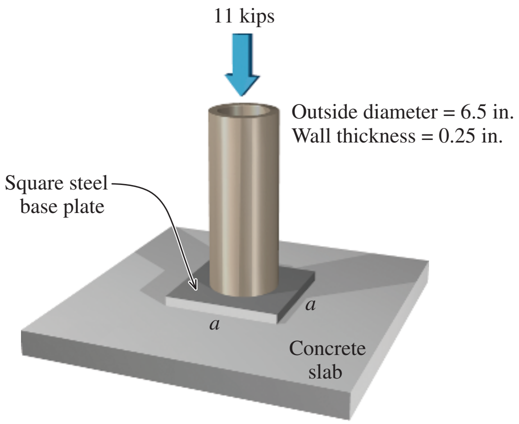


Figure 1

A steel pipe column (6.5 in. outside diameter; 0.25 in. wall thickness) supports a load of 11 kips (**Figure 1**). The steel pipe rests on a square steel base plate, which in turn rests on a concrete slab.

a. Determine the bearing stress between the steel pipe and the steel plate.

b. If the bearing stress of the steel plate on the concrete slab must be limited to 90 psi, what is the minimum allowable plate dimension a?

Plan the Solution

To compute bearing stress, the area of contact between two objects must be determined.

Solution

a. The cross-sectional area of the pipe is required in order to compute the compressive bearing stress between the column post and the base plate. The cross-sectional area of a pipe is given by

$$A_{\text{pipe}} = \frac{\pi}{4}(D^2 - d^2)$$

where D = outside diameter and d = inside diameter. The inside diameter d is related to the outside diameter D by

$$d = D - 2t$$

where t = wall thickness. Therefore, with D = 6.5 in. and d = 6.0 in., the area of the pipe is

$$A_{\text{pipe}} = \frac{\pi}{4}(D^2 - d^2) = \frac{\pi}{4}[(6.5\text{ in.})^2 - (6.0\text{ in.})^2] = 4.9087\text{ in.}^2$$

The bearing stress between the pipe and the base plate is

$$\sigma_b = \frac{F}{A_b} = \frac{11\text{ kips}}{4.9087\text{ in.}^2} = 2.24\text{ ksi}$$

b. The minimum area required for the steel plate in order to limit the bearing stress to 90 psi is

$$\sigma_b \geq \frac{F}{A_b} \qquad \therefore A\ b \geq \frac{F}{\sigma_b} = \frac{(11\text{ kips})(1{,}000\text{ lb/kip})}{90\text{ psi}} = 122.222\text{ in.}^2$$

Since the steel plate is square, its area of contact with the concrete slab is

$$A_b = a \times a \geq 122.222\text{ in.}^2 \qquad \therefore a \geq \sqrt{122.222\text{ in.}^2} = 11.06\text{ in.} \qquad \text{say, 12 in.}$$ **Ans.**

Bearing stresses also develop on the contact surface between a plate and the body of a bolt or a pin. A bearing failure at a bolted connection in a thin steel component is shown in **Figure 1.6**. A tension load was applied upward to the steel component, and a bearing failure occurred below the bolt hole.

