1

Control, Servo-mechanisms and System Regulation

This chapter explores compensator servo-mechanisms and control, correction and proportional control.

1.1. Introduction

1.1.1. *Generalities and definitions*

In all areas of physics, for the research, analysis and understanding of natural phenomena, a stage for modeling and the study of the structure of the physical process is necessary. This has led to the development of modeling, representation and analysis techniques of systems using a fairly general terminology. This terminology is difficult to introduce in a clear manner but the concepts, which it relies upon, will be defined in detail in the following chapters.

A physical process is divided into several components or parts forming a system. For example, this is the case of an engine that consists of an amplifier, power supply, an electromagnetic part and a position and/or speed sensor. The system input is the voltage applied to the amplifier and the output is either the position or the speed of rotation of the motor shaft.

Among the objectives of the control engineer, we can identify modeling, behavior analysis and the regulation or control with the aim of dynamically optimizing the behavior of the system. It should be noted that one preliminary and very important step is the *configuration* of the system before its control. During this step, the automation expert must define sensible choices of sensors, actuators and their placement in the system to optimize the control (control means verification of the good functioning of all sensors, actuators, system and corrector or control law). It is only after this stage that control synthesis finds its place, which might simply be reflected by the use of a conventional controller (proportional, proportional, integral and derivative (PID), phase advance or phase delay or other).

Driving the system or control serves the purpose of ensuring that the variables to be adjusted or system outputs follow a desired trajectory (curve with respect to time in general) or have dynamics defined by the specification requirements, for example temperature control of an oven, fluid flow control or speed and trajectory control of a moving object. When the desired trajectory is reduced to a point, this is referred to as regulation and not as control because the main purpose here is to stabilize the output of the system in a point. The role of control is to allow or to improve the resulting performance of a system, using actuators and sensors available for information acquisition and enabling reaction based on behavior. In general, this can be done using a negative-feedback loop (return or feedback loop) and sometimes a compensation or anticipation chain of the dynamic effects of the system (feedforward or (pre or post) compensation). The operation of a vehicle is according to the block diagram shown in Figure 1.1.

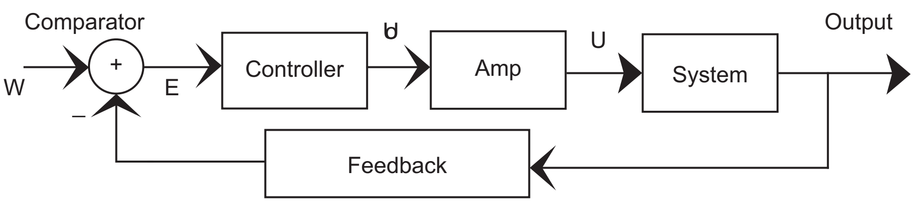

Figure 1.1. *Schematic diagram of a controlled system with compensation and feedback sequence*

In the definition of a control system, we will express transfer functions as follows:

– $H(p)$ transfer of the system to be controlled, p is the Laplace operator;

– $R(p)$ transfer of the sensor or measuring device;

– $C(p)$ transfer of the corrector or servo controller element.

The setpoint is $w(t)$ and the output to control is $y(t)$. The direct chain consists of $C(p)$ and $H(p)$ and $R(p)$ constitute the feedback chain. The difference between output and setpoint is $e(t)$ and is also called control error or trajectory tracking error. In order to simplify the study, we are considering a unity feedback scheme in which $R(p) = 1$.

In general, transfers $H(p)$ and $R(p)$ are known or can be obtained and the objective is to obtain a corrector $C(p)$ that is able to satisfy the performances required for the closed-loop system (transfer from w to y).

For the regulation of the temperature of a speaker to a reference value, it is possible to use one of the following block diagrams.

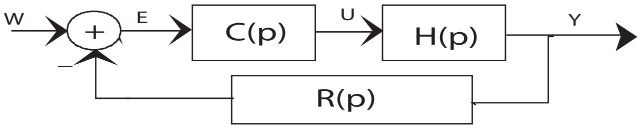

Figure 1.2. *Schematic diagram of a feedback system* $R(p) = 1$

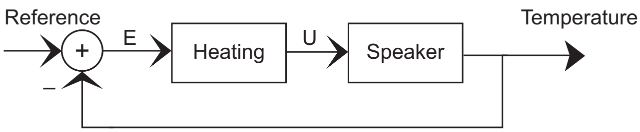

Figure 1.3. *Speed regulation of a motor*

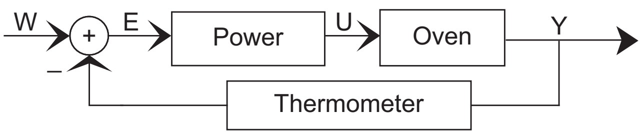

Figure 1.4. *Temperature regulation of an oven*

Vehicle operation follows the principle of the diagram shown in Figure 1.5.

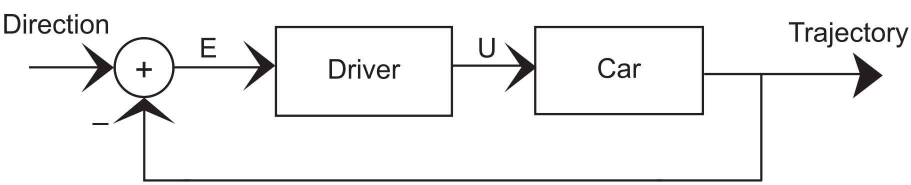

Figure 1.5. *Schematic diagram of the model for vehicle operation*

In this preface, we are going to cover some conventional methods for the design of a control system. This study will serve the purpose of finding a control structure allowing a servo system to be given dynamic characteristics or performances established *a priori* in the definition of the requirements, either in terms of temporal response or in terms of frequency response. In general, the latter is defined to ensure:

– the stability of the controlled system (loop system);

– the smallest possible permanent errors;

– a suitable dynamic behavior: a response quickly reaching its asymptote, the lowest overshoot possible, etc.

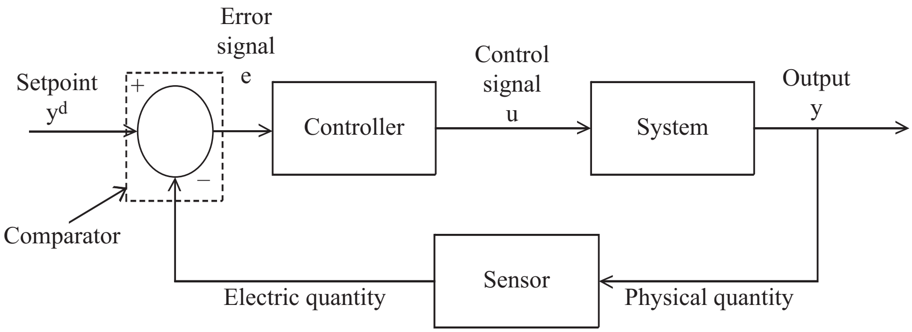

Figure 1.6. *Servo system*

The conventional operation of a servo system is shown in Figure 1.6:

– y^d: the setpoint is an electrical quantity that represents the desired output value of the system;

– ε: the error signal between the setpoint and the actual output of the system;

– u: the control signal generated by the controller;

– y: a physical quantity that represents the system output.

The physical quantity y is measured with a sensor that translates it into an electrical quantity. By means of the comparer, this electric quantity is compared to the setpoint, which is an electric quantity.

A model describing the dynamic behavior (physic) of the open-loop (OL) system is necessary for control synthesis. In general, the accuracy required for modeling is dependent of the finality of the control and the required performance. It should be noted that there are several types of models.

The *simulation model* is useful for the study of behavior and the response of the system to different excitations. It allows that the laws of control be tested and that performance be evaluated before application to the actual system. It has to be as accurate as possible (including disturbance, noises, nonlinearity and all the parts able to be modeled etc.).

The *control model* is usually simpler, sometimes linear, somewhat reduced compared to the simulation model. It is used to infer the appropriate control law so as to minimize complexity (reduction of computation times, ease of implementation, etc.). Consequently, the resulting control law is verified with the simulation model to measure the impact of the dynamic terms neglected in the synthesis stage. If it proves insufficient, either a more complete model is retained or compensators are added.

An ensuing model of the physical system may be empirical, be the result of physical modeling or derived from a process of identification based on information about the observation of the system after excitation. When a representation of the system is available, this is a function of some parameters. The estimation of these parameters from experimental data is the *identification* step.

In linear systems control, modeling is a very important phase. In order to properly control a system, a good model thereof must be known. For example, in order to drive a car, the better its dynamic behavior or model is known (by training), the better it can be controlled at high speed and therefore the better it will be driven. As a result, it will achieve the best performance. The dynamic model is acquired by learning or by system identification.

During the development of an application for automation purposes, we generally follow the following steps:

1) modeling;

2) identification;

3) behavior analysis;

4) controller synthesis;

5) control implementation;

6) analysis and study of the system in closed loop;

7) verification of the performance and eventually repetition of steps (2), (3) or (4).

The modeling stage becomes crucial when the requirements are strict with respect to performances and when the control implemented proves to be complex. In order to introduce the different types of modeling, we will study some examples.

1.1.2. *Control law synthesis*

1.1.2.1. *Specifications and configuration*

Control should enable the closed-loop system to ensure that a certain number of constraints called *specifications* be satisfied. Among the specifications, we can distinguish:

– stability;

– performance;

– robustness.

A *servo-mechanism* can be qualified by its degree of stability, accuracy, response speed, sensitivity to disturbances acting on the system, robustness with regard to disturbances on measures and errors or variations of the characteristic parameters of the system. The accuracy of a control system can be characterized by the maximal amplitude of the position error.

1.1.2.2. *Performances: regulation, disturbance rejection and anticipation*

Disturbance rejection: the process is often subjected to certain inputs considered as being disturbances. The latter must have a minimal effect on the behavior of the system when it is controlled. The *regulation* is the ability of the system to mitigate or even absorb the effects of disturbances.

Trajectory tracking: the loop system must be fast enough, must not present significant overshooting or oscillations in order to correctly follow a *desired trajectory* or setpoint varying in time.

1.1.2.3. *Robustness and parametric uncertainties*

A loop system is said to be robust if its characteristics do not vary much or do not appear too degraded when changing the parameters of the physical system to be controlled or the neglected dynamics during modeling or when disturbances occur. These changes may originate either from the change in characteristics of the system or from the difference between physical system and control model.

Some examples:

– variation in mass of a satellite after fuel consumption;

– aging of a mechanical structure and change in frequency of the natural modes;

– reduced model for the control neglecting the high-frequency dynamics of the physical process;

– external disturbances such as those conveyed by electrical networks and noises in sensors;

– failure occurring in systems that alters their dynamics.

1.1.2.4. *Constraints on control: control system input energy*

Control u is the output of a dynamic system called *controller or control law*, and it may be subjected to constraints (amplitude limits and speed variations, actuators limit, structure limit, etc.). Constraints are sometimes:

– the use of time-invariant linear correction or a simple proportional feedback;

– a control calculated in the discrete domain by a processor using integer or fixed-point representation;

– computation time constraint, limitation of the order of the controller, trajectories continuity and their derivatives up to some order.

Controls admissibility: the amplitudes of signals and control structure must not be too large compared to those physically feasible.

EXAMPLE 1.1.– *Direct current motor with tachometric feedback.*

1.1.3. *Comprehension and application exercises*

1.1.3.1. *Study of a servo-mechanism for the attitude of a satellite*

The aim is to control the attitude of a satellite such to orientate an antenna connected to the satellite with regard to a given axis. The output variable of the system is therefore the attitude $\theta(t)$. For the satellite to start rotate, a thrust $u(t)$ is applied through a nozzle, which produces a couple $\gamma(t) = Lu(t)$ acting on the satellite, where L refers to the distance of the thrust point to the axis of rotation of the satellite. We want to impose direction $\theta^d(t)$ by acting upon $u(t)$. The variable J designates the moment of inertia of the satellite; the dynamic equation is written as:

$$\gamma(t) = J\ddot{\theta}(t) = Lu(t). \qquad [1.1]$$

Hence the transfer function between the input $u(t)$ and output $\theta(t)$,

$$H_o(p) = \frac{\Theta(p)}{U(p)} = \frac{L}{Jp^2}. \qquad [1.2]$$

The system behaves as a double integrator. When a short impulse is given to the system, it will begin to rotate indefinitely (the impulse is integrated twice). Control is achieved using the difference between the desired attitude (setpoint) and the actual attitude (output) to calculate the control $u(t)$ to apply to orientate the antenna. The diagram of the control is shown in Figure 1.7.

We must determine a controller $C(p)$ that connects the error $\varepsilon(p)$ to the control signal $U(p)$. As a first step, we propose a regulation proportional to the error correction ($u(t) = K\varepsilon(t)$), therefore we will write $C(p) = K$, in which K is constant. This control is known as proportional control. The transfer function of the now loop system is given by:

$$\begin{aligned} H(p) &= \frac{\Theta(p)}{\Theta^d(p)} = \frac{C(p)H_o(p)}{1 + C(p)H_o(p)} = \frac{KH_o(p)}{1 + KH_o(p)} = \frac{K\frac{L}{Jp^2}}{1 + K\frac{L}{Jp^2}} \\ &= \frac{K\frac{L}{J}}{p^2 + K\frac{L}{J}}. \end{aligned} \quad [1.3]$$

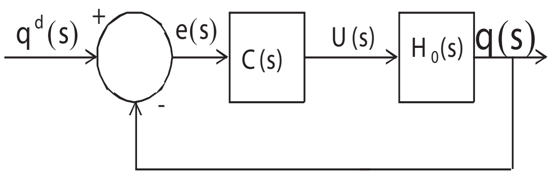

Figure 1.7. *Control diagram*

Suppose that the attitude is initially of 0, and that it is desirable that the satellite assume an attitude of setpoint θ_0. It can be said that the setpoint signal is a Heaviside function of amplitude θ_0, wherefrom

$$\Theta^d(p) = \frac{\theta_0}{p}. \quad [1.4]$$

which gives as output:

$$\Theta(p) = \frac{K\frac{L}{J}}{p(p^2 + K\frac{L}{J})}\theta_0. \quad [1.5]$$

By dividing into simple elements, we get:

$$\theta(t) = \theta_0(1 - \cos(\omega_0 t)) \qquad \text{with} \qquad \omega_0 = \sqrt{K\frac{L}{J}}. \quad [1.6]$$

It can be noted that the attitude of the satellite oscillates around the desired attitude. The result is thus not satisfactory; it is necessary to reconsider the controller to improve the performance of the closed-loop system. The problem comes from the fact that when we assume the value is0, the rotation of the satellite should be slowed down, whereas it is at this moment that the control is zero, since it is proportional to the error. However, it can be observed that when the error is zero, its derivative is maximal (in absolute value). Consequently, the idea is to introduce the error and its derivative in the correction. We then choose a proportional correction and derivative ($u(t) = K_p\varepsilon(t) + K_v\dot{\varepsilon}(t)$). It can be written in a simplified way:

$$C(p) = 1 + Tp. \tag{1.7}$$

The transfer function of the closed-loop system is therefore given by

$$\begin{aligned} H(p) &= \frac{\Theta(p)}{\Theta^d(p)} = \frac{C(p)H_o(p)}{1 + C(p)G(p)} = \frac{(1+Tp)\frac{L}{Jp^2}}{1 + (1+Tp)\frac{L}{Jp^2}} \\ &= \frac{(1+Tp)\frac{L}{J}}{p^2 + T\frac{L}{J}p + \frac{L}{J}}. \end{aligned} \tag{1.8}$$

REMARK 1.1.– *The system using proportional and derivative (PD) control is not physically feasible since the degree of the numerator is greater than the degree of the denominator. On the other hand, a good approximation is always possible to achieve.*

Consider the same regulation conditions (i.e. step response). The output of the system is thus given by

$$\Theta(p) = \frac{(1+Tp)\frac{L}{J}\theta_0}{p(p^2 + T\frac{L}{J}p + \frac{L}{J})}. \tag{1.9}$$

The shape depends on the roots of the following characteristic equation:

$$p^2 + T\frac{L}{J}p + \frac{L}{J} = 0. \tag{1.10}$$

For example, we take $\frac{L}{J} = 10^{-2}$. If $T > 20$, the solutions are real and negative,

$$p_{1,2} = \frac{-10^{-2}T \pm 10^{-1}\sqrt{10^{-2}T - 4}}{2} \tag{1.11}$$

and the response is shown in Figure 1.8.

