CHAPTER 1

Introduction to Dynamics

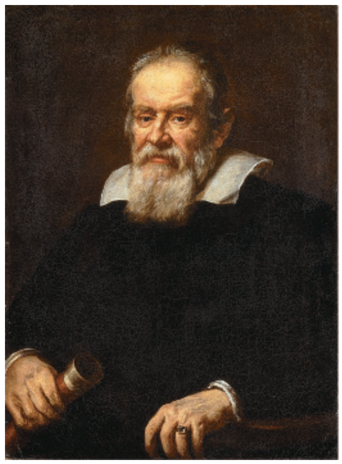
NASA

The International Space Station's Canadarm2 grapples the Kounotori2 H-II Transfer Vehicle as it approaches the station in 2011.

CHAPTER OUTLINE

1/1 History and Modern Applications

Dynamics is that branch of mechanics which deals with the motion of bodies under the action of forces. The study of dynamics in engineering usually follows the study of statics, which deals with the effects of forces on bodies at rest. Dynamics has two distinct parts: *kinematics*, which is the study of motion without reference to the forces which cause motion, and *kinetics*, which relates the action of forces on bodies to their resulting motions. A thorough comprehension of dynamics will provide one of the most useful and powerful tools for analysis in engineering.



© Fine Art Images/SuperStock

Galileo Galilei
Portrait of Galileo Galilei (1564–1642) (oil on canvas), Sustermans, Justus (1597–1681) (school of)/ Galleria Palatina, Florence, Italy/Bridgeman Art Library.

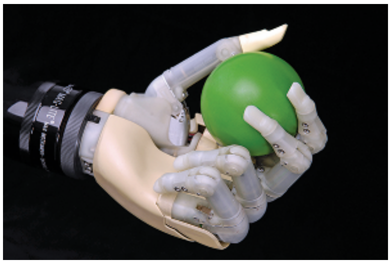
History of Dynamics

Dynamics is a relatively recent subject compared with statics. The beginning of a rational understanding of dynamics is credited to Galileo (1564–1642), who made careful observations concerning bodies in free fall, motion on an inclined plane, and motion of the pendulum. He was largely responsible for bringing a scientific approach to the investigation of physical problems. Galileo was continually under severe criticism for refusing to accept the established beliefs of his day, such as the philosophies of Aristotle which held, for example, that heavy bodies fall more rapidly than light bodies. The lack of accurate means for the measurement of time was a severe handicap to Galileo, and further significant development in dynamics awaited the invention of the pendulum clock by Huygens in 1657.

Newton (1642–1727), guided by Galileo's work, was able to make an accurate formulation of the laws of motion and, thus, to place dynamics on a sound basis. Newton's famous work was published in the first edition of his *Principia*,* which is generally recognized as one of the greatest of all recorded contributions to knowledge. In addition to stating the laws governing the motion of a particle, Newton was the first to correctly formulate the law of universal gravitation. Although his mathematical description was accurate, he felt that the concept of remote transmission of gravitational force without a supporting medium was an absurd notion. Following Newton's time, important contributions to mechanics were made by Euler, D'Alembert, Lagrange, Laplace, Poinsot, Coriolis, Einstein, and others.

Applications of Dynamics

Only since machines and structures have operated with high speeds and appreciable accelerations has it been necessary to make calculations based on the principles of dynamics rather than on the principles of statics. The rapid technological developments of the present day require increasing application of the principles of mechanics, particularly dynamics. These principles are basic to the analysis and design of moving structures, to fixed structures subject to shock loads, to robotic devices, to automatic control systems, to rockets, missiles, and spacecraft, to ground and air transportation vehicles, to electron ballistics of electrical devices, and to machinery of all types such as turbines, pumps, reciprocating engines, hoists, machine tools, etc.



WENN Ltd/Alamy Stock Photo

Artificial hand

Students with interests in one or more of these and many other activities will constantly need to apply the fundamental principles of dynamics.

1/2 Basic Concepts

The concepts basic to mechanics were set forth in Art. 1/2 of *Vol. 1 Statics*. They are summarized here along with additional comments of special relevance to the study of dynamics.