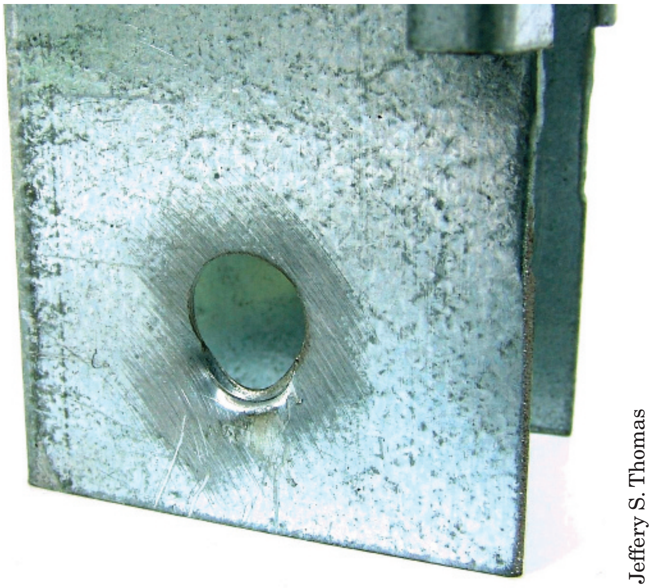


FIGURE 1.6 **Bearing stress failure at a bolted connection.**

The distribution of bearing stresses on a semicircular contact surface is quite complicated, and an average bearing stress is often used for design purposes. This average bearing stress σ_b is computed by dividing the transmitted force by the **projected area** of contact between a plate and the bolt or pin, instead of the actual contact area. This approach is illustrated in the next example.

EXAMPLE 1.8

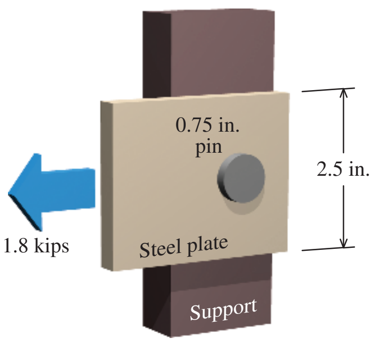
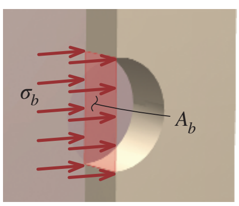


Figure 1



Enlarged view of projected contact area.

Figure 2

A 2.5 in. wide by 0.125 in. thick steel plate is connected to a support with a 0.75 in. diameter pin (**Figure 1**). The steel plate carries an axial load of 1.8 kips. Determine the bearing stress in the steel plate.

Plan the Solution

Bearing stresses will develop on the surface where the steel plate contacts the pin. This surface is the right side of the hole in the illustration. To determine the average bearing stress, the projected area of contact between the plate and the pin must be calculated.

Solution

The 1.8 kip load pulls the steel plate to the left, bringing the right side of the hole into contact with the pin. Bearing stresses will occur on the right side of the hole (in the steel plate) and on the right half of the pin.

Since the actual distribution of bearing stress on a semicircular surface is complicated, an average bearing stress is typically used for design purposes. Instead of the actual contact area, the projected area of contact is used in the calculation.

The figure at the left shows an enlarged view of the projected contact area between the steel plate and the pin (**Figure 2**). An average bearing stress σ_b is exerted on the steel plate by the pin. Not shown is the equal-magnitude bearing stress exerted on the pin by the steel plate.

The projected area A_b is equal to the product of the pin (or bolt) diameter d and the plate thickness t. For the pinned connection shown, the projected area A_b between the 0.125 in. thick steel plate and the 0.75 in. diameter pin is calculated as

$$A_b = dt = (0.75 \text{ in.})(0.125 \text{ in.}) = 0.09375 \text{ in.}^2$$

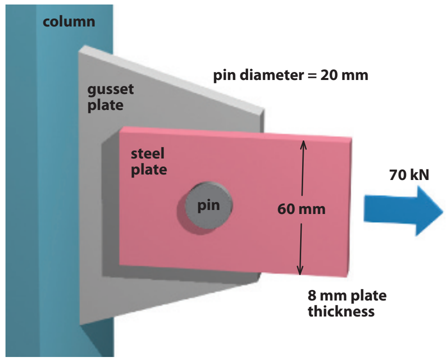
The average bearing stress between the plate and the pin is therefore

$$\sigma_b = \frac{F}{A_b} = \frac{1.8 \text{ kips}}{0.09375 \text{ in.}^2} = 19.20 \text{ ksi}$$ **Ans.**

MecMovies

Example

M1.1 A 60 mm wide by 8 mm thick steel plate is connected to a gusset plate by a 20 mm diameter pin. If a load of $P = 70$ kN is applied, determine the normal, shear, and bearing stresses in this connection.



MecMovies

Exercises

M1.1 For the pin connection shown, determine the normal stress acting on the gross area, the normal stress acting on the net area, the shear stress in the pin, and the bearing stress in the steel plate at the pin.