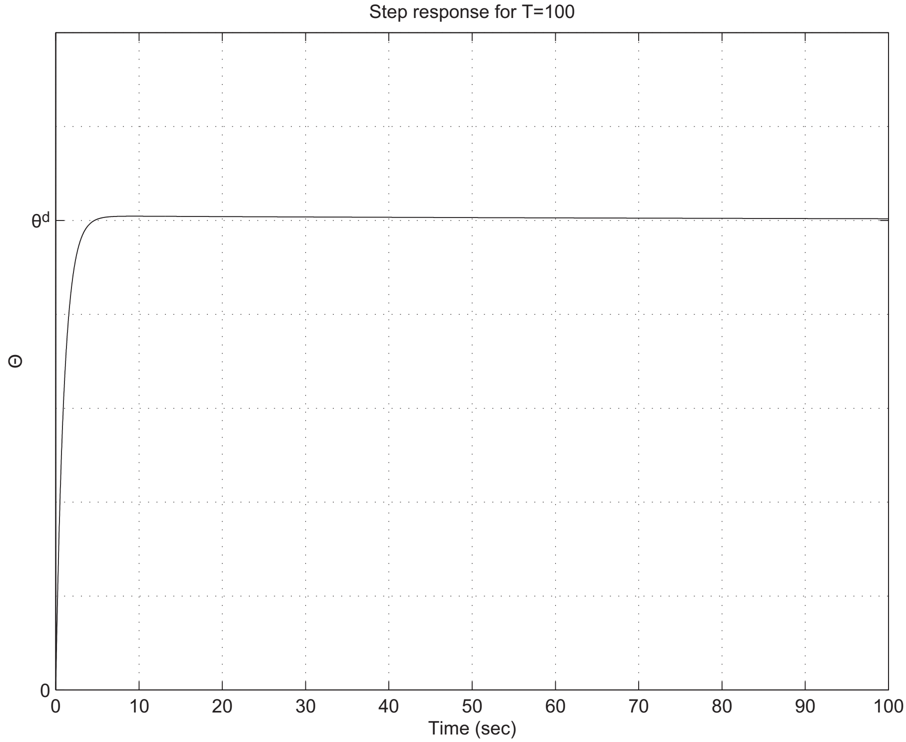

Figure 1.8. *Step response*

For $T = 100$, the roots of the denominator are approximately -1 and -0.01, therefore the dominant term in the response should be the second root (larger time constant $100\,\text{s}$). In fact, it is observed that this is not true, the apparent time constant is of $1\,\text{s}$. The reason is that the numerator of the transfer function has a root equal to $-1/T = -0.01$, which compensates for the effect of the pole for -0.01. We are here confronted with a system that does not have dominant poles.

If we take $T < 20$, solutions are complex conjugate and as a result the response shows damped oscillations. For $T = 10$, we have the response as shown in Figure 1.9.

REMARK 1.2.– *With increasingly smaller values of T, we tend towards an oscillating solution that corresponds to the case of proportional control ($T = 0$).*

It is thus seen that the shape of the response depends completely on the roots of the characteristic equation (poles of $H(s)$) and sometimes depends on the roots of the numerator of the transfer function (zeroes of $H(s)$).

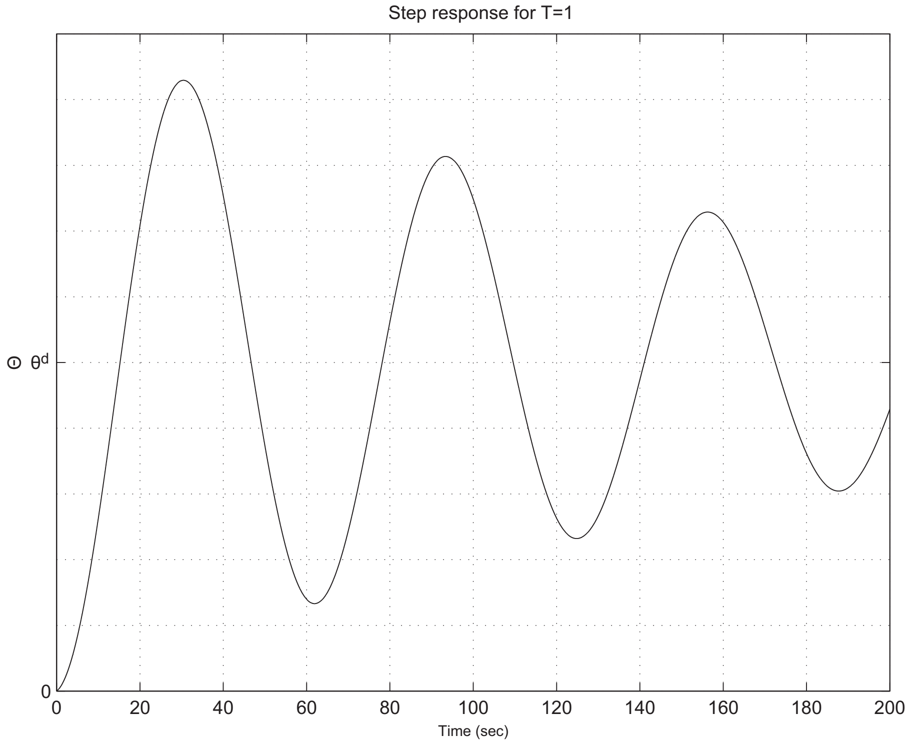

Figure 1.9. *Response with damped oscillations*

In this example, we have highlighted several characteristics of a control:

– the notion of loop that allows a process to be controlled, which in the case being considered could not be controlled in the OL system;

– the notion of control system (controller) that can be more or less adapted to the process to be controlled;

– the influence of poles and zeroes of the transfer function.

1.2. Process control

1.2.1. *Correction in the frequency domain*

As a first step, we are going to focus only on looping a system with a cascade controller (regulation). The use of an anticipation chain and compensators will be addressed farther.

In this section, we are going to cover process control using conventional methods for simple control. These controllers that make use of simple actions are regulated by an approximate study in the frequency domain. Consider a system whose frequency response has a phase margin $\Delta\Phi = \Phi_0$ and a gain margin ΔG. Suppose that these characteristics are not sufficient to provide the desired performance. For a simple system and using Bode, Nyquist or Black–Nichols representations, the observation of its frequency response makes it possible to observe that for improving the performance of a system, it is necessary to make sure that the frequency response of the corrected system passes far away from the critical point $((0dB, -\pi Rad)$, $(0dB, -180°)$ $-1)$. For this purpose, the Bode diagram may inspire two types of corrective actions, one shifting the phase curve upward in the neighborhood of the critical point (phase advance control, to increase the phase margin), the other offsetting the gain downward (phase delay control to increase the gain margin). The two corrections can be achieved using transfer functions of simple controllers and may be combined, but their effectiveness is limited as soon as the order of the system is greater than 2 or 3. Despite the possibility that this type of action can be multiplied, it is preferable to use other synthesis methods, more flexible and more efficient for more complex systems.

1.2.2. *Phase advance controller and PD controller*

The operation principle is that this controller increases the phase of the direct chain to increase the phase margin of the system. The transfer function of a PD controller is written as: $C(p) = K(1 + T_d.p)$. The derivative action is not physically feasible, it must be approximated by $T_d.p \simeq \frac{T_d p}{1+\tau p}$ with $\tau \ll T_d$, which gives us

$$C(p) = K(1 + \frac{T_d p}{1 + \tau p}) = K(\frac{1 + (T_d + \tau)p}{1 + \tau p}). \qquad [1.12]$$

The transfer function of a phase advance controller is defined as follows:

$$C(p) = \frac{1 + aTp}{1 + Tp} \qquad (a > 1). \qquad [1.13]$$

The Bode plot of the phase advance controller is shown in Figure 1.11.

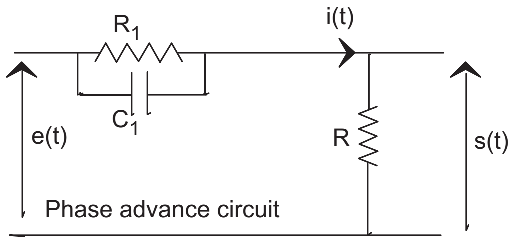

Figure 1.10. *Advance phase control circuit*

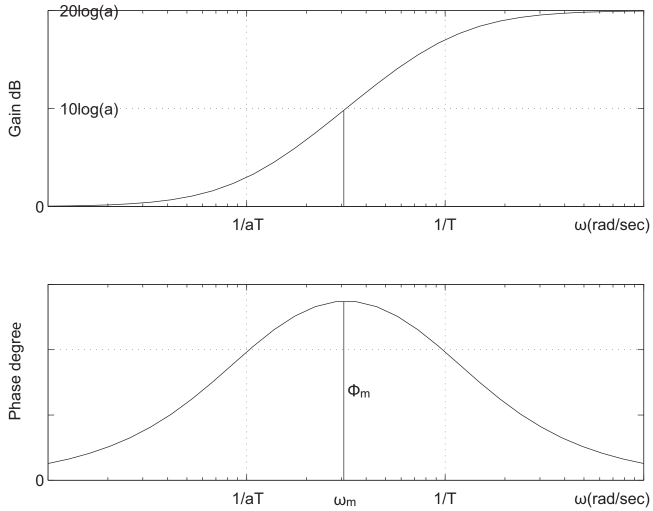

Figure 1.11. *Bode plot of the phase advance controller*

The maximum phase Φ_m of the phase advance controller is obtained for $\omega = \omega_m$, with:

$$\omega_m = \frac{1}{T\sqrt{a}} \qquad [1.14]$$

$$\sin(\Phi_m) = \frac{1-a}{1+a}.$$

We have a phase margin of Φ_0, which means that the controller should add a phase of $\Phi_m = 50° - \Phi_0$. The modulus of the controller is equal to $10\log_{10}(a)$ to $\omega = \omega_m$. As a result, if the controller is calculated to get Φ_m at ω_c, the cutoff pulse of the system corresponding to $0\,\mathrm{dB}$, the new crossing point at $0\,\mathrm{dB}$ would be moved to the right of the starting point and therefore the phase margin would be different from the expected margin. To overcome this problem, ω_m is chosen at the point where the modulus of the system is equal to $-10\log_{10}(a)$, which makes it so that after correction the modulus of the controlled system will cross $0\,\mathrm{dB}$ at $\omega = \omega_m$. The phase margin of the controlled system will be equal to $\Phi_m + 180° - \Phi_{\mathrm{read}}|_{G=-10\log(a)}$.

To determine the coefficients of the controller, the calculated phase margin is overestimated by $5°$ to take into account the fact that we use the asymptotic diagram: $\Phi_m = \Phi_{m(\mathrm{calculated})} + 5°$. After having defined Φ_m, we can derive a by the formula

$$a = \frac{1 + \sin(\Phi_m)}{1 - \sin(\Phi_m)}. \qquad [1.15]$$

Then, since this phase must be placed in $\omega_m = \omega_c = \frac{1}{T\sqrt{a}}$ corresponding to the modulus of the transfer function in the OL system,

$$G = -10\log_{10}(a). \qquad [1.16]$$

This allows us to calculate T,

$$T = \frac{1}{\omega_m\sqrt{a}}. \qquad [1.17]$$

REMARK 1.3.– *The phase advance control increases the bandwidth of the system and as a result the system becomes faster.*

Since the determination of the controller coefficients uses approximations, we must verify the results obtained by printing the Bode plot of $C(p)H_o(p)$. If the phase margin after correction does not match the expected result, this may be caused by too quick a variation of the system phase around the critical point. This variation results in a fall of phase that largely exceeds the $5°$ of margin.

The phase advance controller is not suitable in case of systems having too quick phase variations.

1.2.3. *Phase delay controller and integrator compensator*

The operation principle is that this controller decreases the gain of the direct chain to pulses corresponding to a dephasing shift of the system close to $-\pi\ \mathrm{rad}$. The transfer function of a proportional and integral (PI) controller is written as:

$C(p) = K(1 + \frac{1}{T_i.p}) = K\frac{1+T_i.p}{T_i.p}$. The integral action is often approximated by $\frac{1}{T_i.p} \simeq \frac{1}{\frac{1}{\alpha}+T_i.p}$, which gives us

$$C(p) = K(1 + \frac{1}{\frac{1}{\alpha} + T_i.p}) = K\alpha\frac{(\frac{1}{\alpha} + 1) + T_i.p}{1 + \alpha T_i.p}. \qquad [1.18]$$

The transfer function of a phase advance controller (integral compensator) is defined as follows:

$$C(p) = \frac{1 + aTp}{1 + Tp} \qquad (a < 1). \qquad [1.19]$$

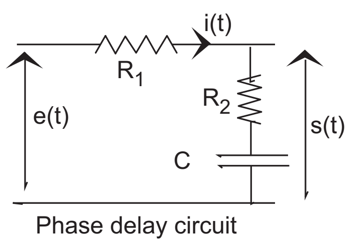

Figure 1.12. *Phase delay controller circuit*

Its Bode plot is given by Figure 1.13.

It can be observed that the phase of the controller is negative and consequently it will delay the phase of the system.

To obtain a desired phase margin of $50°$, we will act this time not upon the phase but upon the modulus so as it passes through $0\,\text{db}$ at pulse ω_c that corresponds to a system phase that is equal to $(\Phi_c = -180° + 50° = -130°)$. As the modulus is cancelled out for ω_c $(\Phi_c = -130°)$, then the phase margin is therefore $\Delta\Phi = 180° - 130° = 50°$. To offset the effect of the phase introduced by the controller, we overestimate by $5°$ or $10°$ the margin, that is to say, instead of taking $\Phi_c = -130°$, we will take $\Phi_c = -125°$.

