1

Overview of the Nucleus

COPYRIGHTED MATERIAL

Overall objective	
To know the general properties of nuclei	
Specific objectives	
To compare the atomic models of Thomson, Perrin and Rutherford	To define the effective scattering cross-section
To compare Rutherford's and Blackett's observations on the first nuclear transmutation reaction	To define the effective elementary scattering cross-section
To know the properties of the isospin operator	To define the separation energy of a nucleon
To know the properties of the spin angular momentum of a nucleus	To determine the separation energy of a neutron from a proton for a given nuclide
To know the fundamental properties of nucleons	To determine the total isospin corresponding to the ground state of a nucleus
To know the expression for the radius of a nucleus assumed to be spherical	To deduce, from the binding energy, the nuclear charge of the most stable isobar
To know the expression for the Sakho unit nuclear radius	To establish the relationship between binding energy and mass defect
To know the principle of a mass spectrograph	To establish the relationship between skin thickness and the diffusivity parameter
To compare the stability of nuclei from their nuclear binding energy	To write the balanced equation of the first nuclear transmutation reaction
To know the effect of nuclear forces on the stability of nuclei	To establish Rutherford's differential effective cross-section

For a color version of all of the figures in this chapter, see www.iste.co.uk/sakho/nuclear1.zip.

To differentiate between *u* and *d* quarks according to the Gell-Mann and Zweig model	To make the analogy between electron gyromagnetic ratio and nuclear gyromagnetic ratio
To differentiate between isospin and nuclear spin	To make the analogy between Bohr magneton and Bohr nuclear magneton
To differentiate between unit nuclear radius and electromagnetic unit radius	To make the analogy between spin multiplicity and isospin multiplicity
To differentiate between isotopes, isobars and isotones	To interpret Chadwick's experiment
To differentiate between mirror nucleus and magic nucleus	To interpret Geiger and Marsden's experiment
To distinguish between valley of stability and line of stability	To interpret Rutherford's scattering experiment
To define a monoisotopic element	To interpret Rutherford's nuclear transmutation experiment
To define a nuclear isomer	To interpret the shape of the Woods–Saxon charge distribution density
To define the nuclear dipole magnetic moment	To interpret the Segrè diagram
To define the nuclear Landé factor	To interpret the Aston curve
To define the skin thickness of a nucleus	To situate the nuclear energy surface or stability valley in the Segrè diagram
To define the atomic mass unit	
Prerequisites	
Material structure	Motion of a charged particle in a uniform magnetic field
Atomic models	Vector product properties
Shell model of electron configurations	Fundamental theorems of the dynamics of the material point
Quantum numbers of the electron	

1.1. Discovery of the electron

1.1.1. *Hittorf and Crookes experiments*

In around 1869, Johann Wilhelm Hittorf studied electric discharge in rarefied gases using a vacuum tube. With the help of a Sprengel pump, Hittorf managed to obtain pressures below 0.001 mbar and found that electric discharges were

accompanied by the emission of glow rays, which he called “cathode rays” [LEP 56, CAR 79, ROU 60, PER 95, SAK 11]. Subsequently, he observed that cathode rays are gifted with the property of being deflected by a magnetic field. But Hittorf stopped at these observations, without providing any physical interpretation.

Still in 1869, Sir William Crookes invented the electronic tube bearing his name [CRO 79, ROU 60, SAK 11]. The *Crookes tube* was a cold cathode tube, that is, a tube that did not have a heating filament as in the case of cathode tubes, designed to generate electrons.

Essentially, a Crookes tube is a bulb containing a gas and is equipped with two electrodes (Figure 1.1). When a voltage of approximately 50,000 volts is applied between the two electrodes and the gas pressure is gradually reduced, a dark space (called Crookes space) fills the tube at around 0.01 millimeters of mercury. Today it is now possible to interpret Crookes’ observations.

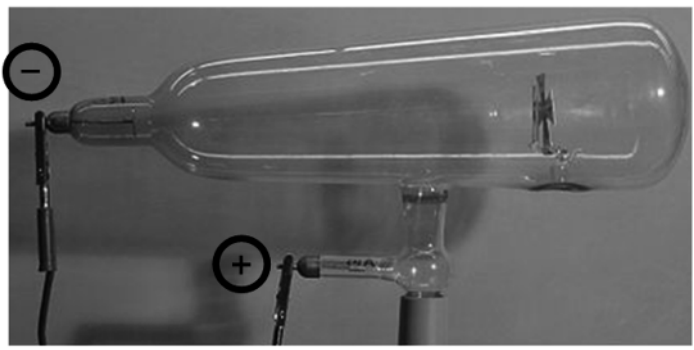


Figure 1.1. *Crookes tube*

Inside the Crookes tube, electrons are generated by ionization of gas molecules excited by the applied continuous voltage. Under the action of the established electric field, the ions created in the tube are accelerated. They collide with gas molecules, knocking electrons off them. The positive ions thus formed are attracted by the cathode. As they strike the latter, they eject a large number of electrons, called cathode rays. As they strike the glass, the electrons excite the atoms in the walls of the tube, thus causing its fluorescence usually in the yellow-green range. The electrons flow in a straight line from the cathode to the anode. This motion is highlighted by the shadow cast by the cross on the fluorescent wall (Figure 1.2). Moreover, cathode radiation has the property of being deflected by a magnetic field. Crookes considered cathode radiation to be the fourth state of matter which he called the radiant state. But what is the nature of this radiation?



Figure 1.2. *Crookes tube wall fluorescence*

Crookes gave the following response to this question in 1885: "cathode radiation consists of negatively charged molecules emitted by the negative electrode". The proof that they are indeed molecules, he cried, is that the action of a magnet makes them deviate from their trajectory, just like particles of iron filings.

Johann Wilhelm Hittorf was a German physicist. He is also known for his work on the interpretation of electrical conductivity in electrolytic solutions in 1859, on the quantitative study of metal ion allotropy in 1865 and on the demonstration of cathode rays in 1869.

Sir William Crookes was a British physicist and chemist. He is most famous for having invented the "Crookes tube" in 1869 which allowed him to highlight cathode rays.

Box 1.1. *Hittorf (1824–1914); Crookes (1832–1919)*

1.1.2. *Perrin and Thomson experiments*

While cathode radiation was an experimental reality that no-one could question, the molecular nature of the famous radiation puzzled many physicists, including Sir Joseph John Thomson [THO 97, ROU 60, CAR 79, PER 95, FAL 87, SAK 11]. The latter, as early as 1881, boldly took the opposite view to Crookes' conception of the nature of cathode radiation. For Thomson, "cathode radiation consists not of molecules but of *particles of pure negative electricity*," which he would later call electrons (a term introduced in 1891 by Stoney [CAR 79]). However, Thomson argued that "particles of pure electricity are not matter". Moreover, he noted, even if experimental observations require particles of pure electricity to be given a mass, then it is only necessary to understand that "this mass is nothing other than their inertia induced by their motion under the action of the magnetic field". However,

Thomson did not say anything regarding the dimension of particles of pure negative electricity.

Challenging the ideas of Crookes, Thomson and others, Perrin suggested that cathode rays should not be considered as being made up of molecules but of "smaller particles charged with pure negative electricity". In 1895, he sought to verify this proposal through experimentation. Perrin then used a Faraday box (a small cylinder capable of trapping cathode rays), in contact with the plate of a positively-charged electroscope (Figure 1.3), to gather the cathode radiation as it exited a Crookes tube.

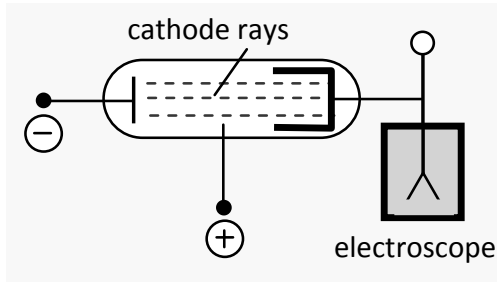


Figure 1.3. *Perrin's simplified experimental set-up*

Perrin found that the famous particles of pure electricity suggested by Thomson roughly neutralized the positive charge of the electroscope. These observations confirmed that the electricity transported by cathode radiation is negative in nature. But it should be noted that, until that date, there was no information on the size of the particles of pure negative electricity. Were they smaller than atoms? This question could be answered by measuring their mass, m. In chronological terms, the measurement of this mass was preceded by the measurement of the electron mass-to-charge ratio, e/m, by Thomson, and by the measurement of the elementary electrical charge, e, by Robert Millikan. Thomson's experiments form the subject of the study that follows. Millikan's experiment is discussed in section 1.1.3.

In 1897, Thomson conducted a series of historical experiments that measured the mass-to-charge ratio of the electron.

First experiment: Thomson studied the possibility of separating the negative electrical charge from the cathode rays by a magnetic field. He built a cathode-ray tube that ends in a pair of cylinders with slots connected to an electrometer. This first experiment showed that the cathode rays are not deflected under the action of a magnetic field. Thomson concluded that the negative charge cannot be separated

from the rays (which tacitly proves that the cathode rays are composed of identical charges).

Second experiment: Thomson studied the action of an electric field on cathode rays. To do this, he built a cathode-ray tube with a deeper vacuum, within which is an electric field created by a voltage applied between two conductive metal plates. He placed a coat of phosphorescent paint at the end of the tube to detect incident rays. Thomson observed that rays are attracted by the positive plate. He thus proved that the electrical charge of cathode rays is negative, in accordance with Perrin's experimental observations.