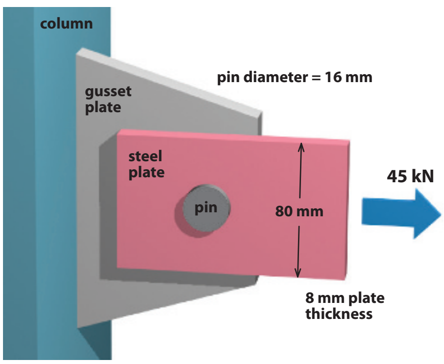


FIGURE M1.1

M1.2 Use normal stress concepts for four introductory problems.

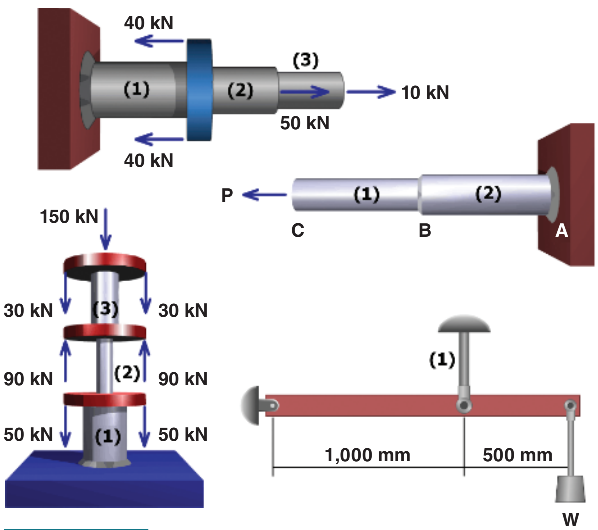


FIGURE M1.2

M1.3 Use shear stress concepts for four introductory problems.

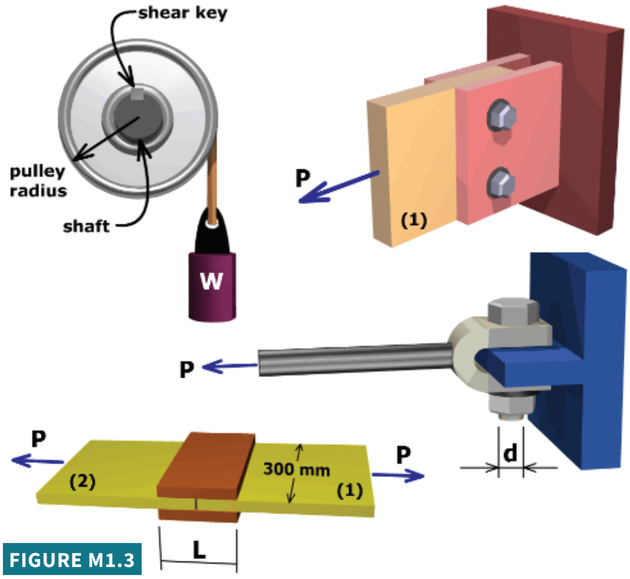


FIGURE M1.3

M1.4 Given the areas and allowable normal stresses for members (1) and (2), determine the maximum load P that may be supported by the structure without exceeding either allowable stress.

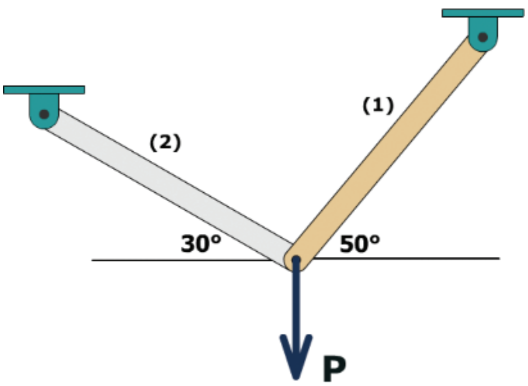


FIGURE M1.4

M1.5 For the pin at C, determine the resultant force, the shear stress, and the minimum required pin diameter for six configuration variations.