The value of a is calculated by measuring the modulus d at pulse ω_c corresponding to a system phase equal to $\Phi_c = -125°$. Thus, by imposing this pulse to the gain of the direct chain $|C(\omega_c)H_o(\omega_c)| = 1$, we therefore obtain the value of a,

$$20\log(a) = -d \Longrightarrow a = 10^{-d/20}. \qquad [1.20]$$

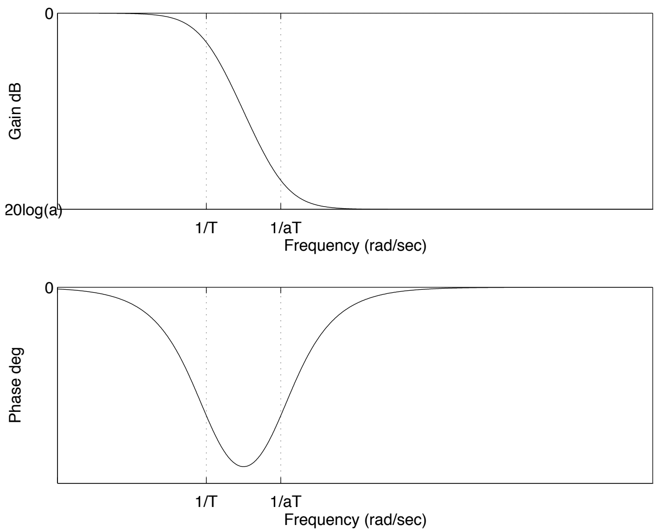

Figure 1.13. *Bode plot of the phase delay controller*

For the other parameter, we choose T in order to not affect the phase around ω_c. To this end, at least a decade is placed between $1/aT$ and the new crossing point ω_c of the modulus at $0\,\mathrm{db}$ after control, which gives

$$\frac{1}{aT} \approx \frac{\omega_c}{10} \Longrightarrow T = \frac{10}{a\omega_c}. \qquad [1.21]$$

REMARK 1.4.– *The main disadvantage of the integral compensator is that it reduces the bandwidth of the system, which makes the system slower.*

It is possible to combine the advantages of the two phase delay and advance controllers by implementing a PID or phase delay and phase advance controller, combining actions: the phase delay part having the purpose to stabilize the system and the phase advance part being designed to accelerate the response (make the system quick).

1.2.4. *Proportional, integral and derivative (PID) control*

The PID controller is a special case of phase advance and phase delay controller or with combined action. It is widely used in the industry. The transfer function of a PID controller is given by:

$$C(p) = K_p(1 + \frac{1}{T_i p} + T_d p). \qquad [1.22]$$

The problem in designing a PID controller is therefore that of determining parameters K_p, T_i and T_d. To illustrate the influence of the choice of each of the parameters, we will study an example.

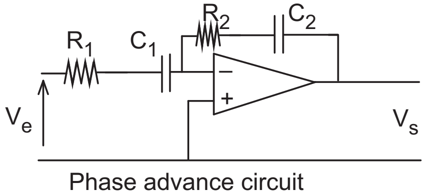

Figure 1.14. *Phase advance circuit*

EXAMPLE 1.2.– *Consider the position control of a direct current motor, whose transfer function is given by*

$$H_o(p) = \frac{100}{p(p+50)}. \qquad [1.23]$$

1.2.4.1. *PD control*

The transfer function of the controller is expressed by

$$C(p) = K_p(1 + T_d p). \qquad [1.24]$$

REMARK 1.5.– *Such a transfer function is not feasible, since the degree of the numerator is smaller than that of the denominator; on the other hand, what we can achieve is a function of the type:*

$$C(p) = K_p(\frac{1 + T_d p}{1 + \tau p}) \qquad [1.25]$$

where τ is small enough such that the influence of the pole $-1/\tau$ is negligible.

The transfer function of a non-loop controlled system is written as:

$$H(p) = H_o(p)C(p) = \frac{100K_p(1 + T_d p)}{p(p + 50)}. \quad [1.26]$$

We have therefore added a zero to the transfer function $H_o(p)$.

First, consider the proportional controller only ($T_d = 0$). The denominator of the transfer function of the loop system (characteristic polynomial) equation is given by

$$P(p) = p^2 + 50p + 100K_p. \quad [1.27]$$

We have:

$$\omega_n^2 = 100K_p; \quad 2\xi\omega_n = 50 \quad \Longrightarrow \quad \omega_n = 10\sqrt{K_p}; \quad \xi = \frac{50}{20\sqrt{K_p}}. \quad [1.28]$$

Therefore, if K_p is increased, ω_n also increases and as a result the speed of the response of the loop system, but the amplitude of the oscillations increases as well (small ξ). For $K_p = 12.5$, we get damping $\xi = \frac{\sqrt{2}}{2}$, but a slow response ($\omega_n = 35.35\,\mathrm{rad/s}$); for $K_p = 100$ the response is fast ($\omega_n = 100\,\mathrm{rad/s}$) but very oscillating because $\xi = 0.25$. It can also be seen that the static error in the velocity is equal to $\frac{50}{100K_p}$; it is improved by increasing the gain. The static error in position does not depend on the controller since the process contains an integration.

The introduction of the term involving a derivative allows for an additional degree of freedom. In effect, the denominator of the transfer function of the loop system becomes

$$P(p) = p^2 + (50 + 100K_pT_d)p + 100K_p, \quad [1.29]$$

that is

$$\omega_n = 10\sqrt{K_p}; \quad \xi = \frac{2.5 + 5K_pT_d}{\sqrt{K_p}}. \quad [1.30]$$

The error of velocity remains equal to $\frac{50}{100K_p}$; the derivative term does not affect the static behavior in speed. By introducing this additional degree of freedom in the controller, it is possible to ensure both large ω_n and ξ.

By taking $K_p = 100$, the same static error can be obtained in velocity as previously, a natural frequency of $\omega_n = 100\,\mathrm{rad/s}$ but also a damping coefficient $\xi = 1$ by choosing $T_d = 0.015\,\mathrm{s}$.

The step response of the controlled loop system is given for different values of K_p and T_d in Figure 1.15.

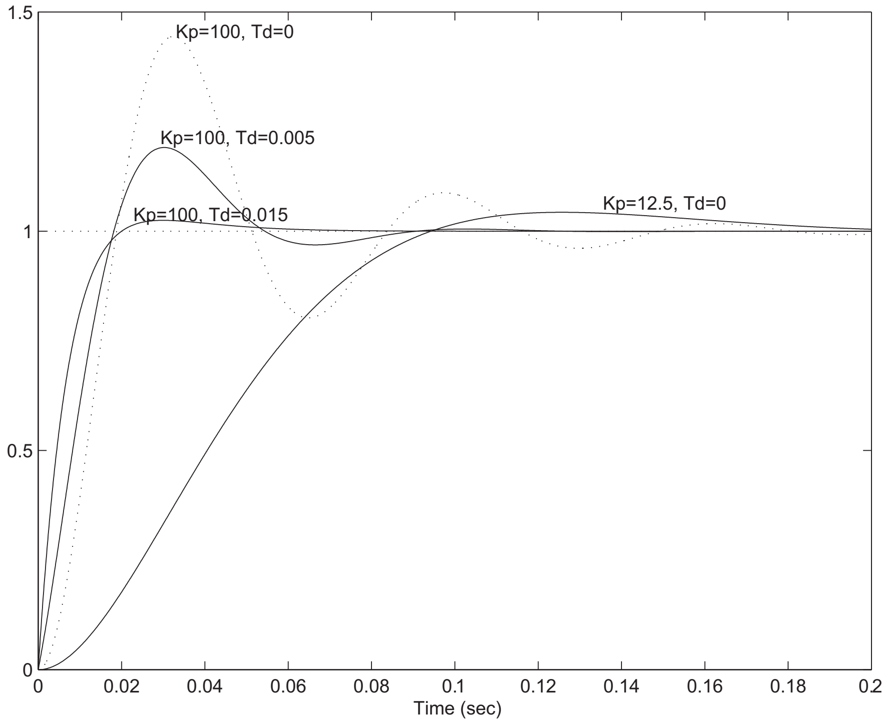

Figure 1.15. *Step response of the corrected loop system*

1.2.4.2. *PI control*

Now consider a controller of the form:

$$C(p) = K_p(1 + \frac{1}{T_i p}) \qquad [1.31]$$

which can be written as:

$$C(p) = \frac{K_p}{T_i} \cdot \frac{1 + T_i p}{p}. \qquad [1.32]$$

Therefore, a zero and a pole in 0 can be added to the system. The addition of the integration reduces the static error. In the case of the PD controller, the static error in velocity imposed the choice of T_d; this is no longer the case here because we have added a pole at the origin, which cancels out the static error in velocity. Thus, the choice of controller parameters will be primarily done based on criteria related to stability and the transient response.

We use the Routh criterion to analyze stability according to the parameters of the corrector. The characteristic equation of the loop system is given by

$$p^3 + 50p^2 + 100K_p p + 100\frac{K_p}{T_i} = 0. \qquad [1.33]$$

The results are presented in Table 1.1.

p^3	1	$100K_p$
p^2	50	$100\frac{K_p}{T_i}$
p^1	$100K_p - 2\frac{K_p}{T_i}$	0
p^0	$100\frac{K_p}{T_i}$	0

Table 1.1. *Routh table results*

We have stability if and only if $\frac{K_p}{T_i} > 0$ and $\frac{1}{T_i} < 50$. Based on this, we need to take $\frac{1}{T_i}$ as small as possible. In effect, the OL transfer function of the controlled system is given by

$$H(p) = C(p)H_o(p) = \frac{100K_p(p + \frac{1}{T_i})}{p^2(p + 50)}. \qquad [1.34]$$

The term in p^2 in the denominator ensures a zero static error in velocity and the fact of choosing the zero as small as possible makes it possible to find approximately the response of the system before integral correction. The responses corresponding to $K_p = 10$, $\frac{1}{T_i} = 0$ ($T_i = \infty$) and $K_p = 10$, $\frac{1}{T_i} = 0.01$ are identical, but the second situation has the advantage of ensuring a permanent zero error in the case of a ramp. The step response of the controlled system is given for different values of K_p and T_i in Figure 1.16.

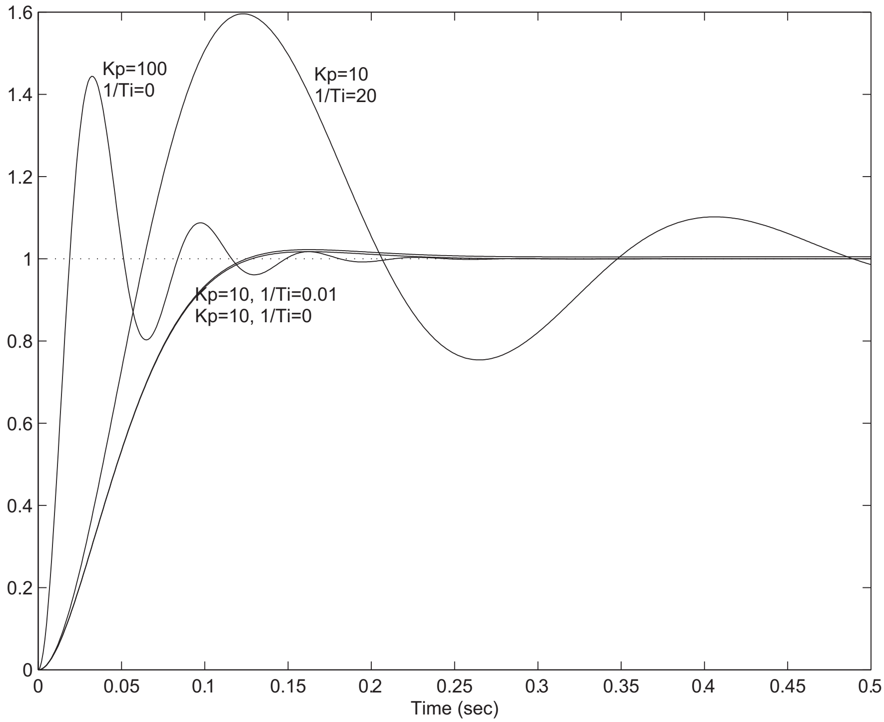

Figure 1.16. *Step response of the controlled system*

REMARK 1.6.– *The response of the system controlled by the PI (*$K_p = 10$*,* $\frac{1}{T_i} = 0.01$*) is slower than with the PD control (*$K_p = 100$*,* $T_d = 0.015$*).*

1.2.4.3. *PID control*

The transfer function of the controller is expressed by

$$C(p) = K_p(1 + \frac{1}{T_i p} + T_d p). \qquad [1.35]$$

This controller allows an approximated implementation using approximations.

There is no general method that can find the best combination of the three actions. There are methods that make it possible to find a first approximation (Ziegler–Nichols method) that then has to be refined according to the data of the problem and to trial and error.

In the case of the correction of the previous system by this type of controller, a good solution consists of recovering $K_p = 100$, $T_d = 0.015$ and adding in the integral controller with $1/T_i = 0.01$ ($T_i = 100$). This combines the speed obtained by the PD with the zero velocity error obtained by the PI.

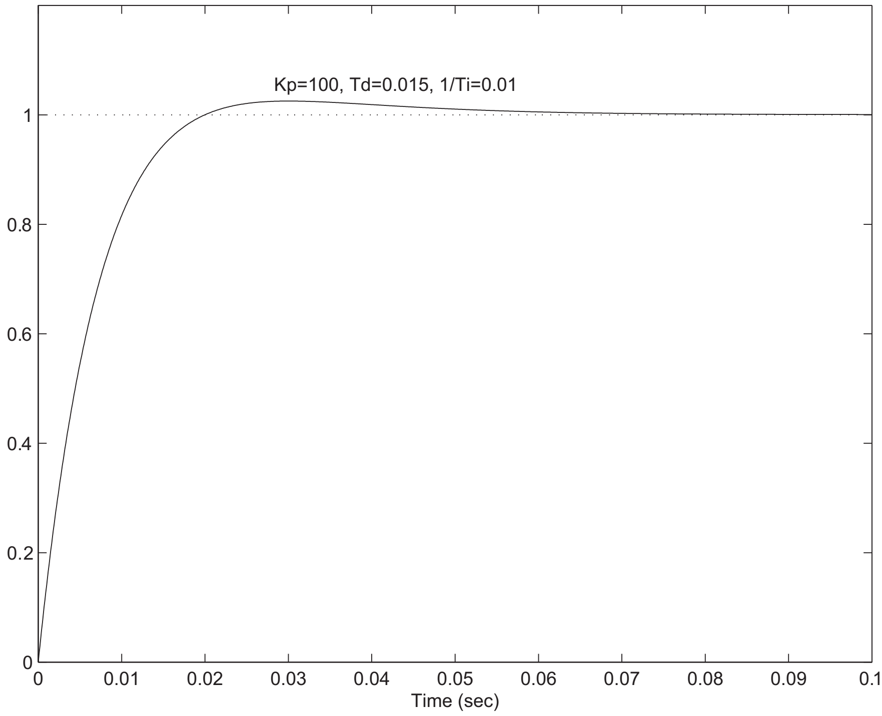

Figure 1.17. *Step response of the controlled system with correction*

In this section we have introduced a conventional method of regulation. This method is based on the knowledge of the frequency response of the system in OL and the determination of the controller consists of improving the gain margin and phase margin relatively to the system looped only by a unity feedback. It thus ensures a robustness margin (at the expense of performance) if the parameters of the transfer function were to change. We have made no assumptions about these possible variations and the knowledge of the transfer function is supposed to be acquired. It can be obtained by identification by using the methods proposed in Chapter 6 or other methods. In the following, we are addressing an example with identification based on the frequency response.