Third experiment: Thomson sought to measure the *electron mass-to-charge ratio, e/m*. Today, Thomson's experiment is replicated by performing more meticulous experiments to determine the value of *e*/*m*.

Electrons in a constant velocity beam penetrate at *O* into a space where uniform electric and magnetic fields can occur simultaneously. In a first experiment, the orthogonal electric and magnetic fields are applied simultaneously so that the motion of the electrons is straight and uniform along *OO*' (Figure 1.4(a)). The electric field is vertical and directed downwards. In a second experiment, the magnetic field is removed; the characteristics of the electric field and the electron velocity vector remain unchanged (Figure 1.4(b)). The set-ups of the two experiments conducted can be schematically presented in parallel, as shown in Figure 1.4.

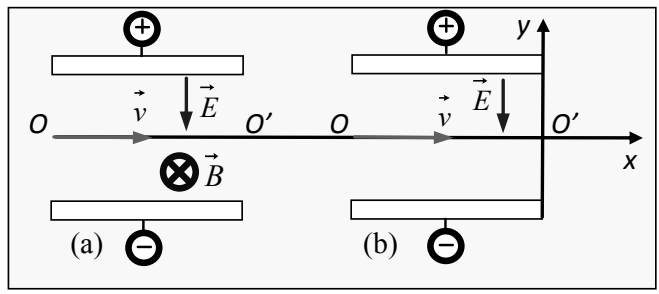


Figure 1.4. *Simplified set-up for measuring the electron mass-to-charge ratio: (a) simultaneous action of the electric field and the magnetic field; (b) action of the electric field alone*

It is then shown that the electron mass-to-charge ratio has the following value:

$$\frac{e}{m} = 1.76 \times 10^{11}\ \mathrm{C \cdot kg^{-1}} \qquad [1.1]$$

APPLICATION 1.1.– Using the device shown in Figure 1.4, show that if Y designates the vertical deviation along the $O'y$ axis, the electron mass-to-charge ratio is given by the expression:

$$\frac{e}{m} = 2\frac{E}{B^2 l^2} Y \qquad [1.2]$$

Then find the result [1.1].

Given data: $E = 50\ \text{kV} \cdot \text{m}^{-1}$; $B = 1$ mT; $OO' = l = 10.0$ cm, $Y = 1.76$ cm.

ANSWER.– Considering Figure 1.4(a), the electrical force is compensated by the magnetic force since the motion is uniform and rectilinear (it is a velocity filter). Let:

$$q\vec{E} = -q\vec{v} \wedge \vec{B} \Rightarrow v = \frac{E}{B} \qquad [1.3]$$

Let us thus determine the equation of the trajectory of an electron in the electric field (Figure 1.4(b)). Using the theorem of the center of inertia gives:

$$\begin{cases} q\vec{E} = m\vec{a} \\ \vec{a} = \dfrac{q\vec{E}}{m} \end{cases} \Rightarrow \begin{cases} a_x = 0 \Rightarrow x = vt \\ a_y = -\dfrac{qE}{m} \Rightarrow y = -\dfrac{1}{2}\dfrac{qE}{m}t^2 \end{cases}$$

Using the last equation system, the equation for the trajectory of an electron within the electric field is written ($q = -e$):

$$y = \frac{1}{2}\frac{eE}{mv^2}x^2 \qquad [1.4]$$

The vertical deviation, Y, along the $O'y$ axis is obtained for $x = OO' = l$. Using [1.4] and taking account of [1.3], we then obtain the following:

$$Y = \frac{e}{m}\frac{B^2}{2E}l^2 \qquad [1.5]$$

Using [1.5], result [1.2] is found.

NOTE.–

$$\frac{e}{m} = 2\frac{5\times10^{4}}{10^{-6}\times10^{-2}} \times 1.76 \times 10^{-2} = 1.76 \times 10^{11}\ \mathrm{C \cdot kg^{-1}}$$

The result [1.1] is indeed found.

The CODATA (Committee on Data and Technology) recommended value is $1.75882001076(53)\times 10^{11}\ \mathrm{C \cdot kg^{-1}}$.

George Johnstone Stoney was an Irish physicist. He introduced the concept of an "electricity atom" or "electricity particle," for which he invented the term "electron" in 1891.

Sir Joseph John Thomson was a British physicist and chemist. He is famous for his work on the study of the structure of matter. He is known primarily for inventing the mass spectrograph which is very useful in the separation of isotopes. Thomson confirmed the existence of the electron, and measured its mass-to-charge ratio in 1897. He was awarded the Nobel Prize in Physics 1906 for his theoretical and experimental research on electrical conductivity in gases.

Jean Perrin was a French physicist. He is known for demonstrating that cathode rays are negatively charged in 1895. In 1901, Perrin envisioned the atom as a solar system formed at the center of a quasi-point-like concentration of positive matter surrounded by a procession of electrons in motion. In addition, in 1908 he measured Avogadro's number. Moreover, in 1919, Perrin was the first to put forward the hypothesis that the transformation of hydrogen into helium was at the origin of energy radiated by the Sun. He was awarded the Nobel Prize in Physics 1926 for his work on the discontinuity of matter and, more specifically, for his discovery of sedimentation equilibrium.

Box 1.2. *Stoney (1826–1911); Thomson (1856–1940); Perrin (1870–1942)*

1.1.3. *Millikan experiment*

In 1909 [MIL 10, ROU 60, CAR 79, PER 95, SAK 10, SAK 11], Robert Andrews Millikan began measuring the *elementary electrical charge*, *e.* The experimental set-up used by Millikan can be schematically presented as indicated in Figure 1.5(a) [SAK 10].

Droplets assimilated to small, homogeneous spheres of radius r, mass m and, in small numbers, are obtained by spraying oil between two plates, P_1 and P_2, through a small orifice in the upper plate, P_1. Between P_1 and P_2, the droplets encounter ions

produced by radiation from a source, *S* (X-ray tube, radium bulb, etc.). Millikan's simplified experimental device is illustrated in Figure 1.5(a).

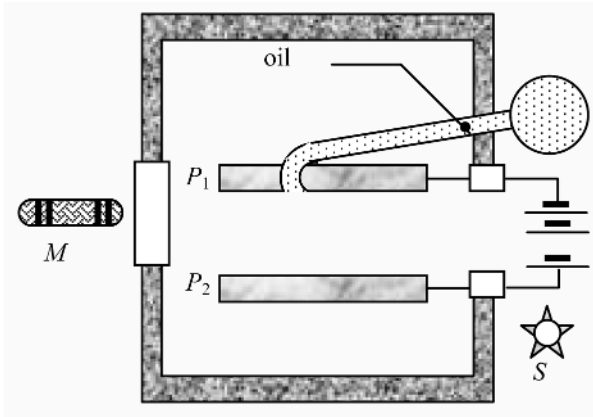


Figure 1.5a. *Millikan's simplified experimental device*

From time to time, an ion attaches itself to one of the droplets. The latter is then electrified and is subjected to the double action of the antagonistic gravity field and electric field reigning between the two plates. The velocity assumed by an electrified droplet depends on its charge, q, its mass, m, its radius, r, and the viscosity coefficient, η, of the air. The motion of the droplets is observed by means of a microscope, *M*. When the velocity of a droplet changes abruptly, it means it has attached an ion. Measuring this velocity for various electric field values (including zero field) and knowing η then enables the absolute value $|q|$ of the charge carried by a droplet to be measured, and the value of the elementary electrical charge, *e*, to be deduced. The value of the elementary electrical charge measured by Millikan is $(4.77 \pm 0.009)\ 10^{-10}$ uemcgs [BIS 19]. This corresponds to the international system value, 1.592×10^{-19} coulombs [BIS 19].

Today, more meticulous experiments produce a far more accurate value than that obtained by Millikan. Using glycerin as oil, the experiment shows that for several observations in an electric field of intensity *E*, the velocities, *v*, acquired by the droplets are distributed according to an arithmetic progression of difference $\Delta v \approx 3 \times 10^{-5}$ m $\cdot$ s $^{-1}$ [SAK 10]. When a droplet attaches the smallest charge, *e*, its velocity varies by Δv. The elementary electrical charge is then given by the expression:

$$e = \frac{6\pi\eta\, r\Delta v}{E} \qquad [1.6]$$

Numerically, for $\eta = 1.83 \times 10^{-5}$ SI; $E = 93.5$ kV $\cdot$ m $^{-1}$ 1; $r = 1.45$ μm [SAK 10], we obtain, knowing that $\Delta v \approx 3 \times 10^{-5}$ m $\cdot$ s $^{-1}$:

$$e = \frac{6\pi \times 1.83 \times 10^{-5} \times 1.45 \times 10^{-6} \times 3 \times 10^{-5}}{9.35 \times 10^{4}} = 1.604833047 \times 10^{-19}$$

Thus:

$$e = 1.604833047 \times 10^{-19} \text{ C}$$

The CODATA recommended value is $1.602179487(40)10^{-19}$ C.

Using value [1.1] for the electron mass-to-charge ratio, the mass, m, of the electron is deduced by considering the previous result: $E = 1.604833047 \times 10^{-19}$ C, which gives $m = 9.11836958 \times 10^{-31}$ kg. The CODATA recommended value is $9.10938215(45)\ 10^{-31}$ kg.