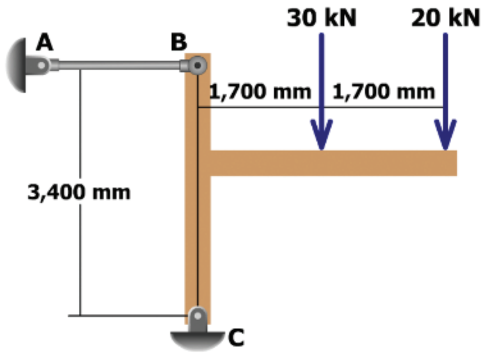


FIGURE M1.5

M1.6 A torque T is transmitted between two flanged shafts by means of six bolts. If the shear stress in the bolts must be limited to a specified value, determine the minimum bolt diameter required for the connection.

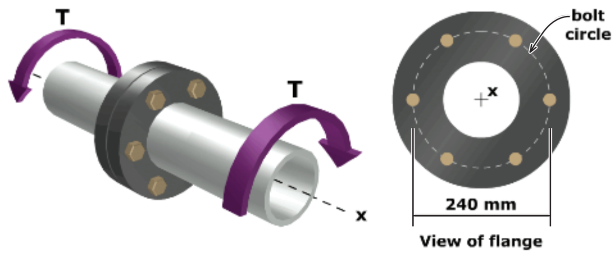


FIGURE M1.6

1.5 Stresses on Inclined Sections

In previous sections, normal, shear, and bearing stresses on planes parallel and perpendicular to the axes of centrically loaded members were introduced. Stresses on planes inclined to the axes of axially loaded bars will now be examined.

Consider a prismatic bar subjected to an axial force P applied to the centroid of the bar (**Figure 1.7a**). Loading of this type is termed **uniaxial**, since the force applied to the bar acts in one direction (i.e., either tension or compression). The cross-sectional area of the bar is A. To investigate the stresses that are acting internally in the material, we will cut through the bar at section a–a. The free-body diagram (Figure 1.7b) exposes the normal stress σ that is distributed over the cut section of the bar. The normal stress magnitude may be calculated from $\sigma = P/A$,

MecMovies 1.11 is an animated presentation of the theory of stresses on an inclined plane.

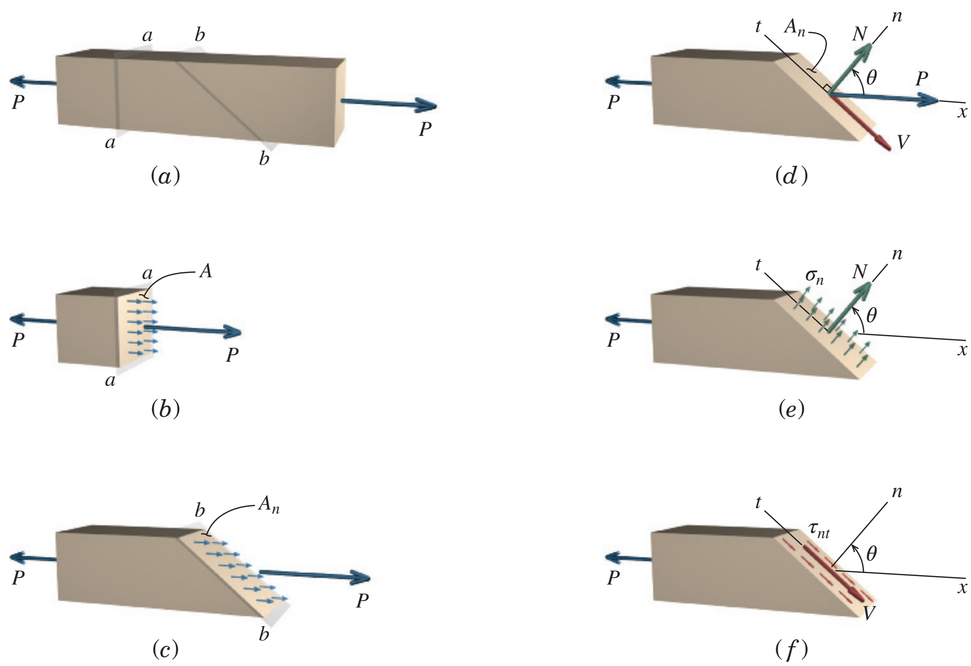


FIGURE 1.7 **(*a*) Prismatic bar subjected to axial force *P*. (*b*) Normal stresses on section *a–a*. (*c*) Stresses on inclined section *b–b*. (*d*) Force components acting perpendicular and parallel to inclined plane. (*e*) Normal stresses acting on inclined plane. (*f*) Shear stresses acting on inclined plane.**