Space is the geometric region occupied by bodies. Position in space is determined relative to some geometric reference system by means of linear and angular measurements. The basic frame of reference for the laws of Newtonian mechanics is the *primary inertial system* or *astronomical frame of reference*, which is an imaginary set of rectangular axes assumed to have no translation or rotation in space. Measurements show that the laws of Newtonian mechanics are valid for this reference system as long as any velocities involved are negligible compared with the speed of light, which is 300 000 km/s or 186,000 mi/sec. Measurements made with respect to this reference are said to be *absolute*, and this reference system may be considered "fixed" in space.

A reference frame attached to the surface of the earth has a somewhat complicated motion in the primary system, and a correction to the basic equations of mechanics must be applied for measurements made relative to the reference frame of the earth. In the calculation of rocket and space-flight trajectories, for example, the

*The original formulations of Sir Isaac Newton may be found in the translation of his *Principia* (1687), revised by F. Cajori, University of California Press, 1934.

absolute motion of the earth becomes an important parameter. For most engineering problems involving machines and structures which remain on the surface of the earth, the corrections are extremely small and may be neglected. For these problems the laws of mechanics may be applied directly with measurements made relative to the earth, and in a practical sense such measurements will be considered *absolute*.

Time is a measure of the succession of events and is considered an absolute quantity in Newtonian mechanics.

Mass is the quantitative measure of the inertia or resistance to change in motion of a body. Mass may also be considered as the quantity of matter in a body as well as the property which gives rise to gravitational attraction.

Force is the vector action of one body on another. The properties of forces have been thoroughly treated in *Vol. 1 Statics*.

A ***particle*** is a body of negligible dimensions. When the dimensions of a body are irrelevant to the description of its motion or the action of forces on it, the body may be treated as a particle. An airplane, for example, may be treated as a particle for the description of its flight path.

A ***rigid body*** is a body whose changes in shape are negligible compared with the overall dimensions of the body or with the changes in position of the body as a whole. As an example of the assumption of rigidity, the small flexural movement of the wing tip of an airplane flying through turbulent air is clearly of no consequence to the description of the motion of the airplane as a whole along its flight path. For this purpose, then, the treatment of the airplane as a rigid body is an acceptable approximation. On the other hand, if we need to examine the internal stresses in the wing structure due to changing dynamic loads, then the deformation characteristics of the structure would have to be examined, and for this purpose the airplane could no longer be considered a rigid body.

Vector and ***scalar*** quantities have been treated extensively in *Vol. 1 Statics*, and their distinction should be perfectly clear by now. Scalar quantities are printed in lightface italic type, and vectors are shown in boldface type. Thus, V denotes the scalar magnitude of the vector $\mathbf{V}$. It is important that we use an identifying mark, such as an underline $\underline{V}$, for all handwritten vectors to take the place of the boldface designation in print. For two nonparallel vectors recall, for example, that $\mathbf{V}_1 + \mathbf{V}_2$ and $V_1 + V_2$ have two entirely different meanings.

We assume that you are familiar with the geometry and algebra of vectors through previous study of statics and mathematics. Students who need to review these topics will find a brief summary of them in Appendix C along with other mathematical relations which find frequent use in mechanics. Experience has shown that the geometry of mechanics is often a source of difficulty for students. Mechanics by its very nature is geometrical, and students should bear this in mind as they review their mathematics. In addition to vector algebra, dynamics requires the use of vector calculus, and the essentials of this topic will be developed in the text as they are needed.

Dynamics involves the frequent use of time derivatives of both vectors and scalars. As a notational shorthand, a dot over a symbol will frequently be used to indicate a derivative with respect to time. Thus, $\dot{x}$ means dx/dt and $\ddot{x}$ stands for d^2x/dt^2.

1/3 Newton's Laws

Newton's three laws of motion, stated in Art. 1/4 of *Vol. 1 Statics*, are restated here because of their special significance to dynamics. In modern terminology they are:

Law I. A particle remains at rest or continues to move with uniform velocity (in a straight line with a constant speed) if there is no unbalanced force acting on it.

Law II. The acceleration of a particle is proportional to the resultant force acting on it and is in the direction of this force.*

Law III. The forces of action and reaction between interacting bodies are equal in magnitude, opposite in direction, and collinear.

These laws have been verified by countless physical measurements. The first two laws hold for measurements made in an absolute frame of reference, but are subject to some correction when the motion is measured relative to a reference system having acceleration, such as one attached to the surface of the earth.