Robert Andrew Millikan was an American physicist. He is best known for his experimental work on the drop of oil sprayed by X-rays, which allowed him to measure elementary electrical charge in 1909. He was awarded the Nobel Prize in Physics 1923 for his work on the "elementary charge of electricity and the photoelectric effect".

Box 1.3. *Millikan (1868–1953)*

APPLICATION 1.2.– Let us consider Figure 1.5(b). When a droplet of radius r and charge q is in motion in the electric field space, demonstrate relationship [1.6]. We will make use of the principle of inertia and will take account of the fact that when a droplet attaches the lowest charge, e, its velocity varies by Δv. The intensity of the *Stokes force* acting on a droplet of velocity v is given: $f = 6\pi\eta r v$, with η the viscosity of the air and r the radius of a droplet. We will produce a diagram showing the forces acting on the droplet concerned.

ANSWER.– Let us consider a droplet of charge q and mass m. It is subjected to its weight, $\vec{P}$, the electrical force, $\vec{F}$, and the Stokes force, $\vec{f}$. With the gravity field and the electric field between the two plates being antagonistic, the electric field is directed upwards (the droplets move from P_1 to P_2 and in the same direction as the Stokes force (Figure 1.5(b)).

With the principle of inertia being verified, the motion of a droplet is uniform and rectilinear along the vertical, that is:

$$\vec{P}+\vec{F}+\vec{f}=m\vec{g}+q\vec{E}-6\pi\eta r\vec{v}=\vec{0} \qquad [1.7]$$

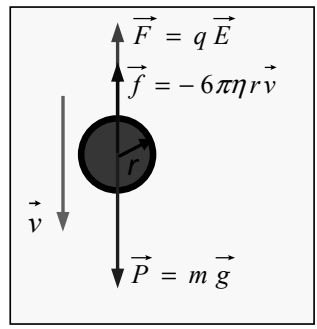


Figure 1.5b. *Uniform drop motion of a droplet of charge q, mass m and radius r.* $\vec{P}$*: weight,* $\vec{F}$*: electrical force,* $\vec{f}$*: Stokes force*

By projecting [1.7] in the direction of the velocity, we obtain:

$$mg - qE - 6\pi\eta rv = 0$$

Which then gives:

$$qE = mg - 6\pi\eta rv \qquad [1.8]$$

From a physical point of view, the discontinuous variations, Δv, in the velocity limit of a droplet correspond to discontinuous variations, Δq, in the charge, q, carried by the droplet. Equation [1.8] then gives:

$$(q + \Delta q)E = mg - 6\pi\eta r\,(v + \Delta v) \Rightarrow |\Delta q| = e = 6\pi\eta r\Delta v/E$$

[1.6] is indeed found.

NOTE.– The following reasoning can be adopted. When the velocity of a droplet varies from v to v', then equation [1.8] is written $qE = 6\pi\eta rv' - 6\pi\eta r\, rv' = 6\pi\eta r\,(v' - v)$. Let: $|q|E = 6\pi\eta r\Delta v$, with $\Delta v = |v' - v|$. Yet the amount of electricity $|q| = ne$. Hence $neE = 6\pi\eta r\Delta v$. Given that the variation in velocity, Δv, is due to the attaching of the elementary charge, then $n = 1$. We thus obtain: $e = 6\pi\eta r\Delta v/E$.

1.2. The birth of the nucleus

1.2.1. *Perrin and Thomson atomic model*

In 1901, Perrin envisioned the atom as a miniature solar system where electrons acting like planets gravitate freely around a positively charged, quasi-point-like center of matter (Figure 1.6). Under the effect of attractive electrostatic force, electrons turn around the quasi-point-like center via elliptic trajectories [DUM 15]. This quasi-point-like center will be identified in the atomic nucleus following the Rutherford scattering experiment.

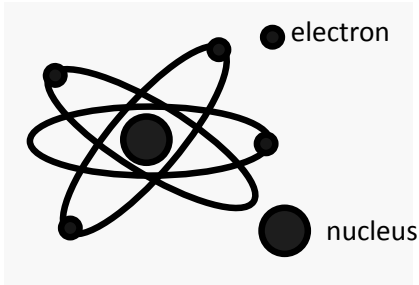
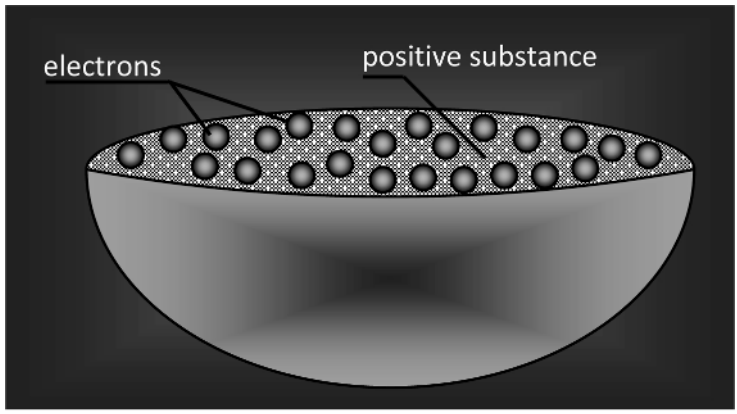


Figure 1.6. *Planetary model of the atom, envisioned by Perrin in 1901*

Through the experiments of Thomson and Millikan, it became known that in addition to atoms, there existed a much smaller particle, called an electron. The internal structure of the quasi-point center in the Perrin atomic model had not yet been elucidated.

In 1902, Thomson envisioned an atomic model that was different to that of Perrin by developing the "raisin bread" theory on the atomic structure [ROU 60, SAK 11]. In this model, electrons are considered to be negative raisins distributed throughout bread in a positive matter, hence this model is known as the plum pudding atom, as shown in Figure 1.7 [SAK 11].

But because of the positive electron–substance attraction and Coulomb electron–electron repulsion forces, the atomic matter as described is unstable. The Thomson atomic model is therefore not suitable, as confirmed by Rutherford in 1911 (see section 1.2.3).



1.2.2. G

In 19 carried
out *parti* ils. The
simplifie resented
below (F ngle, θ,
of partic

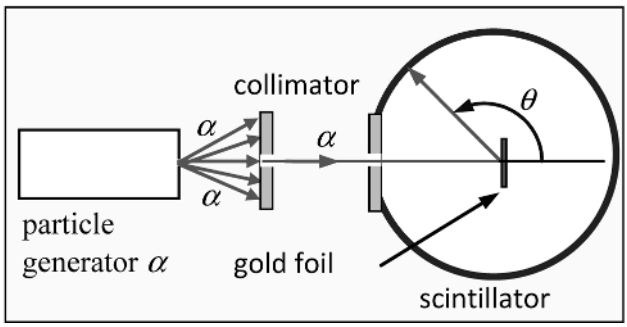


Figure 1.8. *Simplified experimental device by Geiger and Marsden*

The Geiger and Marsden experiments showed that one in 8,000 α particles is scattered at an angle θ greater than 90°. These observations contradicted the Thomson model (Figure 1.7), which predicted small angular deflections by simple scattering, and a very low probability of large multiple scattering deflection.

Johannes Wilhelm "Hans" Geiger was a German physicist. In 1928, together with German physicist Walther Müller (1905-1979), he invented the particle detector known as the Geiger–Müller counter, the operating principle of which he devised in 1913. Geiger is also famous for having established, together with English physicist John Mitchell Nuttall (1890-1958), the law giving the rate of decay as a function of time, known as the Geiger–Nuttall law (see Chapter 3). In addition, in 1908 under Rutherford's supervision (see Box 1.5), Geiger, together with Marsden, carried out the α particle scattering experiment with thin gold strips, which enabled Rutherford to devise the planetary model of the atom.

Sir Ernest Marsden was an English-New Zealand physicist. He is known for his experimental work on α particle scattering by thin gold blades.

Box 1.4. *Geiger (1882–1945); Marsden (1889–1970)*

1.2.3. *Rutherford scattering: Planetary atomic model*

In 1911, Ernest Rutherford [RUT 11, SIV 86, EVA 61, GUY 03, STÖ 07, SAK 11, DUM 15, SAK 19] used radioactive radiation consisting of α particles or helion from a radium source to bombard thin metal foils. This experiment, known as *Rutherford scattering*, enabled him to explain Geiger and Marsden's experimental observations, and find that the atomic structure could not be represented in the static form that Thomson had envisaged in 1902. The experimental set-up created by Rutherford is schematically presented as indicated in Figure 1.9 [SAK 19].

Rutherford's experience makes it possible to make at least two important observations:

First observation: numerous particles pass through the matter without being deflected (although theoretically several deviations should have been observed according to Thomson's model).

Second observation: alpha particles passing close to the "center" of matter are deflected at a large angle.

The first observation allows us to conclude that the positive particles scattered in the Thomson model are concentrated at the "center" of the matter. The second observation, meanwhile, proves that the "center" of the matter pushing alpha particles is a positively charged point-like particle. Rutherford demonstrated that an α particle can be obtained by twice ionizing a helium atom: $He \rightarrow He^{2+} + 2e^{-}$. The α particle is a He^{2+} helium nucleus. The positive "center" of matter was identified as the atomic nucleus. Drawing on the astronomical model of Johannes Kepler

(1571-1630) of the solar system, Rutherford proposed the model of the planetary atom in which electrons gravitate around the nucleus, as shown in Figure 1.10 [SAK 11].