provided that the stress is uniformly distributed. In this case, the stress will be uniform because the bar is prismatic and the force P is applied at the centroid of the cross section. The resultant of this normal stress distribution is equal in magnitude to the applied load P and has a line of action that is coincident with the axes of the bar, as shown. Note that there will be no shear stress τ, since the cut surface is perpendicular to the direction of the resultant force.

Section *a–a* is unique, however, because it is the only surface that is perpendicular to the direction of force P. A more general case would take into account a section cut through the bar at an arbitrary angle. In that regard, consider a free-body diagram along section *b–b* (Figure 1.7*c*). Because the stresses are the same throughout the entire bar, the stresses on the inclined surface must be uniformly distributed. Since the bar is in equilibrium, the resultant of the uniformly distributed stress must equal P even though the stress acts on a surface that is inclined.

The orientation of the inclined surface can be defined by the angle θ between the x axis and an axis *normal* to the plane, which is the n axis, as shown in Figure 1.7*d*. A positive angle θ is defined as a counterclockwise rotation from the x axis to the n axis. The t axis is *tangential* to the cut surface, and the n–t axes form a right-handed coordinate system.

In referencing planes, the orientation of the plane is specified by the normal to the plane. The inclined plane shown in Figure 1.7*d* is termed the *n* face because the *n* axis is the normal to that plane.

To investigate the stresses acting on the inclined plane (Figure 1.7*d*), the components of resultant force P acting perpendicular and parallel to the plane must be computed. Using θ as defined previously, we find that the perpendicular force component (i.e., normal force) is $N = P \cos \theta$ and the parallel force component (i.e., shear force) is $V = -P \sin \theta$. (The negative sign indicates that the shear force acts in the $-t$ direction, as shown in Figure 1.7*d*.) The area of the inclined plane $A_n = A/\cos \theta$, where A is the cross-sectional area of the axially loaded member. The normal and shear stresses acting on the inclined plane (Figures 1.7*e* and 1.7*f*) can now be determined by dividing the force component by the area of the inclined plane:

$$\sigma_n = \frac{N}{A_n} = \frac{P\cos\theta}{A/\cos\theta} = \frac{P}{A}\cos^2\theta = \frac{P}{2A}(1 + \cos 2\theta) \tag{1.8}$$

$$\tau_{nt} = \frac{V}{A_n} = \frac{-P\sin\theta}{A/\cos\theta} = -\frac{P}{A}\sin\theta\cos\theta = -\frac{P}{2A}\sin 2\theta \tag{1.9}$$

Since both the area A_n of the inclined surface and the values for the normal and shear forces, N and V, respectively, on the surface depend on the angle of inclination θ, the normal and shear stresses σ_n and τ_{nt} also depend on the angle of inclination θ of the plane. *This dependence of stress on both force and area means that stress is not a vector quantity*; therefore, the laws of the vector addition do not apply to stresses.

A graph showing the values of σ_n and τ_{nt} as a function of θ is given in **Figure 1.8**. These plots indicate that σ_n is largest when θ is 0° or 180°, that τ_{nt} is largest when θ is 45° or 135°, and also that $\tau_{max} = \sigma_{max}/2$. Therefore, the maximum normal and shear stresses in an axial member that is subjected to an uniaxial tension or compression force applied through the centroid of the member (termed a **centric loading**) are

$$\sigma_{max} = \frac{P}{A} \quad \text{and} \quad \tau_{max} = \frac{P}{2A} \tag{1.10}$$

Note that the normal stress is either maximum or minimum on planes for which the shear stress is zero. It can be shown that the shear stress is always zero on the planes of maximum or minimum normal stress. The concepts of maximum and minimum normal stress and maximum shear stress for more general cases will be treated in later sections of this book.