Newton's second law forms the basis for most of the analysis in dynamics. For a particle of mass m subjected to a resultant force $\mathbf{F}$, the law may be stated as

$$\boxed{\mathbf{F} = m\mathbf{a}} \qquad (1/1)$$

where $\mathbf{a}$ is the resulting acceleration measured in a nonaccelerating frame of reference. Newton's first law is a consequence of the second law since there is no acceleration when the force is zero, and so the particle is either at rest or is moving with constant velocity. The third law constitutes the principle of action and reaction with which you should be thoroughly familiar from your work in statics.

1/4 Units

The International System of metric units (SI) is defined and used in *Vol. 2 Dynamics*. In certain introductory areas, U.S. units are mentioned for purposes of comparison and completeness. Numerical conversion from one system to the other will often be needed in U.S. engineering practice for some years to come. To become familiar with each system, it is necessary to think directly in that system. Familiarity with the new system cannot be achieved simply by the conversion of numerical results from the old system.

Tables defining the SI units and giving numerical conversions between U.S. customary and SI units are included in Table D/5 of Appendix D.

The four fundamental quantities of mechanics, and their units and symbols for the two systems, are summarized in the following table:

Quantity	Dimensional Symbol	SI Units			U.S. Customary Units		
			Unit	Symbol		Unit	Symbol
Mass	M	Base units	kilogram	kg		slug	—
Length	L	Base units	meter*	m	Base units	foot	ft
Time	T	Base units	second	s	Base units	second	sec
Force	F		newton	N	Base units	pound	lb

*Also spelled *metre*.

*To some it is preferable to interpret Newton's second law as meaning that the resultant force acting on a particle is proportional to the time rate of change of momentum of the particle and that this change is in the direction of the force. Both formulations are equally correct when applied to a particle of constant mass.

As shown in the table, in SI the units for mass, length, and time are taken as base units, and the units for force are derived from Newton's second law of motion, Eq. 1/1. In the U.S. customary system the units for force, length, and time are base units and the units for mass are derived from the second law.

The SI system is termed an *absolute* system because the standard for the base unit kilogram (a platinum-iridium cylinder kept at the International Bureau of Standards near Paris, France) is independent of the gravitational attraction of the earth. On the other hand, the U.S. customary system is termed a *gravitational* system because the standard for the base unit pound (the weight of a standard mass located at sea level and at a latitude of 45°) requires the presence of the gravitational field of the earth. This distinction is a fundamental difference between the two systems of units.

In SI units, by definition, one newton is that force which will give a one-kilogram mass an acceleration of one meter per second squared. In the U.S. customary system a 32.1740-pound mass (1 slug) will have an acceleration of one foot per second squared when acted on by a force of one pound. Thus, for each system we have from Eq. 1/1

Omikron/Photo Researchers, Inc.

The U.S. standard kilogram at the National Bureau of Standards.

SI Units	U.S. Customary Units
$(1 \text{ N}) = (1 \text{ kg})(1 \text{ m/s}^2)$	$(1 \text{ lb}) = (1 \text{ slug})(1 \text{ ft/sec}^2)$
$\text{N} = \text{kg} \cdot \text{m/s}^2$	$\text{slug} = \text{lb} \cdot \text{sec}^2/\text{ft}$

In SI units, the kilogram should be used *exclusively* as a unit of mass and *never* force. Unfortunately, in the MKS (meter, kilogram, second) gravitational system, which has been used in some countries for many years, the kilogram has been commonly used both as a unit of force and as a unit of mass.

In U.S. customary units, the pound is unfortunately used both as a unit of force (lbf) and as a unit of mass (lbm). The use of the unit lbm is especially prevalent in the specification of the thermal properties of liquids and gases. The lbm is the amount of mass which weighs 1 lbf under standard conditions (at a latitude of 45° and at sea level). In order to avoid the confusion which would be caused by the use of two units for mass (slug and lbm), in this textbook we use almost exclusively the unit slug for mass. This practice makes dynamics much simpler than if the lbm were used. In addition, this approach allows us to use the symbol lb to always mean pound force.

Additional quantities used in mechanics and their equivalent base units will be defined as they are introduced in the chapters which follow. However, for convenient reference these quantities are listed in one place in Table D/5 of Appendix D.

Professional organizations have established detailed guidelines for the consistent use of SI units, and these guidelines have been followed throughout this book. The most essential ones are summarized in Table D/5 of Appendix D, and you should observe these rules carefully.