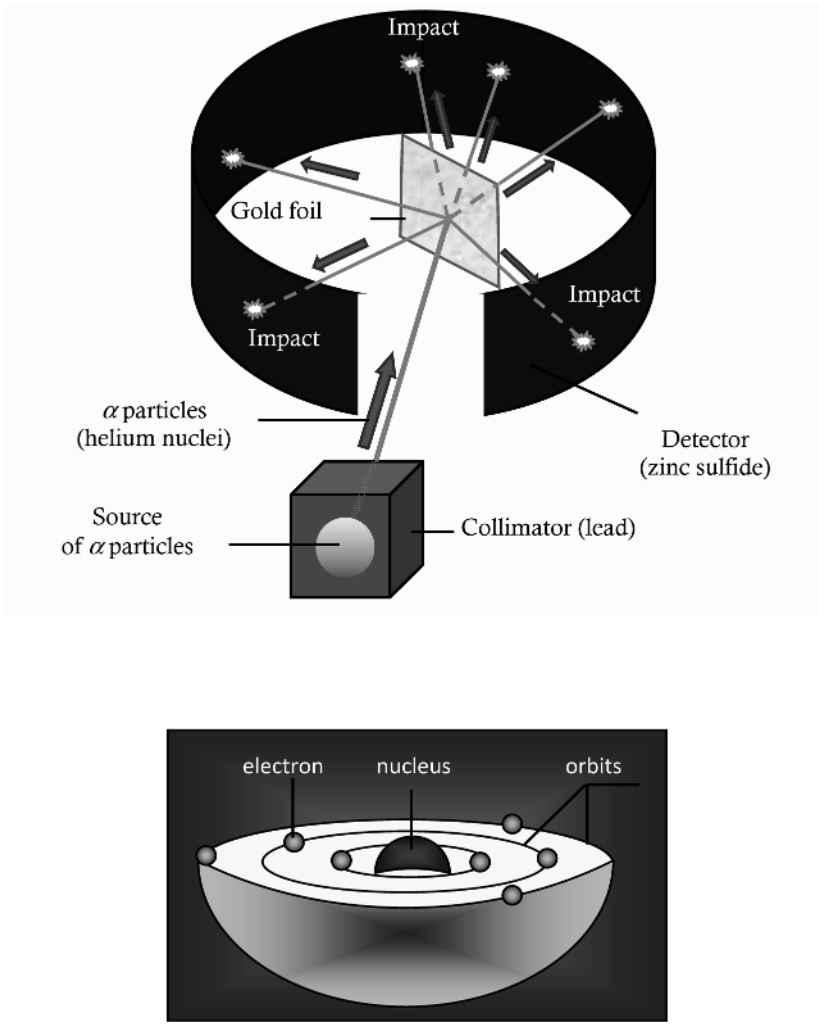


Figure 1.10. *Planetary atomic model envisaged by Rutherford*

Note that the planetary model is unstable. The electrons, as charged particles, are subjected to a centripetal acceleration due to their orbital motion. Yet, according to predictions of classical electrodynamics, any charged particle subjected to acceleration loses energy by radiation. In the planetary model, electrons ought to

lose energy through radiation and eventually fall onto the nucleus in around 10^{-11} s (see exercise 1.7.1). Yet that does not happen.

1.2.4. *Rutherford's differential effective cross-section*

To theoretically interpret these experimental observations, Rutherford developed a quantitative theory of α particle scattering by very thin gold foils, 10^{-5} to 10^{-4} cm thick. These thicknesses made it possible to avoid taking account of potential multiple collisions of α particles with several nuclei in the interpretation of large scattering angles. Thus, the probability that a large scattering angle would result from two or more successive collisions between α particles and gold nuclei is entirely negligible. Moreover, due to low electron mass, the probability that a large scattering angle would result from collisions between the α particles and electrons in the electron cloud of a gold nucleus is also negligible. Thus, under the conditions of the experiment, a large deviation angle should result from a collision between an α particle and a single point-like center, in this case a gold nucleus.

In the general case, let us consider the elastic interaction process of a constant velocity beam of identical particles by a diffuser center (example target nucleus) placed at the origin, *O*, of the coordinates. By definition, the *impact parameter*, *b*, of a particle is the distance from the diffuser center core to the initial direction of the particle [SIV 86, STÖ 07, DUM 15]. The geometry of Rutherford's theoretical model for studying classical scattering is shown in Figure 1.11.

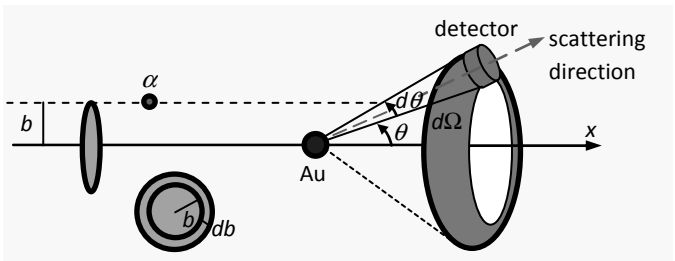


Figure 1.11. *Scattering of a beam of α particles of an angle, θ, in the solid angle, $d\Omega$, b is the impact parameter*

Rutherford's differential effective cross-section is the theoretical expression subject to experimental verification. Let us briefly review some useful definitions.

The *solid angle,* $d\Omega$, between the two cones of revolution, with the same vertex, *O*, the same axis, *Ox,* and with semi-vertex angles, θ and $\theta + d\theta$, is:

$$d\Omega = 2\pi \sin\theta \, \mathrm{d}\theta \qquad [1.9]$$

The particles scattered at a larger angle are those that are deflected under the angles between θ and $\theta + d\theta$ with an impact parameter between b and $b + db$ (the scattering angle is maximum for $b = 0$). With the effective cross-section having the dimension of a surface, let us consider a disk of radius b and axis of revolution *Ox*. The surface of this disk is equal to the *effective scattering cross section* in the direction of angle θ, noted σ, that is: $\sigma = \pi b^2$. The *elementary effective scattering cross-section*, $d\sigma$, is then given by the relationship:

$$d\sigma = 2\pi b db \qquad [1.10]$$

By definition, the *differential effective cross-section* is equal to $\frac{d\sigma}{d\Omega}$. Using the definition relationships [1.9] and [1.10], we then obtain:

$$\frac{d\sigma}{d\Omega} = \left| \frac{b}{\sin\theta} \frac{db}{d\theta} \right| \qquad [1.11]$$

If we use *ze* to designate the *charge of the incident particle of kinetic energy*, *E*, and *Ze* for the charge of the target nucleus, then the impact parameter is equal to:

$$b = \frac{1}{4\pi\varepsilon_0} \frac{zZe^2}{2E} \frac{1}{\tan(\theta/2)} \qquad [1.12]$$

Using [1.12], Rutherford's differential effective cross-section is then written according to [1.11]:

$$\frac{d\sigma}{d\Omega} = \left(\frac{1}{4\pi\varepsilon_0} \right)^2 \left(\frac{zZe^2}{4E} \right)^2 \frac{1}{\sin^4(\theta/2)} \qquad [1.13]$$

The effective cross-section is expressed in *barn* (b) with 1 barn = 10^{-28} m^2 and the solid angle in steradians (sr). The differential cross section [1.13] is then expressed in barn per steradian ($b \cdot sr^{-1}$).

APPLICATION 1.3.– Demonstrate expression [1.13]. Perform the numerical application for α particles of total energy E = 5.5 MeV, scattered under an angle of 90° by a gold foil with atomic number Z = 79. We will take $e = 1.6 \times 10^{-19}$ C; 1 eV = 1.6×10^{-19} J and $(1/4\pi\varepsilon_0) = 9.0 \times 10^9$ SI.

ANSWER.– Using [1.12], we obtain:

$$\frac{db}{d\theta} = \frac{1}{4\pi\varepsilon_0}\frac{zZe^2}{2E}\frac{d}{d\theta}\left(\frac{\cos(\theta/2)}{\sin(\theta/2)}\right) = -\frac{1}{4\pi\varepsilon_0}\frac{zZe^2}{4E}\frac{1}{\sin^2(\theta/2)} \qquad [1.14]$$

Considering that $\sin\theta = 2\sin(\theta/2)\cos(\theta/2)$, we obtain, by considering [1.12]:

$$\frac{b}{\sin\theta} = \frac{1}{4\pi\varepsilon_0}\frac{zZe^2}{2E}\frac{\cos(\theta/2)}{\sin(\theta/2)} \times \frac{1}{2\sin(\theta/2)\cos(\theta/2)}$$

Let:

$$\frac{b}{\sin\theta} = \frac{1}{4\pi\varepsilon_0}\frac{zZe^2}{4E}\frac{1}{\sin^2(\theta/2)} \qquad [1.15]$$

Using [1.14] and [1.15] gives:

$$\frac{b}{\sin\theta} \times \frac{db}{d\theta} = -\frac{1}{4\pi\varepsilon_0}\frac{zZe^2}{4E}\frac{1}{\sin^2(\theta/2)} \times \frac{1}{4\pi\varepsilon_0}\frac{zZe^2}{4E}\frac{1}{\sin^2(\theta/2)}$$

Considering the absolute value of this equation, we find [1.13].