The plot of normal and shear stresses for axial loading, shown in Figure 1.8, indicates that the sign of the shear stress changes when θ is greater than 90°. The magnitude of the shear stress for any angle θ, however, is the same as that for 90° + θ. The sign change merely indicates that the shear force V changes direction.

Significance

Although one might think that there is only a single stress in a material (particularly in a simple axial member), the preceding discussion has demonstrated that

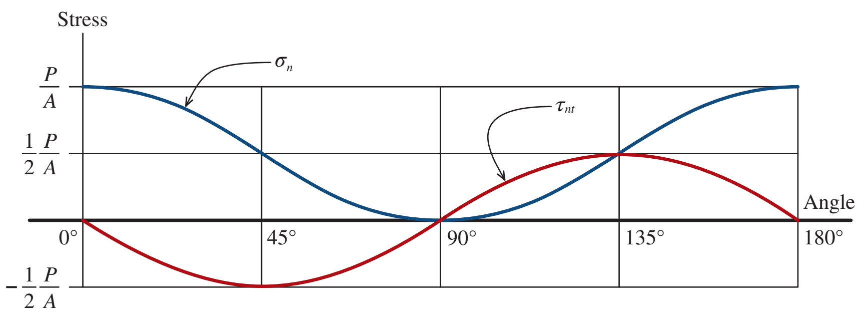


FIGURE 1.8 Variation of normal and shear stress as a function of the orientation θ of the inclined plane.

there are many different combinations of normal and shear stress in a solid object. The magnitude and direction of the normal and shear stresses at any point depend on the orientation of the plane being considered.

Why Is This Important? In designing a component, an engineer must be mindful of all possible combinations of normal stress σ_n and shear stress τ_{nt} that exist on internal surfaces of the object, not just the most obvious ones. Further, different materials are sensitive to different types of stress. For example, laboratory tests on specimens loaded in uniaxial tension reveal that brittle materials tend to fail in response to the magnitude of normal stress. These materials fracture on a transverse plane (i.e., a plane such as section *a–a* in Figure 1.7*a*). Ductile materials, by contrast, are sensitive to the magnitude of shear stress. A ductile material loaded in uniaxial tension will fracture on a 45° plane, since the maximum shear stress occurs on this surface.

1.6 Equality of Shear Stresses on Perpendicular Planes

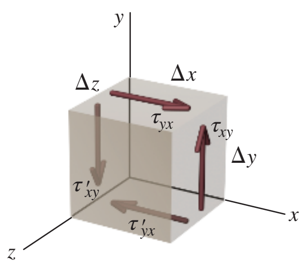


FIGURE 1.9 Shear stresses acting on a small element of material.

If an object is in equilibrium, then any portion of the object that one chooses to examine must also be in equilibrium, no matter how small that portion may be. Therefore, let us consider a small element of material that is subjected to shear stress, as shown in **Figure 1.9**. The front and rear faces of this small element are free of stress.

Equilibrium involves forces, not stresses. For us to consider the equilibrium of this element, we must find the forces produced by the stresses that act on each face, by multiplying the stress acting on each face by the area of the face. For example, the horizontal force acting on the top face of this element is given by $\tau_{yx}\Delta x\Delta z$, and the vertical force acting on the right face of the element is given by $\tau_{xy}\Delta y\Delta z$. Equilibrium in the horizontal direction gives

$$\Sigma F_x = \tau_{yx}\Delta x\Delta z - \tau'_{yx}\Delta x\Delta z = 0 \qquad \therefore \tau_{yx} = \tau'_{yx}$$

Equilibrium in the vertical direction gives

$$\Sigma F_y = \tau_{xy}\Delta y\Delta z - \tau'_{xy}\Delta y\Delta z = 0 \qquad \therefore \tau_{xy} = \tau'_{xy}$$

Finally, taking moments about the z axis gives

$$\Sigma M_z = (\tau_{xy}\Delta y\Delta z)\Delta x - (\tau_{yx}\Delta x\Delta z)\Delta y = 0 \qquad \therefore \tau_{xy} = \tau_{yx}$$

Consequently, equilibrium requires that

$$\tau_{xy} = \tau_{yx} = \tau'_{xy} = \tau'_{yx} = \tau$$

In other words, if a shear stress acts on one plane in the object, then shear stresses of equal magnitude act on three other planes. The shear stresses must be oriented either as shown in Figure 1.9 or in the opposite directions on each face.