1/5 Gravitation

Newton's law of gravitation, which governs the mutual attraction between bodies, is

$$F = G\frac{m_1 m_2}{r^2} \tag{1/2}$$

where $F =$ the mutual force of attraction between two particles

$G =$ a universal constant called the *constant of gravitation*

$m_1, m_2 =$ the masses of the two particles

$r =$ the distance between the centers of the particles

The value of the gravitational constant obtained from experimental data is $G = 6.673(10^{-11})\ \text{m}^3/(\text{kg}\cdot\text{s}^2)$. Except for some spacecraft applications, the only gravitational force of appreciable magnitude in engineering is the force due to the attraction of the earth. It was shown in *Vol. 1 Statics*, for example, that each of two iron spheres 100 mm in diameter is attracted to the earth with a gravitational force of 37.1 N, which is called its *weight*, but the force of mutual attraction between them if they are just touching is only 0.000 000 095 1 N.

Because the gravitational attraction or weight of a body is a force, it should always be expressed in force units, newtons (N) in SI units and pounds force (lb) in U.S. customary units. To avoid confusion, the word "weight" in this book will be restricted to mean the force of gravitational attraction.

Effect of Altitude

The force of gravitational attraction of the earth on a body depends on the position of the body relative to the earth. If the earth were a perfect homogeneous sphere, a body with a mass of exactly 1 kg would be attracted to the earth by a force of 9.825 N on the surface of the earth, 9.822 N at an altitude of 1 km, 9.523 N at an altitude of 100 km, 7.340 N at an altitude of 1000 km, and 2.456 N at an altitude equal to the mean radius of the earth, 6371 km. Thus the variation in gravitational attraction of high-altitude rockets and spacecraft becomes a major consideration.

Every object which falls in a vacuum at a given height near the surface of the earth will have the same acceleration g, regardless of its mass. This result can be obtained by combining Eqs. 1/1 and 1/2 and canceling the term representing the mass of the falling object. This combination gives

$$g = \frac{Gm_e}{R^2}$$

where m_e is the mass of the earth and R is the radius of the earth.* The mass m_e and the mean radius R of the earth have been found through experimental measurements to be $5.976(10^{24})$ kg and $6.371(10^6)$ m, respectively. These values, together with the value of G already cited, when substituted into the expression for g, give a mean value of $g = 9.825\ \text{m/s}^2$.

The variation of g with altitude is easily determined from the gravitational law. If g_0 represents the absolute acceleration due to gravity at sea level, the absolute value at an altitude h is

$$g = g_0 \frac{R^2}{(R + h)^2}$$

where R is the radius of the earth.

*It can be proved that the earth, when taken as a sphere with a symmetrical distribution of mass about its center, may be considered a particle with its entire mass concentrated at its center.

Effect of a Rotating Earth

The acceleration due to gravity as determined from the gravitational law is the acceleration which would be measured from a set of axes whose origin is at the center of the earth but which does not rotate with the earth. With respect to these "fixed" axes, then, this value may be termed the *absolute* value of g. Because the earth rotates, the acceleration of a freely falling body as measured from a position attached to the surface of the earth is slightly less than the absolute value.

Accurate values of the gravitational acceleration as measured relative to the surface of the earth account for the fact that the earth is a rotating oblate spheroid with flattening at the poles. These values may be calculated to a high degree of accuracy from the 1980 International Gravity Formula, which is

$$g = 9.780\ 327(1 + 0.005\ 279 \sin^2 \gamma + 0.000\ 023 \sin^4 \gamma + \cdots)$$

where γ is the latitude and g is expressed in meters per second squared. The formula is based on an ellipsoidal model of the earth and also accounts for the effect of the rotation of the earth.