For α particles, z = 2, we find, numerically:

$$\frac{d\sigma}{d\Omega} = (9\times10^9)^2\left(\frac{2\times79\times1.6\times10^{-19}}{4\times5.5\times10^6}\right)^2\frac{1}{\sin^4(\pi/4)}$$

$$= 4.278 \times 10^{-28} = \text{b} \cdot \text{sr}^{-1}$$

We then find:

$$\frac{d\sigma}{d\Omega} = 4.28\ \text{b} \cdot \text{sr}^{-1}$$

APPLICATION 1.4.– The elastic scattering of an α particle of velocity $\vec{v}$ and kinetic energy E = 6.0 MeV is achieved with a nucleus of gold, Au, that is assumed to be immobile. For a frontal collision, a *minimum approach distance* of $a_{min} = 3.8 \times 10^{-12}$ cm is found. Produce a diagram and then deduct the atomic number of the gold nucleus from the data. We will assume that the α particle comes from the infinite where the potential energy is zero. We will also assume that the {α particle – Au nucleus} system is conservative. The data given for application 1.3 will be used.

ANSWER.– The frontal collision corresponds to a zero-impact parameter (b = 0). We thus obtain the diagram shown below (Figure 1.12).

Rutherford scattering is known to be studied in the context of classical mechanics (the resting energy, m_0c^2, of the particles in the system studied is therefore not taken into account). At infinity, the mechanical energy of the {α – Au} system is equal to the kinetic energy, E, of the α particle. At the distance $r = a_{min}$, the α particle stops then turns back.

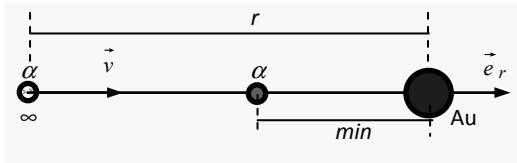


Figure 1.12. *Frontal collision between an α particle and an immobile nucleus of gold, Au*

The mechanical energy of the {α – Au} system is reduced to its potential energy:

$$V(r) = k\frac{zZe^2}{r} \qquad [1.16]$$

In relationship [1.16], the electrical constant $k = 1/4\pi\varepsilon_0 = 9 \times 10^9$ SI. With the {α – Au} system being conservative, we obtain:

$$E = V(r_{min}) = k\frac{zZe^2}{a_{min}} \Rightarrow Z = \frac{a_{min}E}{kze^2} \qquad [1.17]$$

Numerically we obtain from [1.17]:

$$Z = \frac{3.8\times10^{-14}\times6\times10^6}{9\times10^9\times2\times1.6\times10^{-19}} = 79.17 \approx 79$$

Let us now seek to establish an expression deduced from Rutherford's formula [1.13], subject to experimental verification.

If ϕ designates the α *particle flow*, the number, dN_1, of particles scattered by a nucleus per unit time in the solid angle $d\Omega$ is:

$$dN_1 = \phi d.\sigma \qquad [1.18]$$

If I is the intensity of a parallel plane beam of α particles falling on the nucleus, or the number of particles passing per unit time through a surface, S, normal to the beam, then:

$$\phi = \frac{I}{S} \qquad [1.19]$$

According to [1.19], the flow of ϕ particles is expressed in $s^{-1} \cdot cm^{-2}$.

Using [1.19], the number, dN_1, of particles scattered per unit time and per unit area in the solid angle $d\Omega$ is:

$$dN_1 = Id\sigma \qquad [1.20]$$

Let n be the number of nuclei per unit volume. The total number of target nuclei in volume V is then equal to Vn. The total number, dN, of particles scattered per unit time and per unit area in the solid angle $d\Omega$ is then equal to $VndN_1$. Thus, using [1.20]:

$$dN = IVnd\sigma \qquad [1.21]$$

By considering Rutherford's formula [1.13], we then obtain:

$$dN = IVn\left(\frac{1}{4\pi\varepsilon_0}\right)^2\left(\frac{zZe^2}{4E}\right)^2\frac{d\Omega}{\sin^4(\theta/2)} \qquad [1.22]$$

Let us compare the theoretical predictions according to the Rutherford model [1.22] with the experimental observations of Geiger and Marsden for α particles of energy 5.5 MeV, scattered by gold nuclei. The resulting curves are shown in Figure 1.13. Overall, it is noted that for scattering angles less than 140°, the number of α particles scattered as a function of the scattering angle is compatible with the model of the nucleus of highly-concentrated charge density at a point in space. This constitutes experimental evidence of the validity of Rutherford's point-like nucleus model. Moreover, this concordance between theory and experience confirms the planetary atomic model according to Rutherford (Figure 1.10) and overturns the

Thomson atomic model (Figure 1.7). However, the nucleus is not point-like. It has a skin thickness indicating that the mass distribution is not uniform in a nuclear volume of spherical form (see section 1.4.2).

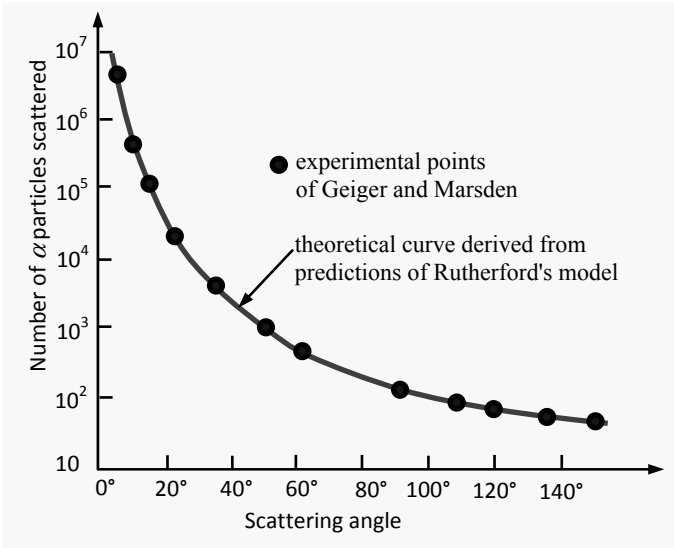


Figure 1.13. *Comparisons of theoretical predictions according to the Rutherford model with the experimental observations of Geiger and Marsden*

Lord Ernest Rutherford of Nelson was a New Zealand-born British physicist and chemist. Rutherford is considered the father of nuclear physics for his notable discoveries in this field. He discovered α radiation (helium nuclei) and β^- radiation (electron). Rutherford also discovered that radioactivity is accompanied by the decay of chemical elements. This won him the Nobel Prize in Chemistry 1908. In 1909, he established, along with British radiochemist Frederick Soddy (1877-1956), the experimental law of radioactive decay of radioelements. Drawing on his past experience of the gold-leaf α particle scattering, achieved by Geiger and Marsden, Rutherford revealed the existence of the atomic nucleus in 1911. This allowed him to put forward the planetary atomic model. Moreover, in 1919, he carried out the first artificial nuclear transmutation reaction, thus paving the way for the study of the properties of the nuclear reactions induced.

Box 1.5. *Rutherford (1871–1937)*

1.3. Composition of the nucleus

1.3.1. *Discovery of the proton*

Rutherford's experiment allowed the existence of the atomic nucleus to be revealed. The α particle corresponding to the nucleus of the helium atom was used in Rutherford's experiment of 1911. However at that time, the composition of the nucleus was not yet known, except for the hydrogen atom whose nucleus is the proton. This enabled Danish physicist Niels Bohr (1885-1962) to develop his semi-classical hydrogen atom theory in 1913 by studying the motion of the electron in the electric field created by the proton charge.

In 1919, Rutherford succeeded in experimentally carrying out the first *reaction of artificial nuclear transmutation* [RUT 19a, RUT 19b, LEP 56, GRO 85, NES 17]. This first *induced nuclear reaction* consisted of bombarding nitrogen gas with α particles. The simplified diagram of the experimental device used by Rutherford is given below (Figure 1.14).

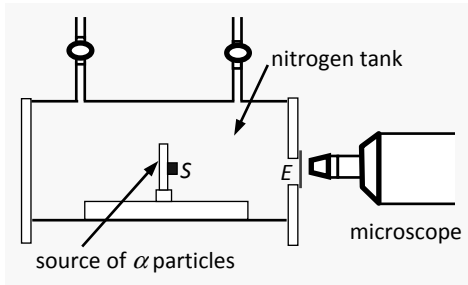


Figure 1.14. *Rutherford's simplified experimental device for conducting the first induced nuclear reaction*

Using an oscilloscope, Rutherford observed scintillations on the zinc sulfide-coated E-screen due to particles emitted as a result of the collision between the α particles and the nitrogen nuclei. He deduced from his observations that in striking nitrogen atoms, alpha particles produced a proton. In 1920, at a conference of the British Association for the Advancement of Science, Rutherford suggested that the nucleus of the hydrogen atom be termed the proton [KRI 19a]. However, Rutherford did not know what had become of the residual nucleus. Thus, he tasked Patrick Blackett (1897-1974), a researcher working under his direction, with identifying the residual nucleus [KRI 19b]. Blackett proceeded to conduct a series of experiments with a Wilson chamber (or fog chamber) and confirmed in 1925 that the particles

emitted in collisions between α particles and nitrogen nuclei are ${}^{1}_{1}H$ protons [BLA 25]. The nuclear transmutation reaction is thus written:

$$ {}^{4}_{2}He + {}^{14}_{7}N \rightarrow {}^{1}_{1}H + {}^{17}_{8}O \qquad [1.23a] $$

The reaction shows the transmutation of the nitrogen nucleus 14 to the oxygen nucleus 17 with production of a proton. For this reason, Rutherford is credited with the discovery of the proton. Between 1930 and 1932, a series of experiments conducted on the study of nuclear transmutation reactions led to the birth of the neutron, the proton's natural companion in the nucleus.