Shear stress arrows on adjacent faces act either toward each other or away from each other. In other words, the arrows are arranged head-to-head or tail-to-tail—never head-to-tail—on intersecting perpendicular planes.

EXAMPLE 1.9

A 120 mm wide steel bar with a butt-welded joint, as shown, will be used to carry an axial tension load of $P = 180$ kN (**Figure 1**). If the normal and shear stresses on the plane of the butt weld must be limited to 80 MPa and 45 MPa, respectively, determine the minimum thickness required for the bar.

Plan the Solution

Either the normal stress limit or the shear stress limit will dictate the area required for the bar. There is no way to know beforehand which stress will control; therefore, both possibilities must be checked. The minimum cross-sectional area required for each limit must be determined. Using the larger of these two results, we will determine the minimum bar thickness. For illustration, this example will be worked in two ways:

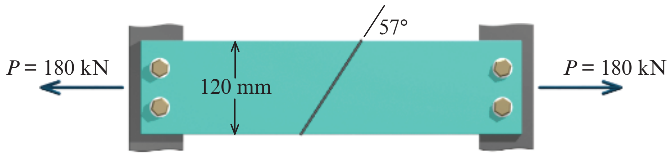


Figure 1

a. by directly using the normal and shear components of force P,

b. by using Equations (1.8) and (1.9).

Solution

(a) Solution Using Normal and Shear Force Components

Consider a free-body diagram (FBD) of the left half of the member (**Figure 2**). Resolve the axial force $P = 180$ kN into a force component N perpendicular to the weld and a force component V parallel to the weld.

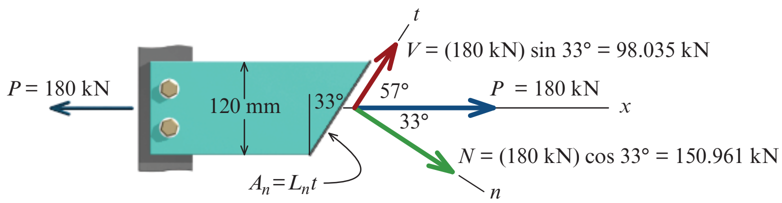


Figure 2

The minimum cross-sectional area A_n of the weld needed to limit the normal stress on the weld to 80 MPa can be computed from

$$\sigma_n \geq \frac{N}{A_n} \qquad \therefore A_n \geq \frac{(150.961 \text{ kN})(1{,}000 \text{ N/kN})}{80 \text{ N/mm}^2} = 1{,}887.013 \text{ mm}^2$$

Similarly, the minimum cross-sectional area A_n of the weld needed to limit the shear stress on the weld to 45 MPa can be computed from

$$\tau_{nt} \geq \frac{V}{A_n} \qquad \therefore A_n \geq \frac{(98.035 \text{ kN})(1{,}000 \text{ N/kN})}{45 \text{ N/mm}^2} = 2{,}178.556 \text{ mm}^2$$

To satisfy both normal and shear stress limits, the minimum cross-sectional area A_n needed for the weld is $A_n \geq 2{,}178.556 \text{ mm}^2$. Next, we can determine the length L_n of the weld along the inclined surface. From the geometry of the surface,

$$\cos 33° = \frac{120 \text{ mm}}{L_n} \qquad \therefore L_n = \frac{120 \text{ mm}}{\cos 33°} = 143.084 \text{ mm}$$

Thus, to provide the necessary weld area, the minimum thickness is computed as

$$t_{\min} \geq \frac{2{,}178.556 \text{ mm}^2}{143.084 \text{ mm}} = 15.23 \text{ mm} \qquad \textbf{Ans.}$$

(b) Solution Using Equations (1.8) and (1.9)

Determine the angle θ needed for Equations (1.8) and (1.9). The angle θ is defined as the angle between the transverse cross section (i.e., the section perpendicular to the applied load) and the inclined surface, with positive angles defined in a counterclockwise direction. Although the butt-weld angle is labeled 57° in the problem sketch, that is not the value needed for θ. For use in the equations, $\theta = -33°$.