The absolute acceleration due to gravity as determined for a nonrotating earth may be computed from the relative values to a close approximation by adding $3.382(10^{-2}) \cos^2 \gamma$ m/s^2, which removes the effect of the rotation of the earth. The variation of both the absolute and the relative values of g with latitude is shown in **Fig. 1/1** for sea-level conditions.*

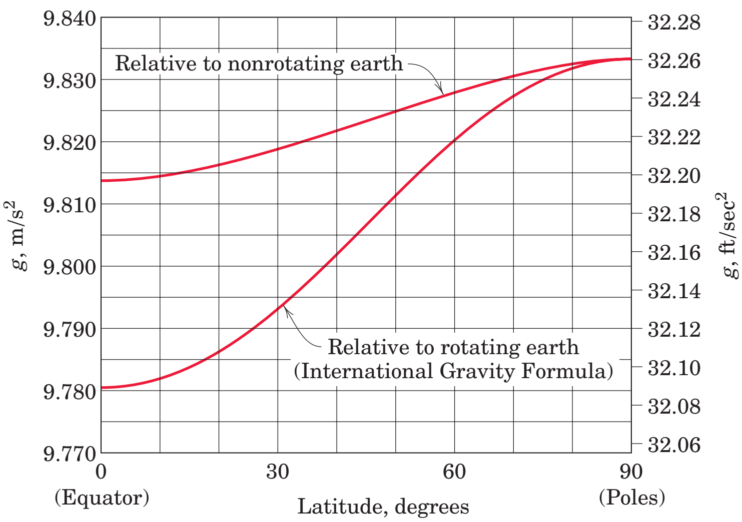


FIGURE 1/1

Standard Value of g

The standard value which has been adopted internationally for the gravitational acceleration relative to the rotating earth at sea level and at a latitude of 45° is 9.806 65 m/s^2 or 32.1740 ft/sec^2. This value differs very slightly from that obtained by evaluating the International Gravity Formula for $\gamma = 45°$. The reason for the

*You will be able to derive these relations for a spherical earth after studying relative motion in Chapter 3.

small difference is that the earth is not exactly ellipsoidal, as assumed in the formulation of the International Gravity Formula.

The proximity of large land masses and the variations in the density of the crust of the earth also influence the local value of g by a small but detectable amount. In almost all engineering applications near the surface of the earth, we can neglect the difference between the absolute and relative values of the gravitational acceleration, and the effect of local variations. The values of 9.81 m/s^2 in SI units and 32.2 ft/sec^2 in U.S. customary units are used for the sea-level value of g.

Apparent Weight

The gravitational attraction of the earth on a body of mass m may be calculated from the results of a simple gravitational experiment. The body is allowed to fall freely in a vacuum, and its absolute acceleration is measured. If the gravitational force of attraction or true weight of the body is W, then, because the body falls with an absolute acceleration g, Eq. 1/1 gives

$$\mathbf{W} = m\mathbf{g} \tag{1/3}$$

The *apparent weight* of a body as determined by a spring balance, calibrated to read the correct force and attached to the surface of the earth, will be slightly less than its true weight. The difference is due to the rotation of the earth. The ratio of the apparent weight to the apparent or relative acceleration due to gravity still gives the correct value of mass. The apparent weight and the relative acceleration due to gravity are, of course, the quantities which are measured in experiments conducted on the surface of the earth.

1/6 Dimensions

A given dimension such as length can be expressed in a number of different units such as meters, millimeters, or kilometers. Thus, a *dimension* is different from a *unit*. The *principle of dimensional homogeneity* states that all physical relations must be dimensionally homogeneous; that is, the dimensions of all terms in an equation must be the same. It is customary to use the symbols L, M, T, and F to stand for length, mass, time, and force, respectively. In SI units force is a derived quantity and from Eq. 1/1 has the dimensions of mass times acceleration or

$$F = ML/T^2$$

One important use of the dimensional homogeneity principle is to check the dimensional correctness of some derived physical relation. We can derive the following expression for the velocity v of a body of mass m which is moved from rest a horizontal distance x by a force F:

$$Fx = \frac{1}{2}mv^2$$

where the $\frac{1}{2}$ is a dimensionless coefficient resulting from integration. This equation is dimensionally correct because substitution of L, M, and T gives

$$[MLT^{-2}][L] = [M][LT^{-1}]^2$$

Dimensional homogeneity is a necessary condition for correctness of a physical relation, but it is not sufficient, since it is possible to construct an equation which is dimensionally correct but does not represent a correct relation. You should perform a dimensional check on the answer to every problem whose solution is carried out in symbolic form.

1/7 Solving Problems in Dynamics

The study of dynamics concerns the understanding and description of the motions of bodies. This description, which is largely mathematical, enables predictions of dynamical behavior to be made. A dual thought process is necessary in formulating this description. It is necessary to think in terms of both the physical situation and the corresponding mathematical description. This repeated transition of thought between the physical and the mathematical is required in the analysis of every problem.