1.3. Some application exercises

1.3.1. *Identification of the transfer function and control*

The transfer function of a system can be determined from its Bode plot. The plots of the modulus and of the phase provide information about whether the system is of minimal or phase non-minimum, which allows us to propose a form of transfer function.

For the Bode plot given by Figure 1.18, we propose the following minimal phase transfer function:

$$H_o(p) = \frac{K}{p(1+\tau_1 p)(1+\tau_2 p)}. \qquad [1.36]$$

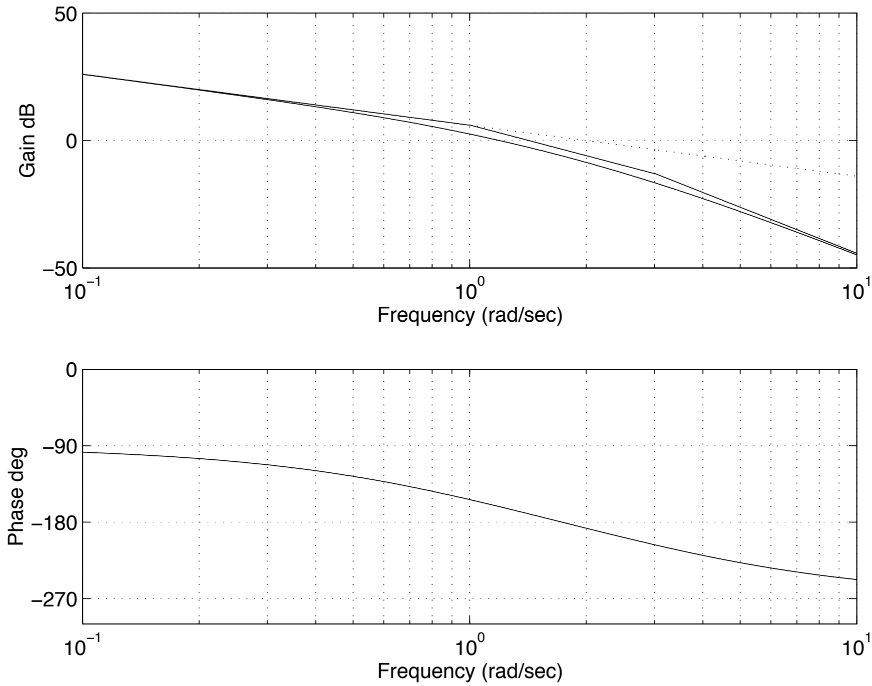

Figure 1.18. *Bode plots of a system*

The integration in the transfer function is justified by the fact that the phase starts from $-90°$ and that the very low frequency modulus follows an asymptote of -20 db/dec. The gain K can be identified by extending the asymptote due to integration. The value of K can be directly obtained at the intersection point of this asymptote with the axis 0 db. The two time constants τ_1 and τ_2 are identified from the pulsations of cutoffs ω_{c_1} and ω_{c_2} corresponding to the intersection points of the asymptotes. It is always possible to verify the results derived by the modulus plot by using the phase plot. For example, it can be verified that the phase starts from $-90°$ (integration) and tends toward $-270° = 3 \times 90°$ (two time constants).

The identified parameters of the transfer function are written as:

$$\left.\begin{array}{lcl} K = 2 & & K = 2 \\ \omega_{c_1} = 1 & \Longrightarrow & \tau_1 = 1 \\ \omega_{c_2} = 3 & & \tau_2 = 1/3 \end{array}\right\} \Longrightarrow H_o(p) = \frac{2}{p(1+p)(1+p/3)}. \qquad [1.37]$$

1.3.1.1. *Calculation of static and dynamic errors*

The static error ε_p of the closed-loop system is zero because the system has an integration in the direct chain ($\varepsilon_p = 0$). The static error in the velocity or dynamic error is calculated in the following manner:

$$\varepsilon_v = \varepsilon(t = +\infty)|_{y^d(t)=tu(t)} = \lim_{p \to 0} p\varepsilon(p)|_{Y^d(p)=1/p^2} = \frac{1}{2}. \qquad [1.38]$$

1.3.1.2. *Stability study*

To study the stability, we calculate the gain and phase margins of the system from the Bode plot:

– the gain margin is $\Delta K = 6$;

– the phase margin is $\Delta\Phi = 18°$.

The behavior of the system in the closed-loop system is not satisfactory, because it shows a very low phase margin and as a result it is very poorly damped. The goal is thus to correct it so as to improve its damping and make it faster (increasing the bandwidth).

1.3.1.3. *Servo-mechanism by phase advance controller*

It is desirable to correct the system to bring the phase margin to $50°$. To this end, we will use two types of controller, phase advance controller and integral compensator (phase delay).

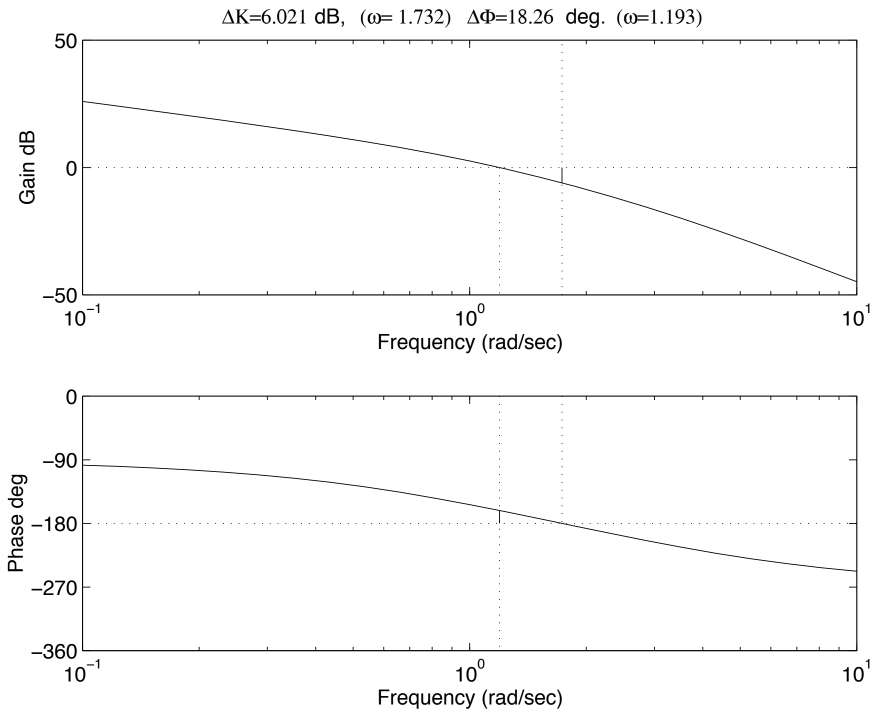

Figure 1.19. *Bode plots of a system with correction*

We have a phase margin of $18°$, which means that the controller should add a phase of $32°$. The transfer function of a phase advance controller is given as follows:

$$C(p) = \frac{1 + aTp}{1 + Tp} \qquad (a > 1). \tag{1.39}$$

The Bode plot of the phase advance controller is shown in Figure 1.20.

The maximum phase Φ_m of the phase advance controller is obtained for $\omega = \omega_m$, with

$$\omega_m = \tfrac{1}{T\sqrt{a}} \quad \sin(\Phi_m) = \tfrac{1-a}{1+a}. \tag{1.40}$$

The modulus of the controller is equal to $10\log_{10}(a)$ for $\omega = \omega_m$. As a result, if we calculate the controller to get Φ_m with ω_c corresponding to $0\,\mathrm{db}$, the new crossing point in $0\,\mathrm{dB}$ would be moved to the right of the starting point and as a result the phase margin would be different from the desired phase margin. To overcome this problem, we choose ω_m at the point where the modulus is equal to$-10\log_{10}(a)$, which makes it so that after correction the modulus of the controlled system will cross $0\,\mathrm{db}$ for $\omega = \omega_m$. However, the gain margin of the controlled system will be equal to $\Phi_m + 180° - \Phi_{\text{read}}|_{G=-10\log(a)}$. The phase margin will be overestimated by $5°$. The calculated Φ_m is $32°$, and therefore we are going to take as new Φ_m $32° + 5° = 37°$. Having set Φ_m, we calculate a:

$$a = \frac{1+\sin(\Phi_m)}{1-\sin(\Phi_m)} = \frac{1+\sin(37°)}{1-\sin(37°)} \Longrightarrow a = 4. \quad [1.41]$$

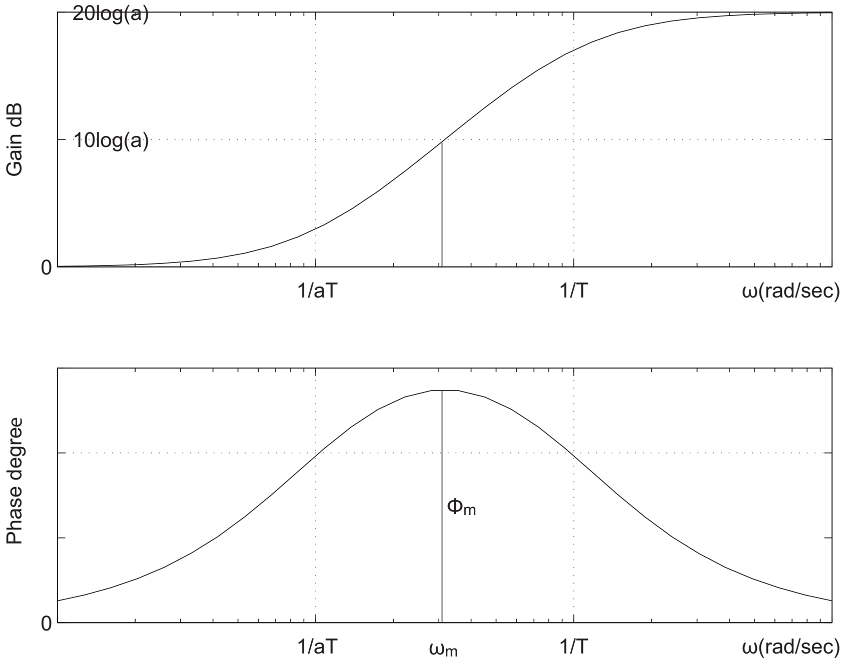

Figure 1.20. *Bode plot of the phase advance controller*

By placing ω_m at ω corresponding to the system modulus that is equal to

$$-10\log_{10}(a) = -10\log_{10}(4) = -6\,\mathrm{db} \Longrightarrow \omega_m = 1.73\mathrm{rd/p}. \quad [1.42]$$

This allows us to calculate T,

$$T = \frac{1}{\omega_m \sqrt{a}} = \frac{1}{1.73\sqrt{4}} = 0.29\,\mathrm{s}. \qquad [1.43]$$

Hence, the following controller:

$$C(p) = \frac{1 + 1.16p}{1 + 0.29p}. \qquad [1.44]$$

The Bode plot of the system after correction is shown in Figure 1.21.

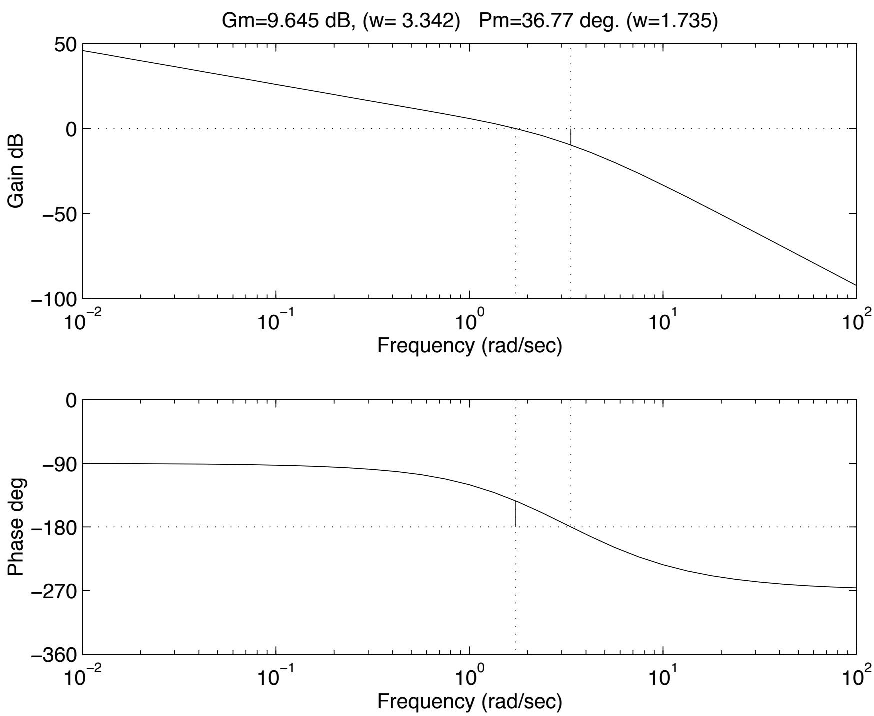

Figure 1.21. *Bode plot of the system after correction*

We can observe that the phase margin after correction does not correspond to the expected result; this is due to the variation of the phase around the critical point that is too fast. This fast variation of the phase causes a phase drop that largely exceeds the $5°$ (in reality $\simeq 18°$ are lost). The advance phase controller is not suitable in the case of excessively fast phase variations.

REMARK 1.7.– *The results in the Bode plot of the system must always be verified after correction.*

1.3.1.4. *Integral compensator control (phase delay controller)*

In order to obtain a desired phase margin of $50°$, this time we are going to act not upon the phase but upon the modulus such that it passes through $0\,\mathrm{db}$ at pulse ω_c that corresponds to a system phase that is equal to $(\Phi_c = -180° + 50° = -130°)$. Since the modulus cancels out for ω_c $(\Phi_c = -130°)$, then the phase margin is therefore $\Delta\Phi = 180° - 130° = 50°$. The integral compensator has the transfer function

$$C(p) = \frac{1 + aTp}{1 + Tp} \qquad (a < 1). \tag{1.45}$$

Its Bode plot diagram is shown in Figure 1.22.