In 1930, German physicist Walther Bothe and his student Herbert Becker conducted transmutation experiments by bombarding atomic targets with α particles [BOT 30, NES 17]. At that time, the available sources of alpha radiation were radium 226 (α of 4.9 MeV) and its descendant, radon 222 (α of 5.6 MeV) or, for polonium 210, emitting α particles of 5.4 MeV.

Using beryllium bombarded by α particles from a source of polonium-210 as a target, the reaction produced penetrating radiation. The latter was interpreted by Bothe and Becker as electromagnetic radiation like the γ radiation known at this time. Thus, they opposed that this radiation was produced by the deexcitation of radiocarbon 13 formed during the transmutation, according to the equation:

$$ {}^{4}_{2}\mathrm{He} + {}^{9}_{4}\mathrm{Be} \rightarrow {}^{13}_{6}\mathrm{C}^{*} \rightarrow {}^{13}_{6}\mathrm{C} + \gamma \qquad [1.23b] $$

Bothe and Becker evaluated γ-ray energy by interposing lead plates of varying thicknesses between the target and the counter. By measuring the attenuation of the gamma-ray flux of known energy, they estimated the energy of the γ photons at around 5 MeV. Unfortunately, this energy was higher than the usual γ photons produced by radioactivity, which are of the order of 1 to 2 MeV.

Intrigued by these results, Irène and Frédéric Joliot-Curie [CUR 32b, CUR 32c] conducted transmutation experiments of boron and beryllium bombarded by α particles, with a view to understanding the real nature of what is known as Bothe–Becker radiation. These experiments showed that the penetrating radiation observed in the Bothe and Becker experiments could eject protons from paraffin. Yet Irène and Frédéric Joliot-Curie did not interpret their observations correctly as they believed that the Compton effect phenomenon had occurred between the light element and gamma rays, whose energy they estimated. At the time, there was no knowledge of any penetrating radiation capable of ejecting particles other than electrons by photoelectric or Compton effect.

In 1932, Sir James Chadwick offered the correct interpretation of Bothe and Becker's observations by conducting a decisive experiment that would solve the enigma of the penetrating radiation observed in the transmutation reactions of boron and beryllium.

Patrick Maynard Stewart Blackett was a British experimental physicist. He is famous for his experiments on the induced nuclear reactions that made it possible, in 1925, to identify the residual nucleus (oxygen) in the transmutation reaction initiated by Rutherford in 1919. He was awarded the Nobel Prize in Physics 1948 for his development of the Wilson cloud chamber and for his discoveries in the fields of nuclear physics and cosmic rays.

Walter Wilhelm Georg Bothe was a German physicist. He is most famous for his contributions to nuclear transmutation reactions. He won half of the Nobel Prize in Physics 1954 for designing the coincidence method (with the other half awarded to the English physicist, Max Born (1882-1970). In nuclear physics, the principle of the coincidence method consists of using sensitive devices to detect the simultaneous (or near-simultaneous) emission of two particles, each collected in a different counter. For more details on this subject see [VUC 55].

Box 1.6. *Blackett (1897–1974); Bothe (1891–1954)*

1.3.2. *Discovery of the neutron*

As noted above, the aim of the Chadwick experiment was to identify radiation penetrating the boron and beryllium transmutation reactions. The experimental set-up used by Chadwick is schematically presented in Figure 1.15.

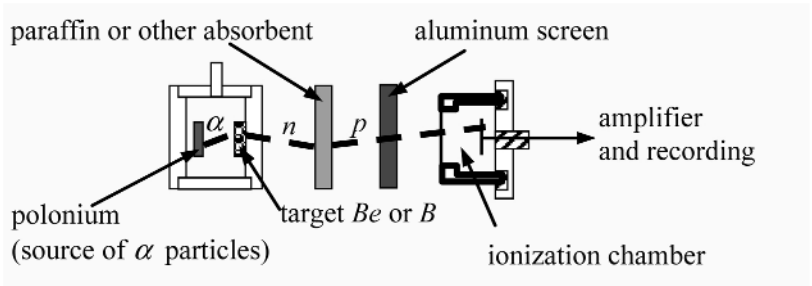


Figure 1.15. *Chadwick's experimental set-up*

Polonium, a source of α particles of 5.3 MeV is introduced into a chamber with a high vacuum. The collision between the α particles and the beryllium producing

penetrating radiation (here neutron n to be identified) in all directions. This radiation then bombards a paraffin screen, inducing the ejection of protons (p). These protons then pass through thin aluminum screens to access an ionization chamber filled with nitrogen gas. The pulse produced by protons and recoil nitrogen nuclei is amplified and recorded. The observed pulses corresponding to protons penetrating into the ionization chamber. The track of the recoil protons is then measured in the aluminum screens interposed between the detector and the paraffin.

In 1920, the concept of the neutron consisting of an intimate combination of a proton and an electron was discussed by Rutherford. But this combination was never confirmed theoretically or experimentally. However, interpreting these observations, Chadwick showed that the measured recoil energies were compatible with the neutron n hypothesis [CHA 32a, CHA 32b, NES 17]. On the basis of this hypothesis, the bombardment of beryllium or boron target nuclei with α particles produced two types of induced nuclear reactions: the ^{9}Be $(\alpha, n)^{12}$C reaction and the ^{11}B $(\alpha, n)^{14}$N reaction. These reactions are explicitly written:

$$^{4}_{2}He + {}^{9}_{4}Be \rightarrow {}^{1}_{0}n + {}^{12}_{6}C \qquad [1.24]$$

$$^{4}_{2}He + {}^{11}_{5}B \rightarrow {}^{1}_{0}n + {}^{14}_{7}N \qquad [1.25]$$

Since the nucleus mass of beryllium 9 was not known in 1932, reaction [1.25] was used by Chadwick. By measuring the energy released by this reaction and using the available masses of ^{11}B, ^{14}N, and ^{4}He, Chadwick found a neutron mass, m_n, between 1.005 u and 1.008 u, i.e. [EVA 61]:

$$1.005 \text{ u} \leq m_n \leq 1.008 \text{ u} \qquad [1.26a]$$

The CODATA recommended value is 1.00866491597(43) u. Although not precise, the Chadwick result is acceptable seeing as the upper limit of neutron mass only differs from the CODATA recommended value by around 0.0007 u.

Chadwick found the neutron he had been seeking for 12 years. He announced his results in a brief note, “Possible Existence of a Neutron”, in the journal *Nature* on February 17 (published on February 27) and detailed his experiments in a longer – and more affirmative – article, “The Existence of a Neutron” on May 10, published on July 1 in *Proceedings of the Royal Society*. Note that he does not speak of “the” neutron, but “a” neutron. The text published by Chadwick is presented in box 1.7. However, it was not immediately accepted that Chadwick had discovered the neutron. Was it a real particle, like the proton or the electron? The problem

was solved in 1933 by Chadwick and the American physicist, Maurice Goldhaber (1911-2011).

In 1931, American chemist Harold Urey (1893-1981) discovered deuterium by demonstrating with his colleagues the existence of heavy water (deuterium oxide, D_2O, a discovery that saw him awarded the Noble Prize in Chemistry 1934). In 1933, Chadwick and Goldhaber succeeded in achieving deuteron photodissociation (deuterium nucleus). By bombarding the deuteron with γ photons of 2.6 MeV, they measured the energy of the protons released. Chadwick and Goldhaber thus determined that the deuteron binding energy was 2.2 MeV. Subsequently, they estimated the neutron mass by drawing on the principle of energy conservation. Let:

$$m_n c^2 = m_d c^2 - m_p c^2 + E_l \qquad [1.26b]$$

In equation [1.26b] m_n is the neutron mass, m_d designates the deuteron mass ($m_d c^2 = 1876.1$), m_p represents the proton mass ($m_p c^2 = 938.3$), and E_l designates the deuteron binding energy ($E_l = 2.2$ MeV). Numerically we find:

$$m_n c^2 = 1876.1 - 938.3 + 2.2 = 940 \text{ MeV}. \qquad [1.26c]$$

The accepted value of the precise neutron mass equals 939.565346 MeV/c^2 (see Table 1.3). This value is in close accordance with result [1.26c] by Chadwick and Goldhaber, since the mass difference is 0.434654 MeV/c^2.

In 1933, the neutron was accepted as a fundamental particle constituting atomic nuclei.

Possible Existence of a Neutron

It has been shown by Bothe and others that beryllium when bombarded by α-particles of polonium emits a radiation of great penetrating power, which has been an absorption coefficient in lead of about 0.3 (cm)$^{-1}$. Recently Mme. Curie-Joliot and M. Joliot found, when measuring the ionization produced by this beryllium radiation in a vessel with a thin window, that the ionization increased when matter-containing hydrogen was placed in front of the window. The effect appeared to be due to the ejection of protons with velocities up to a maximum of nearly 3 x 109 cm. per sec. They suggested that the transference of energy to the proton was by a process similar to the Compton effect, and estimated that the beryllium radiation had a quantum energy of 50 x 106 electron volts. I have made some experiments using the valve counter to examine the properties of this radiation excited in beryllium. The

valve counter consists of a small ionization chamber connected to an amplifier, and the sudden production of ions by the entry of a particle, such as a proton or α-particle, is recorded by the deflexion of an oscillograph. These experiments have shown that the radiation ejects particles from hydrogen, helium, lithium, beryllium, carbon, air, and argon. The particles ejected from hydrogen behave, as regards range and ionizing power, like protons with speeds up to about 3.2 x 10^9 cm. per sec. The particles from the other elements have a large ionizing power, and appear to be in each case recoil atoms of the elements.