One of the greatest difficulties encountered by students is the inability to make this transition freely. You should recognize that the mathematical formulation of a physical problem represents an ideal and limiting description, or model, which approximates but never quite matches the actual physical situation.

In Art. 1/8 of *Vol. 1 Statics* we extensively discussed the approach to solving problems in statics. We assume therefore, that you are familiar with this approach, which we summarize here as applied to dynamics.

Approximation in Mathematical Models

Construction of an idealized mathematical model for a given engineering problem always requires approximations to be made. Some of these approximations may be mathematical, whereas others will be physical. For instance, it is often necessary to neglect small distances, angles, or forces compared with large distances, angles, or forces. If the change in velocity of a body with time is nearly uniform, then an assumption of constant acceleration may be justified. An interval of motion which cannot be easily described in its entirety is often divided into small increments, each of which can be approximated.

As another example, the retarding effect of bearing friction on the motion of a machine may often be neglected if the friction forces are small compared with the other applied forces. However, these same friction forces cannot be neglected if the purpose of the inquiry is to determine the decrease in efficiency of the machine due to the friction process. Thus, the type of assumptions you make depends on what information is desired and on the accuracy required.

You should be constantly alert to the various assumptions called for in the formulation of real problems. The ability to understand and make use of the appropriate assumptions when formulating and solving engineering problems is certainly one of the most important characteristics of a successful engineer.

Along with the development of the principles and analytical tools needed for modern dynamics, one of the major aims of this book is to provide many opportunities to develop the ability to formulate good mathematical models. Strong emphasis is placed on a wide range of practical problems which not only require you to apply theory but also force you to make relevant assumptions.

Application of Basic Principles

The subject of dynamics is based on a surprisingly few fundamental concepts and principles which, however, can be extended and applied over a wide range of conditions. The study of dynamics is valuable partly because it provides experience

Key Concepts **Method of Attack**

An effective method of attack is essential in the solution of dynamics problems, as for all engineering problems. Development of good habits in formulating problems and in representing their solutions will be an invaluable asset. Each solution should proceed with a logical sequence of steps from hypothesis to conclusion. The following sequence of steps is useful in the construction of problem solutions.

1. Formulate the problem:
 (*a*) State the given data.
 (*b*) State the desired result.
 (*c*) State your assumptions and approximations.

2. Develop the solution:
 (*a*) Draw any needed diagrams, and include coordinates which are appropriate for the problem at hand.
 (*b*) State the governing principles to be applied to your solution.
 (*c*) Make your calculations.
 (*d*) Ensure that your calculations are consistent with the accuracy justified by the data.
 (*e*) Be sure that you have used consistent units throughout your calculations.
 (*f*) Ensure that your answers are reasonable in terms of magnitudes, directions, common sense, etc.
 (*g*) Draw conclusions.

The arrangement of your work should be neat and orderly. This will help your thought process and enable others to understand your work. The discipline of doing orderly work will help you to develop skill in problem formulation and analysis. Problems which seem complicated at first often become clear when you approach them with logic and discipline.

in reasoning from fundamentals. This experience cannot be obtained merely by memorizing the kinematic and dynamic equations which describe various motions. It must be obtained through exposure to a wide variety of problem situations which require the choice, use, and extension of basic principles to meet the given conditions.

In describing the relations between forces and the motions they produce, it is essential to define clearly the system to which a principle is to be applied. At times a single particle or a rigid body is the system to be isolated, whereas at other times two or more bodies taken together constitute the system.

The definition of the system to be analyzed is made clear by constructing its ***free-body diagram***. This diagram consists of a closed outline of the external boundary of the system. All bodies which contact and exert forces on the system but are not a part of it are removed and replaced by vectors representing the forces they exert on the isolated system. In this way, we make a clear distinction between the action and reaction of each force, and all forces on and external to the system are accounted for. We assume that you are familiar with the technique of drawing free-body diagrams from your prior work in statics.

Numerical versus Symbolic Solutions

In applying the laws of dynamics, we may use numerical values of the involved quantities, or we may use algebraic symbols and leave the answer as a formula. When numerical values are used, the magnitudes of all quantities expressed in their particular units are evident at each stage of the calculation. This approach is useful when we need to know the magnitude of each term.