The normal and shear stresses on the inclined plane can be computed from

$$\sigma_n = \frac{P}{A}\cos^2\theta \quad \text{and} \quad \tau_{nt} = -\frac{P}{A}\sin\theta\cos\theta$$

According to the 80 MPa normal stress limit, the minimum cross-sectional area required for the bar is

$$A_{\min} \geq \frac{P}{\sigma_n}\cos^2\theta = \frac{(180 \text{ kN})(1{,}000 \text{ N/kN})}{80 \text{ N/mm}^2}\cos^2(-33°) = 1{,}582.58 \text{ mm}^2$$

Similarly, the minimum area required for the bar, on the basis of the 45 MPa shear stress limit, is

$$A_{\min} \geq -\frac{P}{\tau_{nt}}\sin\theta\cos\theta = -\frac{(180 \text{ kN})(1{,}000 \text{ N/kN})}{45 \text{ N/mm}^2}\sin(-33°)\cos(-33°) = 1{,}827.09 \text{ mm}^2$$

Note: Here we are concerned with force and area *magnitudes*. If the area calculations had produced a negative value, we would have considered only the absolute value.

To satisfy both stress limits, the larger of the two areas must be used. Since the steel bar is 120 mm wide, the minimum bar thickness must be

$$t_{\min} \geq \frac{1{,}827.09 \text{ mm}^2}{120 \text{ mm}} = 15.23 \text{ mm} \qquad \textbf{Ans.}$$

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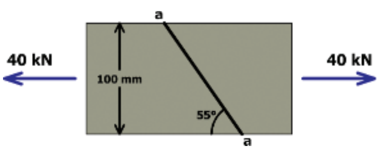
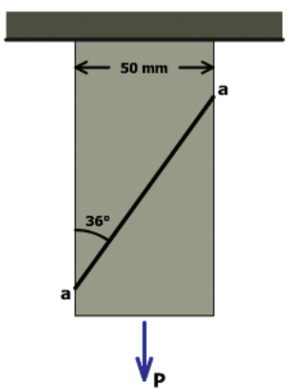
Example

M1.12 The steel bar shown has a 100 mm by 25 mm rectangular cross section. If an axial force of P = 40 kN is applied to the bar, determine the normal and shear stresses acting on the inclined surface *a–a*.



M1.13 The steel bar shown has a 50 mm by 10 mm rectangular cross section. The allowable normal and shear stresses on the inclined surface must be limited to 40 MPa and 25 MPa, respectively. Determine the magnitude of the maximum axial force of P that can be applied to the bar.



Exercises

M1.12 The bar shown has a rectangular cross section. For a given load P, determine (1) the force components perpendicular and parallel to section a–a, (2) the inclined surface area, and (3) the normal and shear stress magnitudes acting on surface a–a.

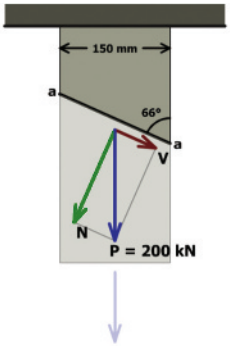


FIGURE M1.12

M1.13 The bar shown has a rectangular cross section. The allowable normal and shear stresses on inclined surface a–a are given. Determine (1) the magnitude of the maximum axial force P that can be applied to the bar and (2) the actual normal and shear stresses acting on inclined plane a–a.

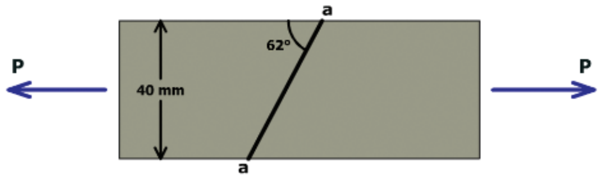


FIGURE M1.13