If we ascribe the ejection of the proton to a Compton recoil from a quantum of 52 x 10^6 electron volts, then the nitrogen recoil atom arising by a similar process should have an energy not greater than about 400,000 volts, should produce not more than about 10,000 ions, and have a range in air at N.T.P. of about 1.3 mm. Actually, some of the recoil atoms in nitrogen produce at least 30,000 ions. In collaboration with Dr. Feather, I have observed the recoil atoms in an expansion chamber, and their range, estimated visually, was sometimes as much as 3 mm at N.T.P. These results, and others I have obtained in the course of the work, are very difficult to explain on the assumption that the radiation from beryllium is a quantum radiation, if energy and momentum are to be conserved in the collisions.

The difficulties disappear, however, if it be assumed that the radiation consists of particles of mass 1 and charge 0, or neutrons. The capture of the α particle by the Be^9 nucleus may be supposed to result in the formation of a C^{12} nucleus and the emission of the neutron. From the energy relations of this process the velocity of the neutron emitted in the forward direction may well be about 3 x 10^9 cm. per sec. The collisions of the neutron with the atoms through which it passes give rise to the recoil atoms, and the observed energies of the recoil atoms are in fair agreement with this view. Moreover, I have observed that the protons ejected from hydrogen by the radiation emitted in the opposite direction to that of the exciting α-particle appear to have a much smaller range than those ejected by the forward radiation. This again receives a simple explanation of the neutron hypothesis.

If it be supposed that the radiation consists of quanta, then the capture of the α-particle by the Be9 nucleus will form a C^{13} nucleus. The mass defect of C^{13} is known with sufficient accuracy to show that the energy of the quantum emitted in this process cannot be greater than about 14 x 10^6 volts. It is difficult to make such a quantum responsible for the effects observed. It is to be expected that many of the effects of a neutron in passing through matter should resemble those of a quantum of high energy, and it is not easy to reach the final decision between the two hypotheses. Up to the present, all the evidence is in favor of the neutron, while the quantum hypothesis can only be upheld if the conservation of energy and momentum be relinquished at some point.

J. Chadwick, Cavendish Laboratory, Cambridge, Feb. 17. [CHA 32b]

Sir James Chadwick was a British physicist. He is particularly well known for his historical experiment that enabled the discovery of the neutron in 1932. This discovery saw him receive the Nobel Prize in Physics 1935. This discovery permitted an understanding of the internal structure of nuclei and paved the way for the study of various induced nuclear reactions, such as fission.

Box 1.7. *Chadwick (1891–1974)*

1.3.3. *Internal structure of nucleons:* u *and* d *quarks*

Aside from the electrical charge, the proton and neutron are spin-1/2 fermions like the electron. Their mass difference is equal to 0.00138 u. Thus the idea of German physicist Werner Heisenberg (1901-1976) was accepted, namely that proton and neutron are the two possible states of the same heavy particle, called the *nucleon*. Thus, the atomic nucleus consists of nucleons, i.e. the protons and neutrons. The nucleus of the hydrogen atom consists of a single nucleon, the proton. The α particle corresponding to the nucleus of the helium atom consists of four nucleons: two protons and two neutrons.

With Chadwick's discovery of the neutron in 1932, the internal structure of the nucleus was established. But the internal structure of nucleons remained a mystery. What do they consist of? This question would remain unanswered until 1964.

Developing their theory on elementary particles, in 1964 Murray Gell-Mann and George Zweig independently assumed the existence of particles that would be the constituent elements of all others [LEP 56, GRO 85, PAG 85, STÖ 07, SEC 10]. Gell-Mann called these elements *quarks*. This concept is derived from the literary work of Irish novelist and poet James Joyce (1882-1941) in his book, *Finnegans Wake*, which includes the line, "Three quarks for Muster Mark," where half-real, half-imaginary objects are presented three by three. In the Gell-Mann and Zweig model, the existence of the *u* ("up") *quark* with a charge of (2/3) *e* and the *d* ("down") *quark* with a charge of (–1/3) *e*, to explain the internal structure of the nucleons is assumed. Associated with these quarks, each of spin-1/2, are two *antiquarks*, noted $\overline{u}$ and $\overline{d}$, respectively.

In order to satisfy the *principle of electric charge conservation*, the proton has a *udu*-type structure, three-quark internal configuration and the neutron has a *udd*-type structure (Figure 1.16).

In both the neutron and the proton, the strong interaction between quarks is due to an exchange of particles called *gluons*, noted *g* (spin $s = 1$, mass $m = 0$). These exchanged particles thus ensure the cohesion of the nucleons.

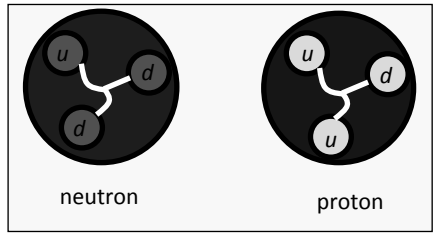


Figure 1.16. *Internal proton and neutron structures according to the Gell-Mann and Zweig quark model*

In particle physics, two types of particle systems can be distinguished:

– the *elementary system*: a quantum object fully characterized by its mass and spin;

– the *elementary particle*: an elementary system in which one cannot distinguish between the elementary constituents of given mass and spin.

The proton and neutron are thus elementary systems (they are considered elementary particles while ignoring their internal structure) whereas quarks are elementary particles (their internal structure not being known at this time). Whereas the proton is a stable particle with an estimated lifetime of more than 1.2×10^9 years, the neutron is an unstable particle with a radioactive period of 885.7 s [STÖ 07].

For a given nucleus, a mass number or nucleon number is defined and noted A. By definition, $A = Z + N$, with Z the proton number and N the neutron number of the nucleus considered. If X designates the symbol of the chemical element considered, then a nucleus is symbolized by writing ${}^{A}_{Z}\mathrm{X}$. This conventional notation of nuclei then enables understanding of the writing of the reagents and products of the induced nuclear reactions, [1.23] and [1.24].

Murray Gell-Mann was an American physicist. He is famous most notably for having formulated the theory of quarks which he considered to be the constituent elementary particles of all the other particles. He was awarded the Nobel Prize in Physics 1969.

George Zweig was a Russian-American physicist. In particle physics he developed quark theory, independently from Gell-Mann. We thus speak of the Murray–Zweig quark model. He later went on to study neurobiology.

Box 1.8. *Gell-Mann (1929–2019); Zweig (born 1937)*

1.3.4. *Isospin*

In 1932, Heisenberg introduced the concept of *isospin*, or *isobaric spin*, to characterize the two possible states of the nucleon. Isospin is noted I [STÖ 15] or T [EVA 61, SEC 10]. To avoid confusion with nuclear spin, I [EVA 61, BRÖ 01], reserved for the nucleus (see section 1.3.5), we will use the notation T below to designate isospin.

In a "hypothetical isospin space", the nucleon is characterized by the isospin vector, $\vec{T}$, which is an operator of components T_x, T_y, and T_z. By making the analogy with electronic spin, taking the two possible values, $m_s = \pm 1/2$, it is agreed that the projection, Tz, of the *isospin operator* along a preferred direction, Oz, in an isospin space takes the values:

$$T_z = +1/2 \text{ for the proton; } T_z = -1/2 \text{ for the neutron} \qquad [1.27]$$

According to the *rule of addition of angular momenta*, the total isospin, $\vec{T}$, resulting from the coupling of the respective individual isospins, $\vec{T}_1$ and $\vec{T}_2$, of two nucleons, (1) and (2), and its projection, T_z along a reference direction, Oz, in the *isospin space* satisfy the relationships:

$$\vec{T} = \vec{T}_1 + \vec{T}_2 \; ; T_z = T_{z1} + T_{z2} \qquad [1.28]$$

For the nucleon, the introduction of isospin is based on the fundamental assumption that *pp* (proton–proton), *pn* (proton–neutron), or *nn* (neutron–neutron) nuclear interaction is the same because of the *charge independence* of nuclear forces. By analogy to the spin multiplicity $(2S + 1)$ leading to the singlet state, $S = 0$, and triplet state, $S = 1$, the number of states in the isospin space is equal to the *isospin multiplicity* $(2T + 1)$. The following nucleon states and systems are thus obtained.

– For $T = 0$, $T_z = 0$: singlet state corresponding to the *pn* nucleonic system.

– For $T = 1$: triplet state. Three nucleon systems are obtained: *pp* ($T_z = +1$); *pn* ($T_z = 0$) and *nn* ($T_z = -1$).

APPLICATION 1.5.– Is the *ground state of the deuteron* a singlet state or a triplet state? We will substantiate the answer.

ANSWER.– The deuteron (deuterium nucleus) consists of a proton ($T_{z1} = +1/2$) and a neutron ($T_{z2} = -1/2$). This gives $T_z = 0$ and $T = 0$: the ground state of the deuteron is a singlet state.

Generally, for an arbitrary nucleus of mass number $A = N + Z$, the projection, Tz, of the total isospin quantum number is given by the relationship:

$$T_z = \frac{1}{2}(N - Z) \qquad [1.29]$$

Relationship [1.29] shows that T_z depends on the excess neutrons ($N - Z$). Therefore, the ground state of even-even nuclei is a singlet state.