The symbolic solution, however, has several advantages over the numerical solution:

1. The use of symbols helps to focus attention on the connection between the physical situation and its related mathematical description.

2. A symbolic solution enables you to make a dimensional check at every step, whereas dimensional homogeneity cannot be checked when only numerical values are used.
3. We can use a symbolic solution repeatedly for obtaining answers to the same problem with different units or different numerical values.

Thus, facility with both forms of solution is essential, and you should practice each in the problem work.

In the case of numerical solutions, we repeat from *Vol. 1 Statics* our convention for the display of results. All given data are taken to be exact, and results are generally displayed to three significant figures, unless the leading digit is a one, in which case four significant figures are displayed. An exception to this rule occurs in the area of orbital mechanics, where answers will generally receive an additional significant figure because of the necessity of increased precision in this discipline.

Solution Methods

Solutions to the various equations of dynamics can be obtained in one of three ways.

1. Obtain a direct mathematical solution by hand calculation, using either algebraic symbols or numerical values. We can solve the large majority of the problems this way.
2. Obtain graphical solutions for certain problems, such as the determination of velocities and accelerations of rigid bodies in two-dimensional relative motion.
3. Solve the problem by computer. A number of problems in *Vol. 2 Dynamics* are designated as *Computer-Oriented Problems*. They appear at the end of the Review Problem sets and were selected to illustrate the type of problem for which solution by computer offers a distinct advantage.

The choice of the most expedient method of solution is an important aspect of the experience to be gained from the problem work. We emphasize, however, that the most important experience in learning mechanics lies in the formulation of problems, as distinct from their solution per se.

1/8 Chapter Review

This chapter has introduced the concepts, definitions, and units used in dynamics, and has given an overview of the approach used to formulate and solve problems in dynamics. Now that you have finished this chapter, you should be able to do the following:

1. State Newton's laws of motion.
2. Perform calculations using SI and U.S. customary units.
3. Express the law of gravitation and calculate the weight of an object.
4. Discuss the effects of altitude and the rotation of the earth on the acceleration due to gravity.
5. Apply the principle of dimensional homogeneity to a given physical relation.
6. Describe the methodology used to formulate and solve dynamics problems.

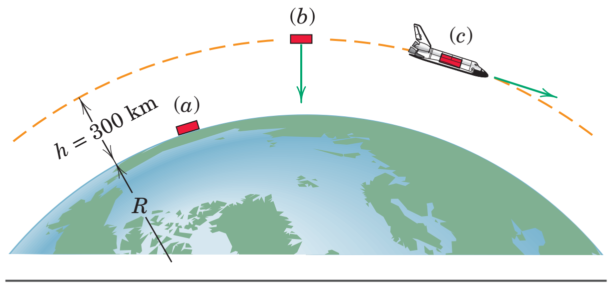
Just after liftoff, there is a multitude of critical dynamic events taking place for this jetliner.

SAMPLE PROBLEM 1/1

A space-shuttle payload module has a mass of 50 kg and rests on the surface of the earth at a latitude of 45° north.

(*a*) Determine the surface-level weight of the module in both newtons and pounds, and its mass in slugs.

(*b*) Now suppose the module is taken to an altitude of 300 kilometers above the surface of the earth and released there with no velocity relative to the center of the earth. Determine its weight under these conditions in both newtons and pounds.



(*c*) Finally, suppose the module is fixed inside the cargo bay of a space shuttle. The shuttle is in a circular orbit at an altitude of 300 kilometers above the surface of the earth. Determine the weight of the module in both newtons and pounds under these conditions.

For the surface-level value of the acceleration of gravity relative to a rotating earth, use $g = 9.80665$ m/s^2 (32.1740 ft/sec^2). For the absolute value relative to a nonrotating earth, use $g = 9.825$ m/s^2 (32.234 ft/sec^2). Round off all answers using the rules of this textbook.

Solution. (*a*) From relationship 1/3, we have

$[W = mg]$ $\qquad W = (50 \text{ kg})(9.80665 \text{ m/s}^2) = 490 \text{ N}$ ① *Ans.*

Here we have used the acceleration of gravity relative to the rotating earth, because that is the condition of the module in part (*a*). Note that we are using more significant figures in the acceleration of gravity than will normally be required in this textbook (9.81 m/s^2 and 32.2 ft/sec^2 will normally suffice).