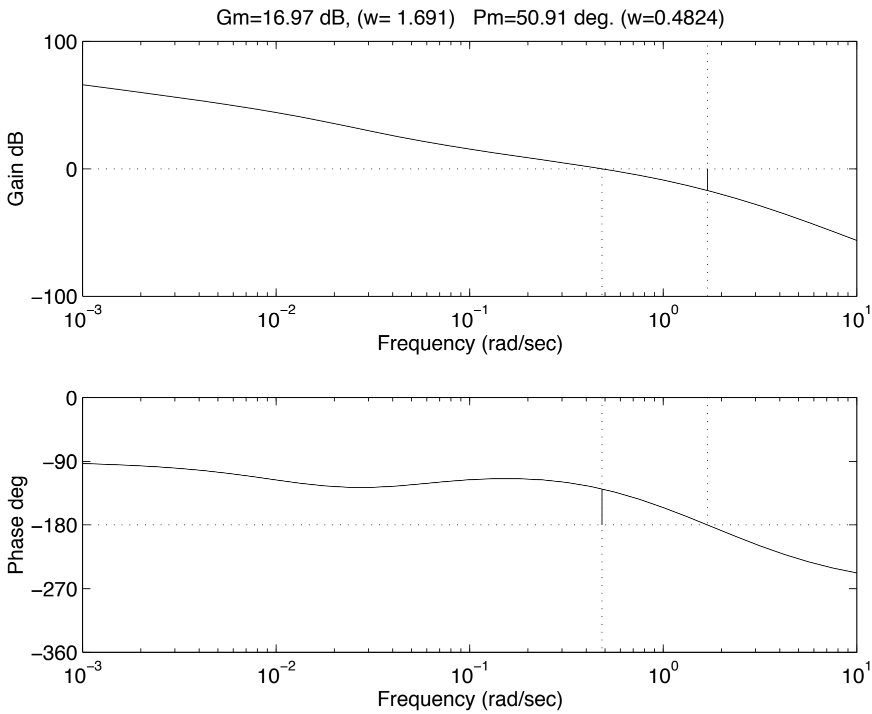

Figure 1.22. *Bode plot of the system after correction*

It can be observed that the phase of the controller is negative and consequently it will delay the phase of the system; as a result, the calculated phase margin will be affected. To compensate for this effect, we add a margin of $5°$, that is to say, instead of taking $\Phi_c = -130°$, we will choose $\Phi_c = -125°$ and by placing ω_m we will manage to not lose more than these $5°$ because of the controller.

The value of a is calculated by measuring the modulus d at ω_c $(\Phi_c = -125°)$ and by making

$$20\log(a) = -d \Longrightarrow a = 10^{-d/20}. \qquad [1.46]$$

We choose T so as to not affect the phase around ω_c. For this purpose, we put a decade between $1/aT$ and ω_c, the impulse of the modulus crossing $0\,\mathrm{db}$ after control, which makes

$$\frac{1}{aT} = \frac{\omega_c}{10} \Longrightarrow T = \frac{10}{a\omega_c}. \qquad [1.47]$$

The phase $\Phi = \Phi_c = -125°$ is obtained for $\omega = 0.486$, which yields a modulus $d = 11.30$. Hence, the value of a is given by:

$$a = 10^{-11.30/20} = 0.27. \qquad [1.48]$$

The value of T is given by

$$T = \frac{10}{a\omega_c} = \frac{10}{0.27 \times 0.486} = 76.20\,\mathrm{s}. \qquad [1.49]$$

Hence the following controller is obtained:

$$C(p) = \frac{1 + 20.57p}{1 + 76.20p}. \qquad [1.50]$$

The Bode plot of the system after control is shown in Figure 1.23.

The phase margin is maintained, and it is correctly written as $\Delta\Phi \simeq 50°$. The advantage of this controller is its simplicity, but its main drawback is that it reduces the bandwidth of the system.

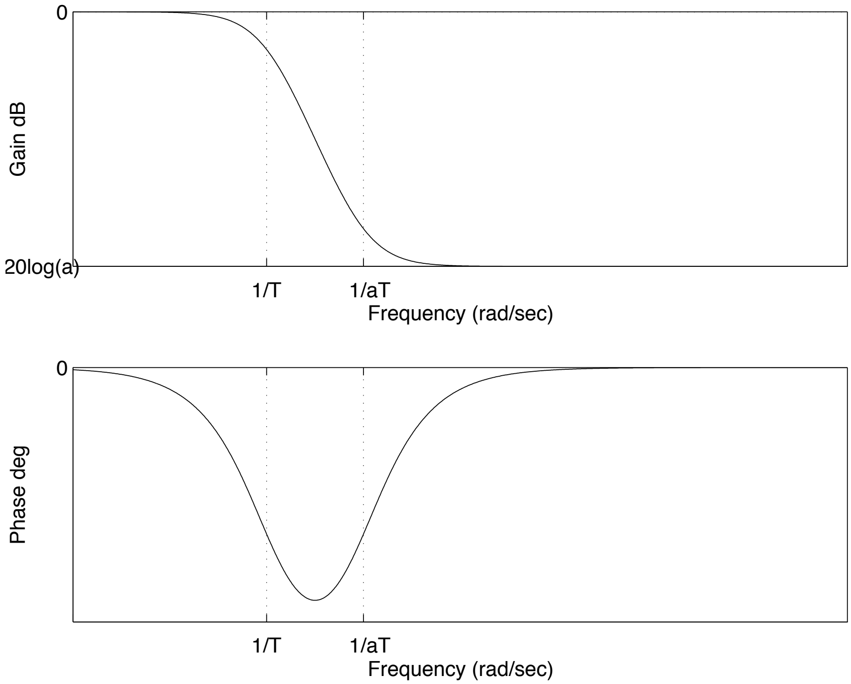

Figure 1.23. *Bode plot with integral compensator*

1.3.2. *PI control*

For the system represented by its transfer function,

$$H_o(p) = \frac{2}{(1+0.5p)^2}. \qquad [1.51]$$

The following requirements must be satisfied:

– zero static position error;

– bandwidth $\omega_c \geq 4\,\mathrm{rd/s}$ (cutoff impulse);

– phase margin $\simeq 50°$.

The Bode plot of this system is shown in Figure 1.24.

The cutoff pulse $\omega_c = 2\,\mathrm{rd/s}$ ($|H(j\omega_c)| = 0\,\mathrm{db}$), and the phase margin $\Delta\Phi = 90°$.

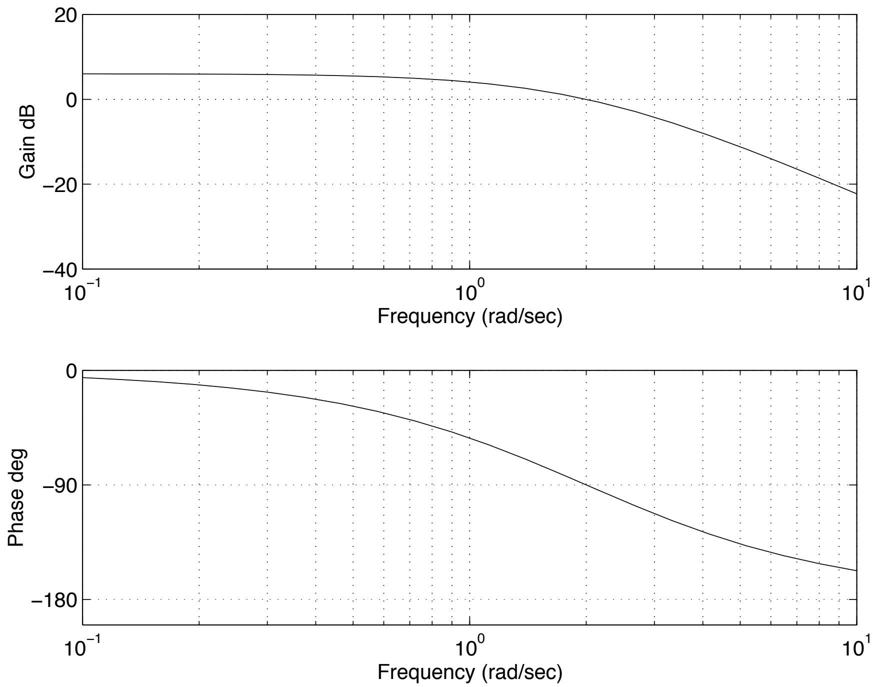

Figure 1.24. *Bode plot of the system without PI control*

The static position error is non-zero; to cancel, it is necessary to introduce an integration in the controller. The controller that we thus propose is a PI controller:

$$C(p) = K(1 + \frac{1}{T_i p}) = K\left(\frac{1 + T_i p}{T_i p}\right) = \frac{K}{T_i} \cdot \frac{1 + T_i p}{p}. \qquad [1.52]$$

The choice of $\frac{1}{T_i}$ is made so as to compensate the effect on the integration phase. The phase around ω_c should be unchanged ($\approx$ the system phase). We will therefore place $\frac{1}{T_i}$ a decade further than ω_c: $\frac{1}{T_i} = \frac{\omega_c}{10} \Longrightarrow T_i = \frac{10}{\omega_c} = \frac{10}{2} = 5\,\mathrm{s}$. The Bode diagram of the system controlled by means of the following controller ($\frac{K}{T_i} = 1$) is shown in Figure 1.25:

$$C_1(p) = \frac{1 + T_i p}{p}. \qquad [1.53]$$

The position error is zero because there is integration in the direct chain; we are now going to determine the parameters of the controller in order to meet the specifications. The choice of T_i has been done so as to offset the phase effect of the integration, it then just suffices to calculate the second parameter K to ensure a phase margin of $\simeq 50°$ and a bandwidth of at least $4\,\mathrm{rd/s}$ ($\omega_c \geq 4\,\mathrm{rd/s}$).

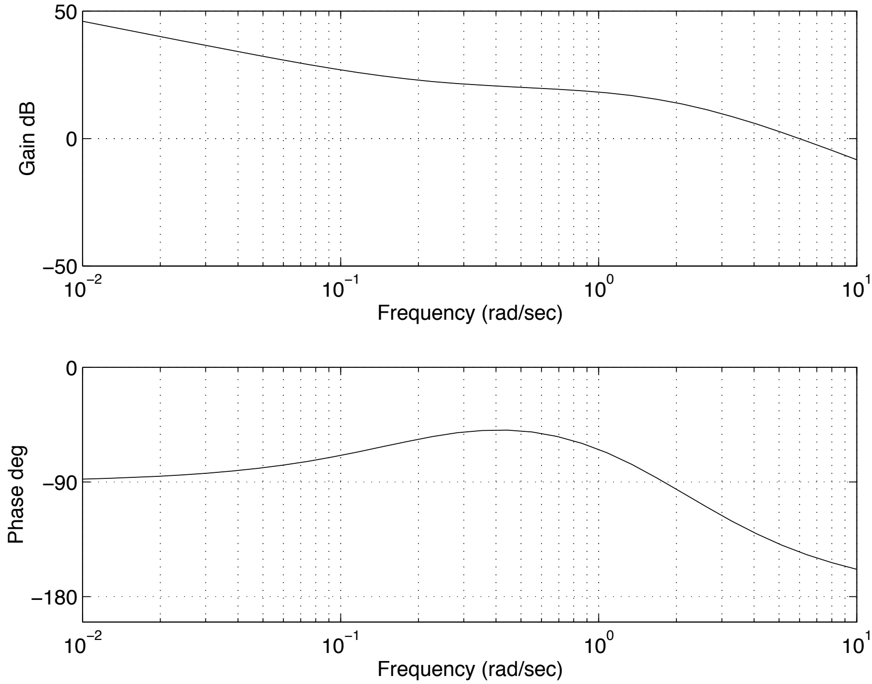

Figure 1.25. *Bode plot of the system with control*

The quantity $\frac{K}{T_i}$ is calculated by measuring the modulus d of $|C_1(j\omega_c')H_o(j\omega_c')|$ with ω_c' the new cut-off pulse corresponding to a phase $\Phi_c = -130°$ (phase margin $\approx 50°$) and by imposing:

$$20\log(\frac{K}{T_i}) = -d \Longrightarrow K = T_i \cdot 10^{-d/20}. \qquad [1.54]$$

The phase $\Phi = \Phi_c = -130°$ is obtained for $\omega = \omega_c = 4.02\,\mathrm{rd/s}$, which yields a modulus $d = 5.94$. Hence, the value of K: $K = 5 \cdot 10^{-5.94/20} = 2.5$ and the expression of the controller

$$C(p) = 2.5(1 + \frac{1}{5p}). \qquad [1.55]$$

The Bode plot of the system after control is shown in Figure 1.26. We correctly verify on the plot that the bandwidth is of $4\,\mathrm{rd/s}$ and that the phase margin is $50°$. The position static error is zero due to the integration in the direct chain. The requirements of the specifications are thus properly satisfied.

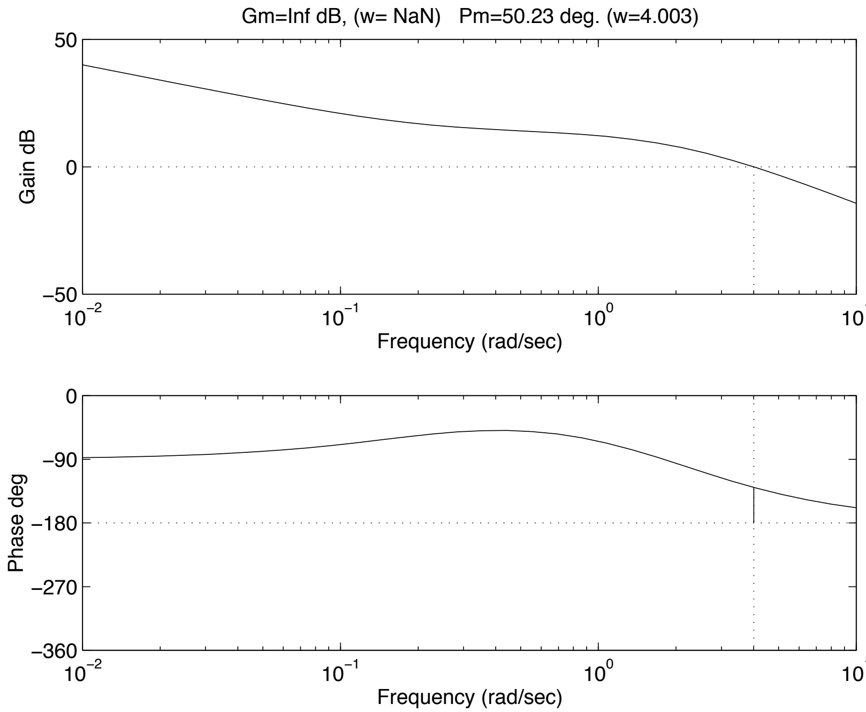

Figure 1.26. *Bode plot of the system with control*

1.3.3. *Phase advance control*

For the system represented by its transfer function

$$H_o(p) = \frac{K(1+p)}{p(1+0.2p)(1+0.05p)}, \qquad [1.56]$$

The following requirements must be satisfied:

– zero static error;

– a gain of about $30\,\mathrm{db}$ for $\omega = 2\,\mathrm{rd/s}$;

– phase margin $\simeq 50°$.

We want a gain of 30 for $\omega = 2\,\mathrm{rd/s}$. This allows us to define the value of K. We write:

$$\begin{aligned} 20\log_{10}\left(|H(j\omega)|_{\omega=2}\right) &= 20\log_{10}\left(\left.\frac{K\sqrt{1+\omega^2}}{\omega\sqrt{1+0.04\omega^2}\sqrt{1+0.0025\omega^2}}\right|_{\omega=2}\right) \\ &= 30\,\mathrm{db}. \end{aligned} \quad [1.57]$$

This equation can be rewritten by replacing ω by its value

$$20\log_{10}(1.033K) = 30 \Longrightarrow K = 30.6 \qquad \text{(that is } K = 30\text{).} \quad [1.58]$$

The new transfer function of the system is

$$H(p) = \frac{30(1+p)}{p(1+0.2p)(1+0.05p)}. \quad [1.59]$$

The Bode plot of this system is shown in Figure 1.27.

The controller that we propose is a phase advance corrector because the dynamic accuracy is imposed by the value of K (gain of $30\,\mathrm{db}$ at $\omega = 2\,\mathrm{rd/s}$). Consequently, we cannot use an integral compensator (phase delay). The transfer function of the controller is as follows:

$$C(p) = \frac{1+aTp}{1+Tp} \qquad (a > 1). \quad [1.60]$$

The phase margin is $\Delta\Phi = 25°$, hence the phase to be added is $\Phi_m = 25° + 5° = 30°$. The value of a is given by

$$a = \frac{1+\sin(\Phi_m)}{1-\sin(\Phi_m)} = \frac{1+\sin(30°)}{1-\sin(30°)} = 3 \qquad \text{(that is } a = 4\text{).} \quad [1.61]$$

By placing ω_m at ω corresponding to the system modulus that is equal to

$$-10\log_{10}(a) = -10\log_{10}(4) = -6\,\mathrm{db} \Longrightarrow \omega_m = 76\,\mathrm{rd/s}. \quad [1.62]$$

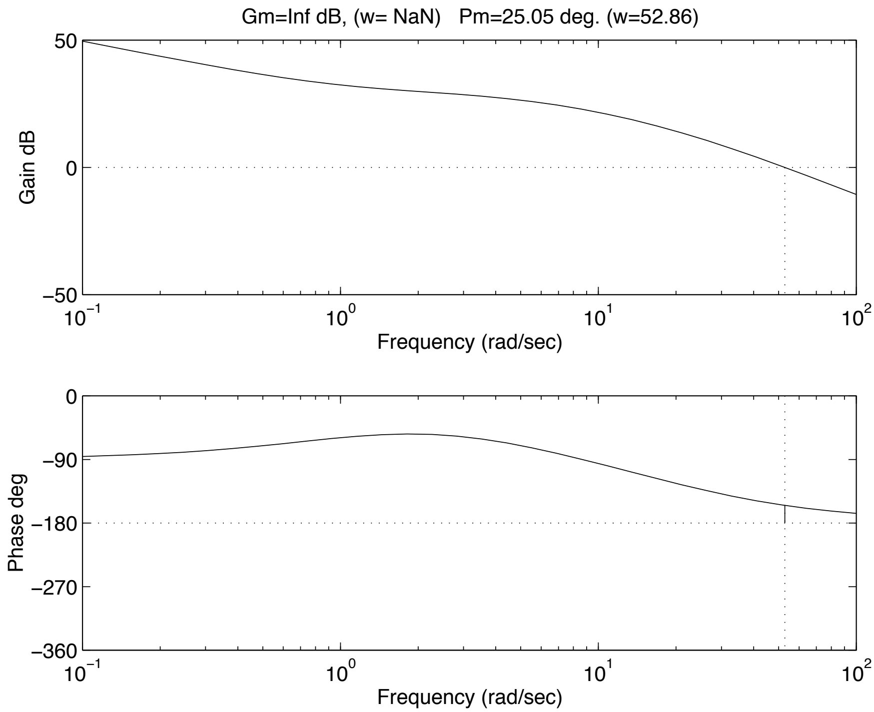

Figure 1.27. *Outline of the Bode plot of the system without phase advance control*

This allows us to calculate T,

$$T = \frac{1}{\omega_m \sqrt{a}} = \frac{1}{76\sqrt{4}} = 0.0066\,\mathrm{s}. \qquad [1.63]$$

Hence, the following controller:

$$C(p) = \frac{1 + 0.0263p}{1 + 0.0066p}. \qquad [1.64]$$

The outline of the Bode plot of the system after being controlled is shown in Figure 1.28 in which we verify that the specifications are properly satisfied.

The cases of nonlinear systems will be addressed in the following chapters and examples of nonlinear systems using linearization methods will be given.

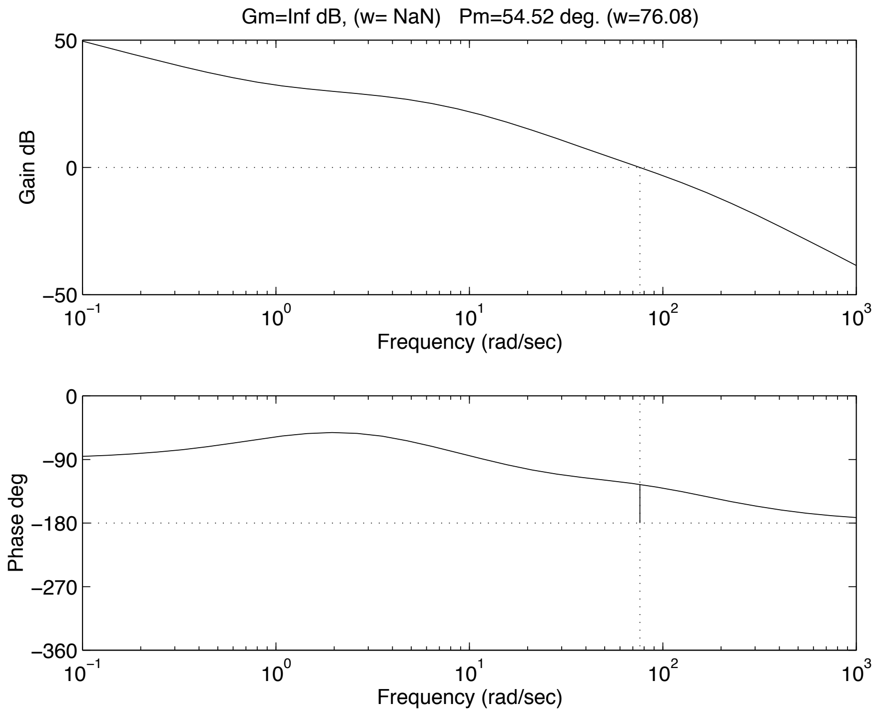

Figure 1.28. *Outline of the Bode plot of the system with phase advance control*

1.4. Some application exercises

EXERCISE 1.– A device, whose transfer function is as follows, is controlled by a controller placed in cascade with the system with a unity feedback loop. $H_o(p) = \frac{1}{p(p+1)}$:

1) Let $C(p) = K$.

a) Express the parameters of the transfer function of the system in the closed-loop system.

b) For $K = 1$, what can be said of its step response $y(t)$ to unit step function, and about permanent errors of positions ε_p and velocity ε_v?

c) For $K = 9$, what can be said of $y(t)$, of ε_p and of ϵ_v? Specify the effects due to the increase in K. Following Bode's method, determine the gain and phase margins. What can be observed from these values?

2) We want to find a permanent velocity error $\epsilon_v < 10\%$ and a phase margin of about $50°$.

a) Determine the parameters of a controller to be inserted.

b) Verify the result using the Nyquist method by plotting the frequency response with and without control.

EXERCISE 2.– Figure 6.21 shows the frequency response of a system plotted according to Bode's method:

1) Determine the transfer function $H_o(p)$ of the system.

2) What are the position ϵ_p and velocity errors ϵ_v?

3) What are the gain ΔG and phase margins $\Delta\Phi$? The behavior of the system is not considered satisfactory. Why?

4) We want to bring the phase margin to $50°$. Study the serial compensation using phase advance and phase delay controllers. Compare both methods.

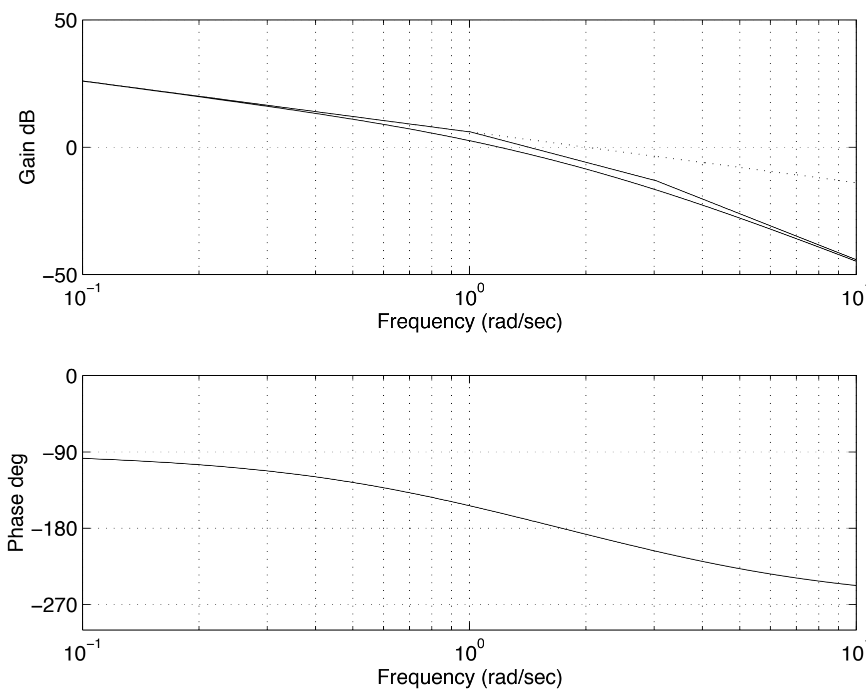

Figure 1.29. *Frequency response of the unknown system*

EXERCISE 3.– An OL system is described by Bode, see Figure 1.30.

$$H_o(p) = \frac{2}{(1 + 0.5p)^2}. \quad [1.65]$$

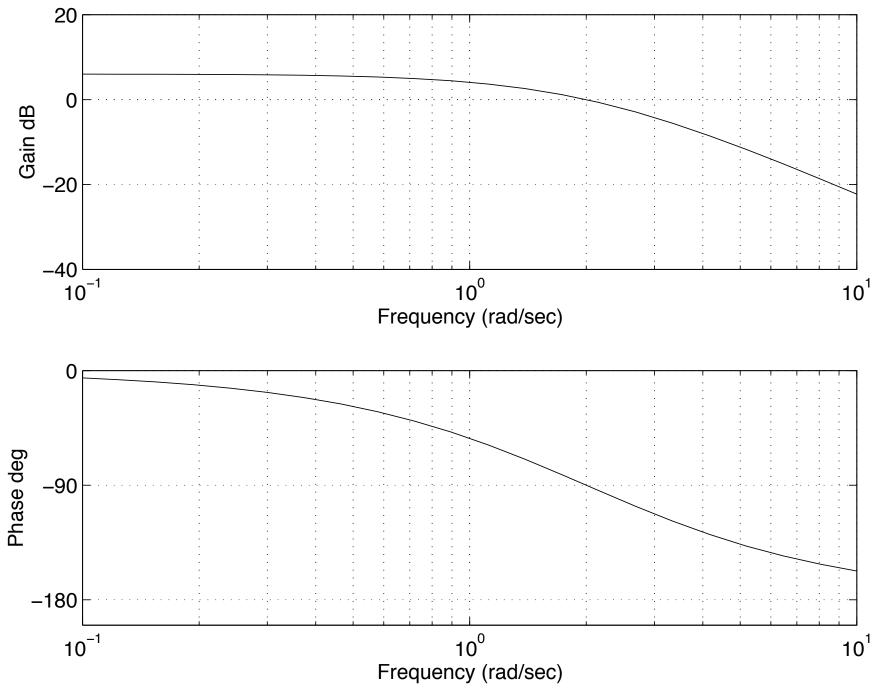

Figure 1.30. *Bode plot of* $H(p) = \frac{2}{(1+0.5p)^2}$

Determine the parameters of a controller to be inserted in series to obtain the following performance:

1) zero static (position) error;

2) bandwidth $\omega_c \geq 4$ rad/s;

3) phase margin $\simeq 50^o$.

EXERCISE 4.– Let a system be described by its transfer function in OL (Bode, Figure 1.31):

$$H_o(p) = \frac{K(1+p)}{p(1+0.2p)(1+0.05p)}. \tag{1.66}$$

This process is inserted in a control chain with unity back. The desired performances are:

1) zero static (position) error;

2) a gain of about $30\,\mathrm{db}$ for $\omega = 2\,\mathrm{rd/s}$;

3) phase margin $\simeq 50^{\mathrm{o}}$.

Determine the parameters of a controller to be inserted in the sequence to satisfy these conditions.

Bode plot of

$$H(s) = \frac{K(1+s)}{s(1+0.2s)(1+0.05s)}. \tag{1.67}$$

1.5. Application 1: stabilization of a rigid robot with pneumatic actuator

The model of a robot for manipulation with two degrees of freedom is defined by

$$\tau = M(q)\ddot{q} + C(q,\dot{q})\dot{q} + F_v\dot{q} + G(q). \tag{1.68}$$

The system variables are vectors of dimension 2, respectively, representing positions, velocities and accelerations, q, $\dot{q}$,$\ddot{q}$. We define the following parameters for this robot:

– $M(q)$ is the matrix of inertia of dimension 2×2;

– $G(q)$ is the vector of gravity effects;

– $C(q,\dot{q})\dot{q}$ represents the centrifugal and Coriolis forces;

– F_v is the coefficient of viscous friction at the axis level;

– τ represents the couples applied at the axis level of the robot.

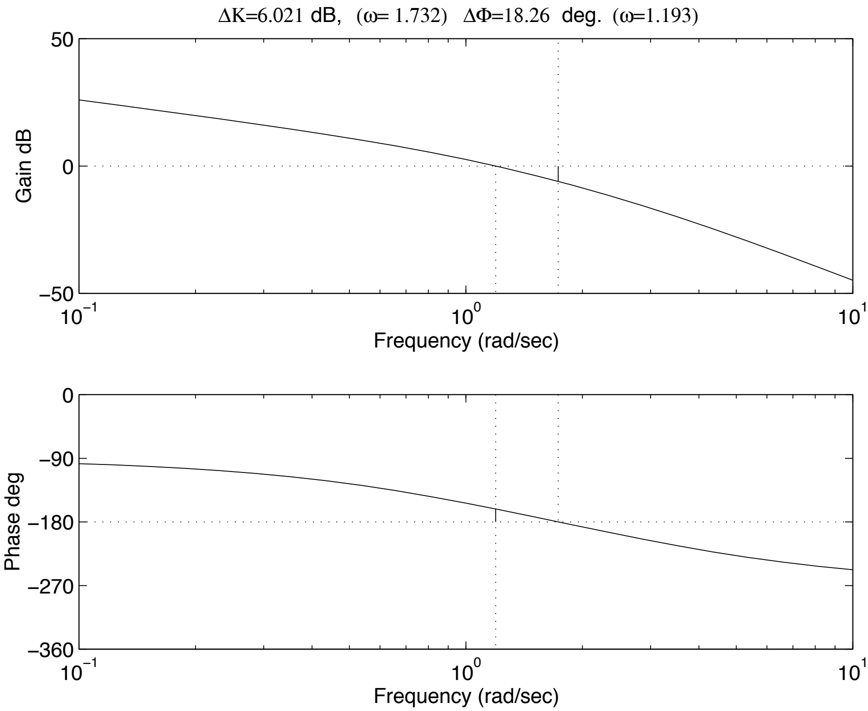

Figure 1.31. *Bode plot of* $\frac{H(p)}{K} = \frac{(1+p)}{p(1+0.2p)(1+0.05p)}$

Pneumatic actuators used for this robot can be represented by the differential equation linking the couple τ with the control voltage u and the velocity of the axes $\dot{q}$ as it follows:

$$\dot{\tau} + B\tau + E\dot{q} = Ju. \qquad [1.69]$$

First, to simplify the study, it will be assumed that the essential terms of the dynamic model: $M(q) = M$, $C(q,\dot{q})\dot{q} = C_o\dot{q}$, $G(q) = G_o$ are constant. The first part of the study concerns the first axis of the robot. In other words, variables $q, \dot{q}$,$\ddot{q}$ and u can be regarded as scalars.