From the table of conversion factors in Table D/5 of Appendix D, we see that 4.4482 newtons is equal to 1 pound. Thus, the weight of the module in pounds is

$$W = 490 \text{ N}\left[\frac{1 \text{ lb}}{4.4482 \text{ N}}\right] = 110.2 \text{ lb}$$ ② *Ans.*

Finally, its mass in slugs is

$[W = mg]$ $$m = \frac{W}{g} = \frac{110.2 \text{ lb}}{32.1740 \text{ ft/sec}^2} = 3.43 \text{ slugs}$$ ③ *Ans.*

As another route to the last result, we may convert from kilograms slugs. Again using Table D/5, we have

$$m = 50 \text{ kg}\left[\frac{1 \text{ slug}}{14.594 \text{ kg}}\right] = 3.43 \text{ slugs}$$

We recall that 1 lbm is the amount of mass which under standard conditions has a weight of 1 lb of force. We rarely refer to the U.S. mass unit lbm in this textbook series, but rather use the slug for mass. The sole use of slug, rather than the unnecessary use of two units for mass, will prove to be powerful and simple in U.S. units.

HELPFUL HINTS

① Our calculator indicates a result of 490.3325 . . . newtons. Using the rules of significant figure display used in this textbook, we round the written result to three significant figures, or 490 newtons. Had the numerical result begun with the digit 1, we would have rounded the displayed answer to four significant figures.

② A good practice with unit conversion is to multiply by a factor such as $\left[\frac{1 \text{ lb}}{4.4482 \text{ N}}\right]$, which has a value of 1, because the numerator and the denominator are equivalent. Be sure that cancellation of the units leaves the units desired—here the units of N cancel, leaving the desired units of lb.

③ Note that we are using a previously calculated result (110.2 lb). We must be sure that when a calculated number is needed in subsequent calculations, it is obtained in the calculator to its full accuracy (110.2316 . . .). If necessary, numbers must be stored in a calculator storage register and then brought out of the register when needed. We must not merely punch 110.2 into our calculator and proceed to divide by 32.1740—this practice will result in loss of numerical accuracy. Some individuals like to place a small indication of the storage register used in the right margin of the work paper, directly beside the number stored.

SAMPLE PROBLEM 1/1 (CONTINUED)

(b) We begin by calculating the absolute acceleration of gravity (relative to the nonrotating earth) at an altitude of 300 kilometers.

$$\left[g = g_0 \frac{R^2}{(R+h)^2}\right] \qquad g_h = 9.825\left[\frac{6371^2}{(6371+300)^2}\right] = 8.96 \text{ m/s}^2$$

The weight at an altitude of 300 kilometers is then

$$W_h = mg_h = 50(8.96) = 448 \text{ N} \qquad \textit{Ans.}$$

We now convert W_h to units of pounds.

$$W_h = 448 \text{ N}\left[\frac{1 \text{ lb}}{4.4482 \text{ N}}\right] = 100.7 \text{ lb} \qquad \textit{Ans.}$$

As an alternative solution to part (b), we may use Newton's universal law of gravitation. In SI units,

$$\left[F = \frac{Gm_1m_2}{r^2}\right] \qquad W_h = \frac{Gm_em}{(R+h)^2} = \frac{[6.673(10^{-11})][5.976(10^{24})][50]}{[(6371+300)(1000)]^2}$$

$$= 448 \text{ N}$$

which agrees with our earlier result. We note that the weight of the module when at an altitude of 300 km is about 90% of its surface-level weight—it is *not* weightless. We will study the effects of this weight on the motion of the module in Chapter 3.

(c) The weight of an object (the force of gravitational attraction) does not depend on the motion of the object. Thus the answers for part (c) are the same as those in part (b).

$$W_h = 448 \text{ N} \quad \text{or} \quad 100.7 \text{ lb} \qquad \textit{Ans.}$$

This Sample Problem has served to eliminate certain commonly held and persistent misconceptions. First, just because a body is raised to a typical shuttle altitude, it does not become weightless. This is true whether the body is released with no velocity relative to the center of the earth, is inside the orbiting shuttle, or is in its own arbitrary trajectory. And second, the acceleration of gravity is not zero at such altitudes. The only way to reduce both the acceleration of gravity and the corresponding weight of a body to zero is to take the body to an infinite distance from the earth.