The parameters of the system (considered linear invariant) are the following:

$g = 9.81\,;\ l1 = 0.11\,; l2 = 0.15\,;\ I_1 = 0.07\,\mathrm{kgm}^2;\ I_2 = 0.025\,\mathrm{kgm}^2; m_1 = 0.6\,\mathrm{kg};\ m_2 = 0.4\,\mathrm{kg};$

$E = 5I_{d2}$; $B = 10I_{d2}$; $J = 100I_{d2}$, I_{d2} is the identity matrix of dimension 2.

$s = \sin(q_2)$; $c = \cos(q_2)$;

$mll = 2m_2l_1l_2$; $mlc = 2m_2l_1l_2c$; $mls = 2m_2l_1l_2s$;

$c_{12} = \cos(q_1 + q_2)$; $s_{12} = \sin(q_1 + q_2)$;

Components of the inertia matrix: M_{ij}: let $A = 4m_2l_1^2 + I_1 + I_2$;

$M_{11} = A + 4m_2l_1l_2c + m_1l_1^2 + m_2l_2^2$; $M_{12} = I_2 + 2m_2l_1l_2c + m_2l_2^2$;

$M_{21} = I_2 + 2m_2l_1l_2c + m_2l_2^2$; $M_{22} = I_2 + m_2l_2^2$;

Components of the matrix C: $C_{11} = -mls\dot{q}_2$; $C_{12} = -mls(\dot{q}_1 + \dot{q}_2)$; $C_{21} = mls\dot{q}_1$; $C_{22} = 0$;

Componants of vector G: $G_1 = (m_1 + 2m_2)gl_1 sin(q_1) + m_2gl_2s_{12}$; $G_2 = m_2gl_2s_{12}$.

To simplify the preliminary study of the control of this robot, we will focus on the first axis only and as a first step, couplings and nonlinearities, which can be considered as disturbance inputs, will be neglected. Next, we will be able to consider as a nominal model the one obtained when functioning around angular positions $q_1 = q_2 = 0$ and movements of small amplitudes.

$$M = \begin{pmatrix} 0.1570 & 0.0472 \\ 0.0472 & .0340 \end{pmatrix}; C_o = 0; \text{ and } \; G_o = 0; \; F_v = 0. \qquad [1.70]$$

$$J = \begin{pmatrix} 100 & 0 \\ 0 & 100 \end{pmatrix}; \; B = \begin{pmatrix} 10 & 0 \\ 0 & 10 \end{pmatrix}; \; E = \begin{pmatrix} 5 & 0 \\ 0 & 5 \end{pmatrix}. \qquad [1.71]$$

1.5.1. *Conventional approach*

For the study below, the system equation will be taken as: $\tau = M_o\ddot{q}$ and $\dot{\tau} + B\tau + E\dot{q} = Ju$, with $M_o = 0.157$, $B = 10$, $J = 100$, and $E = 5$.

1) *Write in the form of a single differential equation the model of the first axis of the robot with its actuator.*

We shall express the model of the first axis of the robot with its actuator in the form of a single differential equation. The system equations: $\tau = M_o\ddot{q}$ and $\dot{\tau} + B\tau + E\dot{q} = Ju$, with $M_o = 0.157$, $B = 10$, $J = 100$, and $E = 5$ can be written describing $\tau = M_o\ddot{q}$ and substituting in the other equation. This gives us: $Ju = M_o\dddot{q} + BM_o\ddot{q} + E\dot{q} = 0.157\dddot{q} + 1.57\ddot{q} + 5\dot{q} = 100u$.

2) *Express the system transfer functions for the velocity* $H_1(p) = \frac{V(p)}{U(p)}$ *and for position* $H_2(p) = \frac{q(p)}{U(p)}$*, with* $v(t) = \frac{dq(t)}{dt}$ *the rotation velocity of the axis. Determine the poles and zeros of these two transfer functions.*

From the above equation, the transfer functions of the system are derived using Laplace transformation and considering zero initial conditions,

$$H_1(p) = \frac{V(p)}{U(p)} = \frac{100}{0.157p^2 + 1.57p + 5} \qquad [1.72]$$

and

$$H_2(p) = \frac{q(p)}{U(p)} = \frac{100}{p(0.157p^2 + 1.57p + 5)}. \qquad [1.73]$$

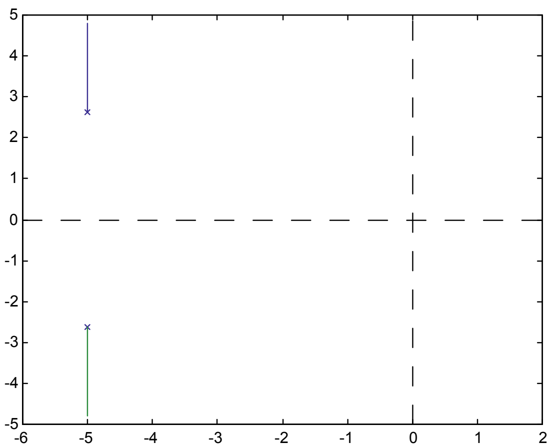

Figure 1.32. *Locus of the roots of* $H_1(p)$

3) *Plot the Nyquist locus of the transfer function* $H_o(p) = K_p.\ H_2(p) = K_p \frac{q(p)}{U(p)}$ *and analyze the stability of the system with a unity feedback loop for this position control.*

Nyquist locus of the transfer function

$$H_o(p) = K.H_2(p) = K\frac{q(p)}{U(p)} = \frac{100K}{p(0.157p^2 + 1.57p + 5)}. \qquad [1.74]$$

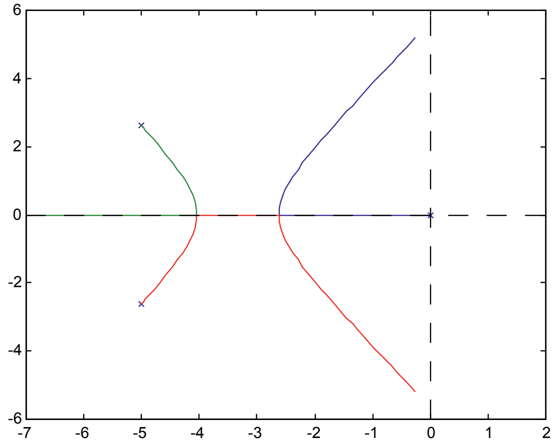

Figure 1.33. *Locus of the roots of* $H_2(p)$

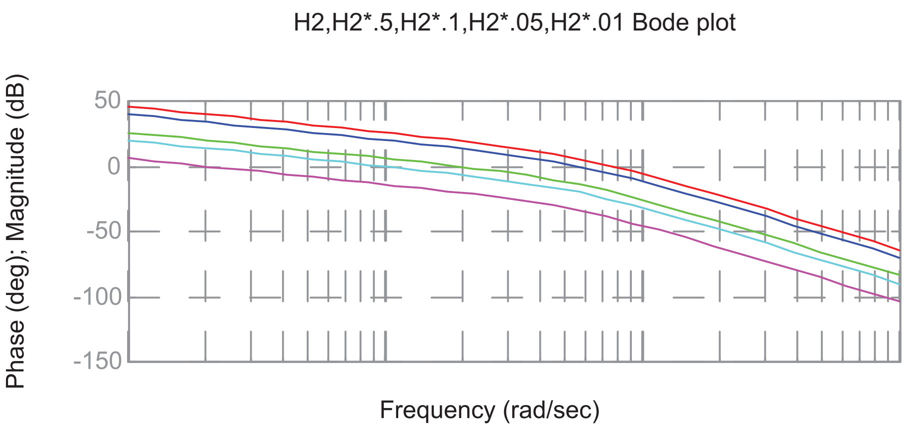

Figure 1.34. *Bode plot of* $K.H_2(p)$*. For a color version of this figure, see www.iste.co.uk/femmam/signals.zip*

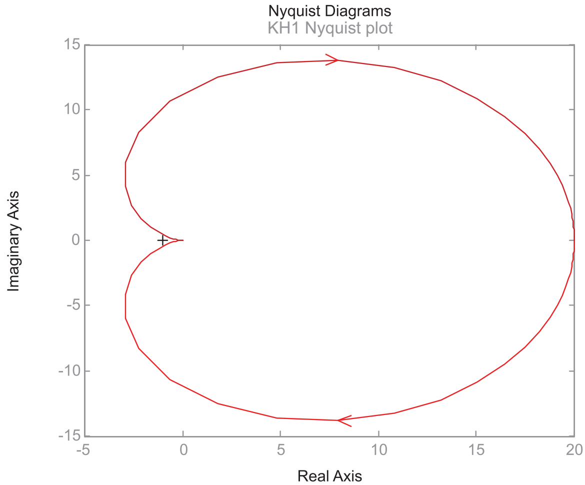

Figure 1.35. *Nyquist locus for* $K = 1(H_1)$

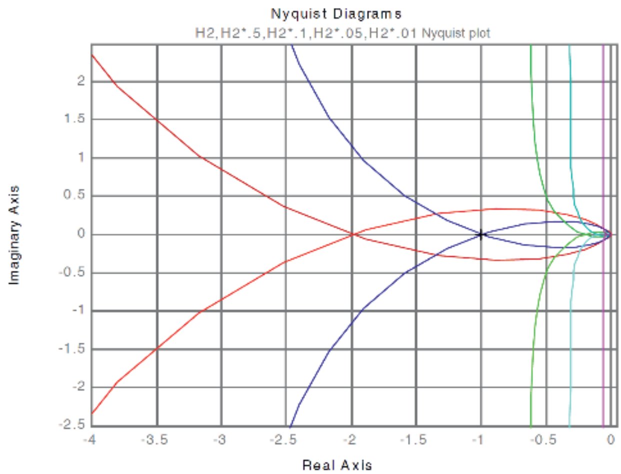

Figure 1.36. *Nyquist loci for* $H_2(p) * Ki$*. For a color version of this figure, see www.iste.co.uk/femmam/signals.zip*

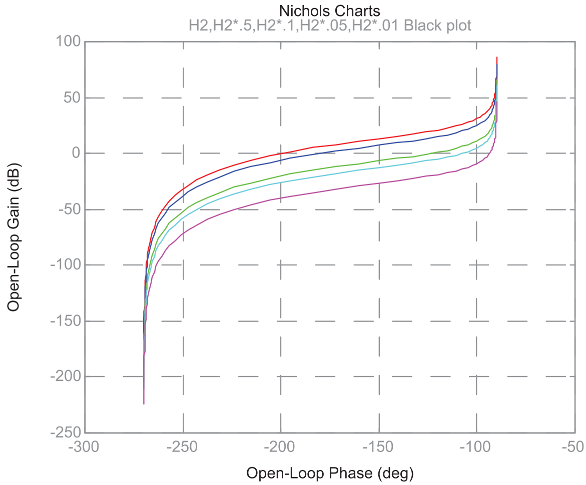

Figure 1.37. *Black plot for $K.H_2(p)$. For a color version of this figure, see www.iste.co.uk/femmam/signals.zip*

4) The stability of the system with a unity feedback loop is guaranteed if $K < 0.5$.

5) *First, we want to control the velocity of this axis; the system is then considered as defined by* $H_1(p) = \frac{V(p)}{U(p)}$.

a) *By applying the Routh criterion, analyze the stability of the system, whose velocity is looped with a proportional controller of gain* K_v $(C(p) = K_v)$ with a unity feedback. Determine the conditions on the gain of a proportional feedback ensuring the stabilization of the system.

Velocity control of $H_1(p) = \frac{V(p)}{U(p)} = \frac{100}{(0.157p^2+1.57p+5)}$.

The system looped with a proportional controller of gain K $(C(p) = K)$ with a unity feedback has a transfer

$$G_1(p) = \frac{100K}{0.157p^2 + 1.57p + 5 + 100K}. \qquad [1.75]$$

We apply the Routh criterion to

$$0.157p^2 + 1.57p + 5 + 100K. \qquad [1.76]$$

6) a) The condition on gain K ensuring the stability of the system in the closed-loop system is: $K > -0.05$.

$line\ p^2$	0.157	$5+100K$
$line\ p^1$	1.57	0
$line\ p^0$	$5+100K$	0

Table 1.2. *Routh table results*

b) *Plot the Bode graph and recall the definitions of phase margin of gain margin and static gain.*

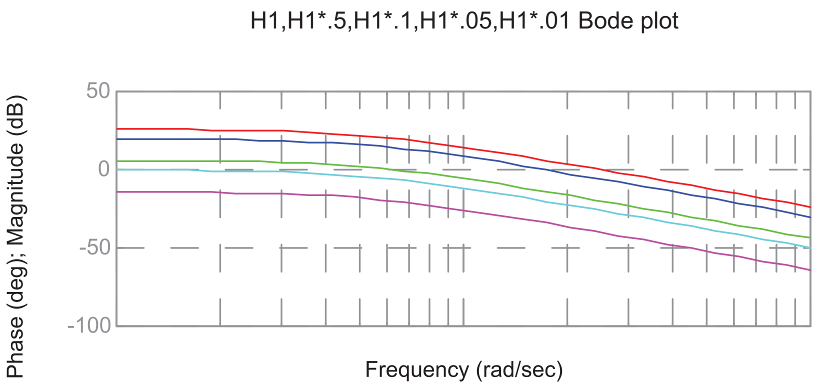

Figure 1.38. *Bode plot of $K.H_1(p)$. For a color version of this figure, see www.iste.co.uk/femmam/signals.zip*

7) a) *Plot the Nyquist locus of the transfer function $H(p) = K_v.H_1(p) = K_v\frac{V(p)}{U(p)}$ and verify the previous results.*

Nyquist locus of the transfer function

$$H(p) = K.H_1(p) = K\frac{V(p)}{U(p)} = \frac{100K}{0.157p^2+1.57p+5}. \qquad [1.77]$$

b) *Can a margin be obtained with phase of 45°? What is the order of magnitude of the gain margin? Justify the answers.*

If the gain is correctly chosen, a phase margin greater than 45° can be obtained as well as an infinite gain margin regardless of the order of magnitude of K.

c) *What can be said about the static error of position ε_p and about the system controlled in velocity ε_v?*

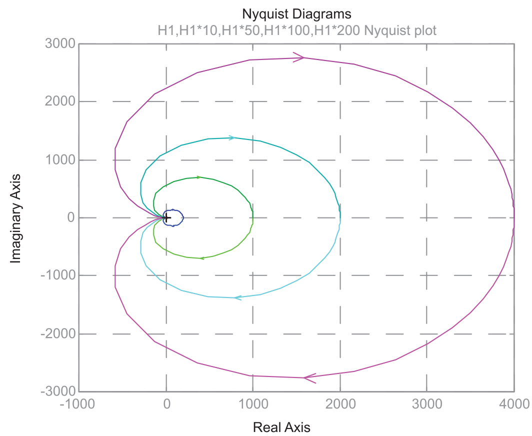

Figure 1.39. *Nyquist locus for velocity $K.H_1(p)$. For a color version of this figure, see www.iste.co.uk/femmam/signals.zip*

The static error in position $\varepsilon_p = \lim_{p\to 0} \frac{1}{0.157p^2+1.57p+5+100K} = \frac{1}{5+100K}$ and the static error in velocity $\varepsilon_v = \lim_{p\to 0} \frac{p}{0.157p^2+1.57p+5+100K} = 0$.

Some observations: in order to stabilize the system, it is more interesting to implement a first loop for the velocity feedback using a control $u = K_v(v-\dot{q})$ and then to consider the system having as input v and as output angular position q. The transfer function becomes: $H_o(p) = \frac{100K_v}{0.157p^2+1.57p+5+100K_v}\frac{1}{p}$. Here, the Nyquist locus for such a system is represented for $K_v = 1$, then for $K_v = 10$. Note the difference with the result of Question 3. Calculate the gain margin and the phase margin in these two cases and conclude on the difference and the significance of the velocity feedback (differential term).

8) The objective is to complete this servo with a position loop, after the velocity feedback giving a phase margin of $45°$ *(this allows us to set the value of K_v).*

a) Express the transfer function $H_3(p)$ of the system that results therefrom (considering as the output the position q).

$$H_3(p) = \frac{100K_v}{0.157p^2 + 1.57p + 5 + 100K_v}\frac{1}{p}. \qquad [1.78]$$

b) By applying the Routh criterion, analyze the stability of the system, in which position is looped back with a proportional controller of gain K_p with a unity feedback. Determine the conditions of the gain K_p that ensure the stabilization of the system.

c) Plot the Bode chart and the Nyquist locus of the transfer function $H_o(p) = K_p.H_3(p)$. Conclude on the stability of the system using the Nyquist criterion.

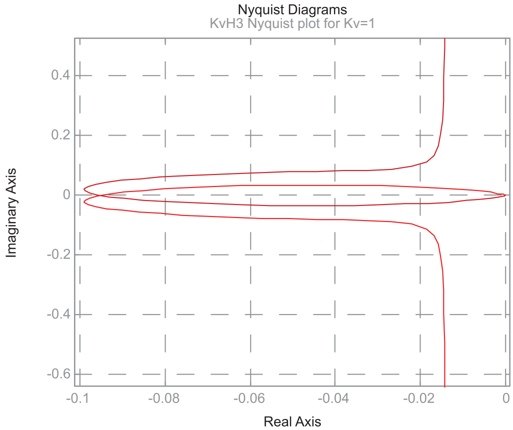

Figure 1.40. *Nyquist locus for the system position after velocity feedback $K_v = 1$ and $K = 1$*

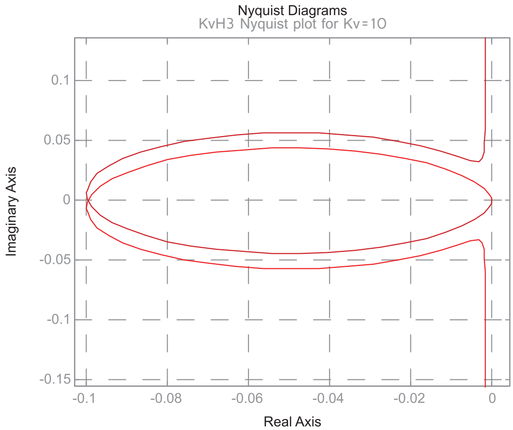

Figure 1.41. *Nyquist locus for the system position after velocity feedback $K_v = 10$ and $K = 1$*

d) Determine the optimal values that can be obtained for the phase margin and the gain margin of the system looped in this manner.

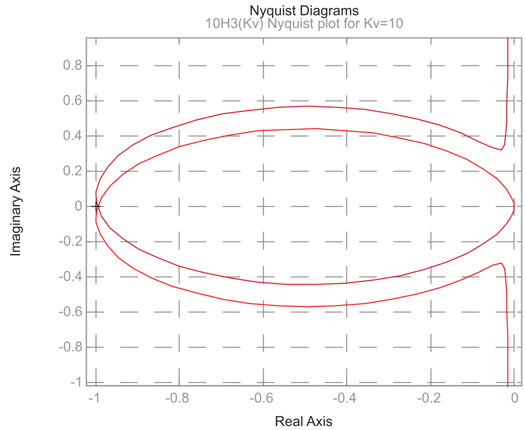

Figure 1.42. *Nyquist locus for the system position after velocity feedback $K_v = 10$ and $K = 10$*

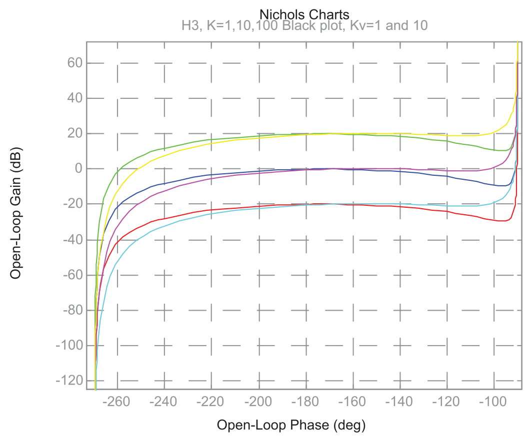

Figure 1.43. *Black plot for $Kv = 1$ and 10 with $K = 1$, 10 and 100*

e) Express static position ε_p and velocity ε_v errors of the system controlled in position.

f) For the position control of the system, is an acceleration feedback loop necessary and what would its contribution be in this case?

9) Compare and discuss the results of Questions 3–5 in the cases that follow:

a) what would be the effect due to a velocity sensor of transfer function $H_c(p) = \frac{1}{1+Tp}$?

b) G_o the effect of gravitation is materialized by a non-zero constant;

c) the coefficient of frictions F_v was not zero;

d) C_o originating from the Coriolis effect and centrifugal is non-zero and variable (see definition of C_{11} at the beginning of the text);

e) M_o varies according to the angular position;

f) G_o the effect of gravitation is not null and variable as well as C_o and F_v;

10) *In this section, the study of the control of the system is achieved in the state space*:

a) Give two state–space representations for the model of this axis, one of which under a controllable canonical form. Voltage u will be considered as input and as output the angular position q on the one hand and the angular velocity $\dot{q}$ on the other hand.

b) Study of the system in the state space: state–space representations for the model of this axis. In the case where the output is the angular velocity $\dot{q}$,

$$\begin{aligned} H_1(p) &= \frac{100}{0.157p^2 + 1.57p + 5} = \frac{100/.157}{p^2 + 10p + 5/.157} \\ &= \frac{636.94}{p^2 + 10.0p + 31.847}. \end{aligned} \quad [1.79]$$

$$\dot{X} = \begin{bmatrix} -10 & -31.847 \\ 1 & 0 \end{bmatrix} X + \begin{bmatrix} 636.94 \\ 0 \end{bmatrix} u \quad \text{and} \quad y = \begin{bmatrix} 0 & 100 \end{bmatrix} X \quad [1.80]$$

$$\dot{X} = \begin{bmatrix} 0 & -10 \\ 1 & -31.847 \end{bmatrix} X + \begin{bmatrix} 1 \\ 0 \end{bmatrix} u \quad \text{and} \quad y = \begin{bmatrix} \beta_1 & \beta_2 \end{bmatrix} X \quad [1.81]$$

For the case where the output is the angular position q,

$$H_2(p) = \frac{100}{p(0.157p^2 + 1.57p + 5)} = \frac{636.94}{p^3 + 10p^2 + 31.847p}. \quad [1.82]$$

$$\dot{X} = \begin{bmatrix} -10 & 31.847 & 0 \\ 1 & 0 & 0 \\ 0 & 1 & 0 \end{bmatrix} X + \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix} u \text{ and } y = \begin{bmatrix} 0 & 0 & 636.94 \end{bmatrix} X \quad [1.83]$$

$$\dot{X} = \begin{bmatrix} 0 & 0 & -10 \\ 1 & 0 & -31.847 \\ 0 & 1 & 0 \end{bmatrix} X + \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix} u \text{ and } y = \begin{bmatrix} \beta_1 & \beta_2 & \beta_3 \end{bmatrix} X \qquad [1.84]$$

c) Give the state–space representation of the system having the state vector $x = \begin{pmatrix} q \\ \dot{q} \\ \ddot{q} \end{pmatrix}$.

$$0.157\dddot{q} + 1.57\ddot{q} + 5\dot{q} = 100u \Rightarrow \dddot{q} = -10\ddot{q} - 31.847\dot{q} + 636.94u \qquad [1.85]$$

$$\dot{X} = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 0 & -31.847 & -10 \end{bmatrix} X + \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix} u \text{ and } y = \begin{bmatrix} 1 & 0 & 0 \end{bmatrix} X \qquad [1.86]$$

d) Express the characteristic equation of the system.

e) Velocity control: the system is velocity looped, with a state feedback $u = -L_1 x$, determine the gains l_1 and l_2 of vector L_1, *in order for the closed-loop system to have a natural frequency* $\omega_o = 10\,rad/s$ and a damping $\xi = 1$ (corresponding to the characteristic equation $p^2 + 2\omega_o p + \omega_o^2$).

f) The system (having the angular position on output) is position looped on a state feedback $u = -Lx$, determine the gains l_1 and l_2 of vector L, such that the closed-loop system has a natural frequency $\omega_o = 10\,rad/s$ and a damping $\xi = 1$ (corresponding to the characteristic equation $(p + 10)(p^2 + 2\omega_o p + \omega_o^2)$).

11) Compare both approaches of the position control of the system, considering disturbances, couplings and variations of the parameters.

12) Start the study again considering this time both mobile axes simultaneously and non-diagonal matrices and with variable coefficients.

1.6. Application 2: temperature control of an oven

The study consists of two parts: modeling and identification on the one hand and control on the other hand.

1.6.1. *Modeling and identification study*

In the case of thermal processes, the most often applied modeling and identification technique consists of finding a model describing the behavior of a system from experimental measurements. Most often, the measure chosen is the reading of the system response to a step setpoint (in the case of the figure in multiples of 500°). The step response of the system as a function of time, if the step function is applied at date t = 1 s, is shown in Figure 1.44 for the case of an empty oven, half-loaded and with a full load. It is desired to derive its transfer function when empty (half-loaded and in full load). Modeling is the most important step in the study of an automated system. In order to control or regulate a given system, it is first necessary to have its model in order to study it in simulation. Once simulation results are very satisfactory, we will apply the control laws proposed in simulation on the real system. On the other hand, if the results with the real system are not acceptable, the modeling will imperatively have to be reviewed. After modeling, the second step consists of analyzing the behavior of the system using its model. This behavior analysis allows us to elaborate a control strategy for the system taking into account the performance restrictions imposed by the specifications and of the physical limitations of the real system. To illustrate this approach, we propose the following organization chart (Figure 1.45).

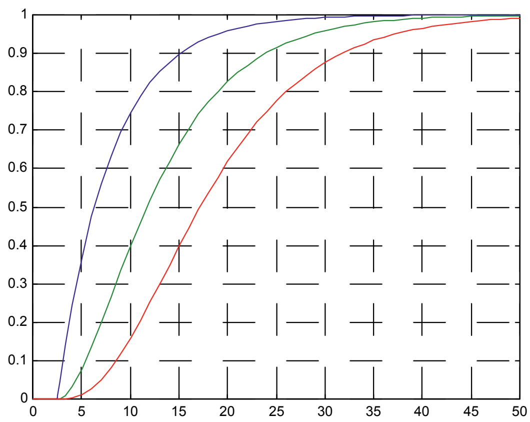

Figure 1.44. *Model describing the behavior of a system*

The modeling is achieved by writing the physical equations that describe the behavior of the system. In the continuous case, these equations are differential equations and in the discrete case recurrence equation. To these equations we apply transformations (Laplace transform: continuous; Z-transform: discrete) to shift from the time domain to the frequency domain where the analysis of the behavior of the system is more interesting. In the case where the system cannot be described by physical equations, it is always possible that an approximate model be proposed, through identification. This model should best describe the behavior of the system. It can be obtained from the identification by analogy to known systems, using the responses to test signals or by using an identification that includes the optimization of the error criterion. In this case, the algorithm uses data that correspond to the input and output signals of the system. These data must be rich enough in excitation to cover the entire spectrum of the system. The block diagram is shown in Figure 1.46.

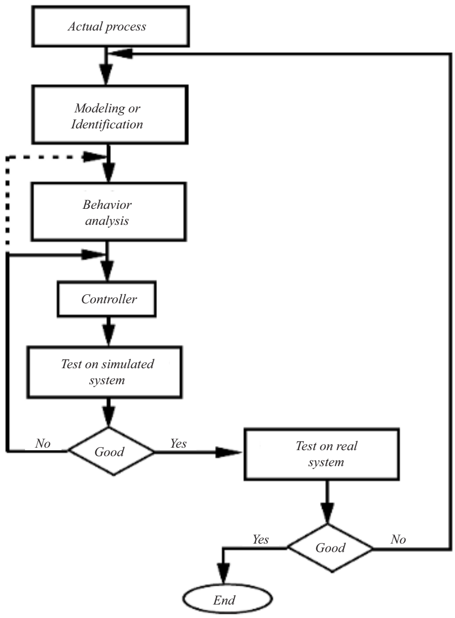

Figure 1.45. *Organigram of the modeling stages for the study of an automated system*

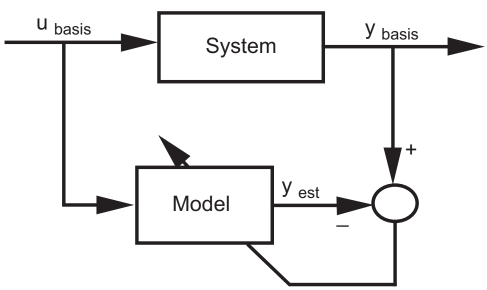

Figure 1.46. *Approximate model for identification*

Representation choice

The delay

The time constant(s)

$$H_o(p) = \frac{Ke^{-\tau}}{(1+Tp)}. \quad [1.87]$$

$$H_o(p) = \frac{Ke^{-\tau}}{(1+Tp)^n}. \quad [1.88]$$

Parametrization choice

Transfer function

State–space representation

Discrete representations

Identification method choice

Comparative method

Strejc method

Broida method

Least squares method

and at the end of this application as practical work we should consider control and performance improvement.