APPLICATION 1.6.– Determine the *total isospin*, T, corresponding to the ground state of the following isobar nuclei: ^{14}C ($Z = 6$), ^{14}N ($Z = 7$) and ^{14}O ($Z = 8$).

ANSWER.– Using formula [1.29], we obtain the following for the nuclei, respectively: ^{14}C, ^{14}N and ^{14}O: $T_z = +1$, $T_z = 0$ and $T_z = -1$. The ground state of the carbon-14 and oxygen-14 nuclei has isospin $T = 1$, while $T = 0$ for the nitrogen-14 nucleus.

1.3.5. *Nuclear spin*

By analogy to the electron's own rotational motion characterized by the spin angular momentum, the atomic nucleus is associated with a *nuclear spin angular momentum* noted I. Similarly, by analogy to the magnetic quantum number of spin, m_s, taking $(2s + 1)$ values with s the spin of the electron, the magnetic quantum number of nuclear spin, m_I, taking $(2I + 1)$ values with I the *nuclear spin*.

Nuclear spin, I, is the sum of the nucleon spins and their possible orbital angular momenta in the nucleus. The proton and neutron are *fermions* of spin $I = 1/2$. According to *Pauli's exclusion principle*, two nucleons belonging to the same nucleus can have only parallel spin orientations if they are different (*pn*), or antiparallel if they are identical (*PP* or *nn*). For a given nucleus with mass number A, I is integer if A is even and half-integer if A is odd. For example, for deuteron $^{2}_{1}$H (*pn*), $I = 1$, for lithium $^{7}_{3}$Li (*pp nn pn n*), $I = 3/2$ and for cobalt $^{59}_{27}$Co , $I = 7/2$.

1.3.6. *Nuclear magnetic moment*

As a general rule, it is assumed that any charged particle following a closed curve is a source of magnetic field. At a very far distance from the particle, the magnetic field can be considered as that created by a *magnetic dipole* placed at the location of the charged particle [SAK 20].

As with electrons, the nucleus is associated with a *spin angular momentum,* $\vec{I}$, which is an observable. The square $\vec{I}^2$ and one of these projections, for example I_z, following a preferred direction (here *Oz*) have determined values equaling, respectively:

$$\hbar^2 I(I+1) \text{ and } M_I\hbar .$$

The *nuclear magnetic quantum number,* M_I, then takes the values:

$$-I, -(I+1), \ldots\ldots, +(I-1), +I$$

This is a total of $(2I + 1)$ values. By definition, the *nuclear dipole magnetic moment*, noted $\overrightarrow{M}_N$, is given by the relationship:

$$\overrightarrow{M}_N = g_I \gamma_I \vec{I} \qquad [1.30]$$

In relationship [1.30], g_I is the *nuclear Landé factor* and the magnitude γ_I is the *nuclear gyromagnetic ratio* [BIÉ 06, SAK 20] given by the following expression (this is the analogue of the *electron gyromagnetic ratio* $\gamma = -e/2m$):

$$\gamma_I = \frac{e}{2m_p} \qquad [1.31]$$

In relationship [1.31], m_p is the proton mass.

Likewise, we define the *nuclear magneton*, also called the *nuclear Bohr magneton*, noted μ_N (by analogy to the *Bohr magneton* $\mu_B = -\gamma\hbar = e\hbar/2m$). The nuclear magneton is given by the expression:

$$\mu_N = \gamma_I \hbar = \frac{e\hbar}{2m_p} \qquad [1.32]$$

Using [1.31] and [1.32], the dipole magnetic moment [1.32] is written:

$$\overrightarrow{M}_N = g_I \frac{\mu_N}{\hbar} \vec{I} \qquad [1.33]$$

Noting that $\mu_N/\mu_B = m/m_p$ and $m_p/m = 1836.1526715$, then we obtain:

$$\mu_N = 5.05078317 \times 10^{-27} \text{ J} \cdot \text{T}^{-1}$$

For the proton:

$$I = 1/2;\ g_I = 5.5883;\ \mu_p = 2.7928473375\ \mu_N \qquad [1.34]$$

Although electrically neutral, a dipole magnetic moment, μ_n, is associated with the *neutron's spin angular momentum*. Thus, for the neutron:

$$I = 1/2;\ g_I = -3.8263;\ \ \mu_n = -1.91304272\ \mu_N \qquad [1.35]$$

Table 1.1 summarizes the fundamental properties of nucleons.

Properties	Proton	Neutron
Mass	$1.672621637(83)\times 10^{-27}$ kg	$1.674927211(84)\times 10^{-27}$ kg
Charge	$1.602179487(40)\times 10^{-19}$ C	0
Spin	½	½
Isospin	+ ½	– ½
Radioactive period	$\sim 2.1\times 10^{29}$ years	(885.7 ± 08) s
Nuclear Landé factor	5.5883	– 3.8263
Dipole magnetic moment	2.7928473375 μ_N	– 1.91304272 μ_N

Table 1.1. *Fundamental properties of nucleons*

1.4. Nucleus dimensions

1.4.1. *Nuclear radius*

Most nuclei are considered to be spherical. In this approximation, the nuclear radius, R, given by the relationship [EVA 61, MIS 96, BRÖ 01, STÖ 07, SEC 10, DUM 15, SAK 18a]:

$$R = r_0 A^{1/3} \qquad [1.36]$$

In relationship [1.36], r_0 is a nuclear radius whose value depends on the nucleus model adopted:

– for the *nuclear model with constant nucleon density*, the nucleus charge is considered to be uniformly distributed within the nuclear volume, r_0 is called the *unit nuclear radius* within the framework of classical mechanics and worth:

$$r_0 = 1.45\ \times 10^{-15}\ \text{m} = 1.45\ \text{fm} \qquad [1.37]$$

– for the nuclear model, it is considered that the nucleus charge is distributed uniformly at the surface of the nucleus; r_0 is called the *electromagnetic unit radius* within the framework of quantum mechanics and is worth:

$$r_0 = 1.2 \times 10^{-15} \text{ m} = 1.2 \text{ fm} \qquad [1.38]$$

The first theoretical expression for the unit nuclear radius was established in 2018 by the author as part of the nuclear model with constant nucleon density. The *Sakho unit nuclear radius* is given by the expression [SAK 18a]:

$$r_0 = \frac{\alpha^2}{(1+N/Z)} a_0 \qquad [1.39]$$

In relationship [1.39], α is the *fine-structure constant* and a_0 represents the *Bohr radius*.

For most nuclei, $A \approx 2Z$ ($N = Z$). The unit nuclear radius is then written:

$$r_0 = \frac{\alpha^2}{2} a_0 \qquad [1.40]$$

Based on $\alpha^2 = 5.325 \times 10^{-5}$ and $a_0 = 5.2917 \times 10^{-11}$ m, relationship [1.40] gives:

$$r_0 = 1.41 \times 10^{-15} \text{ m} = 1.41 \text{ fm} \qquad [1.41]$$

The theoretical result [1.41] is very much consistent with the experimental results, $r_{0\text{exp}} = 1.40 \times 10^{-15}$ m = 1.40 fm obtained on the scattering of α particles by light nuclei Li, Be, Mg and Al [POL 35] and on the scattering of fast neutrons by nuclei [DAY 53].

NOTE.– The radii of exotic nuclei do not confirm relationship [1.36]. An *exotic nucleus* is a nucleus created artificially in particle accelerators.

APPLICATION 1.7.– Using [1.36], give a framing of the nuclear radii, R_X, corresponding to natural nuclei. Let $r_0 = 1.2$ fm.

ANSWER.– The lightest nucleus is that of the hydrogen atom: $R_H = 1.20$ fm. The heaviest natural nucleus is uranium 238: $R_U = 1.2 \times 10^{-15} \times (238)^{1/3} = 7.4$ fm. Thus, for a natural nucleus of radius R_X, we have:

$$(1.0 < R_X < 8.0) \text{ fm} \qquad [1.42]$$

APPLICATION 1.8.– Estimate the *density of nuclear matter*. We will compare with the mass of 1 m^3 of water and then draw a conclusion. The mass of a nucleus, $m \approx$ Au, with u the atomic mass unit.

Let: $r_0 = 1.2$ fm; $u = 1.66 \times 10^{-27}$ kg.

ANSWER.– The density of a nucleus assumed to be spherical of radius $R = r_0 A^{1/3}$ is:

$$\rho = \frac{m}{V} = \frac{Au}{(4/3)\pi R^3} = \frac{3u}{4\pi r_0^3} \qquad [1.43]$$

NOTE.– $\rho \approx 2.3 \times 10^{17}$ kg · m^{-3}

Given that 1 m^3 of water weighs 1,000 kg, 1 m^3 of nuclear matter weighs around 230 million billion kilograms.

CONCLUSION.– nuclear material is extremely dense.

1.4.2. *Nuclear density, skin thickness*

As noted in section 1.4.1, the value of the radius, r_0, depends on the adopted nucleus model according to whether the nuclear mass is uniformly distributed in volume or surface. As a result, the nuclear density is not constant for a given nucleus.

With nuclear rays varying approximately between 1 and 8 fm [1.42], to probe the inside of the nuclei, the probe particles must have a de Broglie wavelength of the order of 1 fm. To achieve this, it is possible to envisage conducting experiments of, for example, photon, electron or neutron scattering by target nuclei. The information obtained on the inside of the nucleus will then depend on the nature of the probe particle used and the probe-nucleon particle interaction.

Scattering experiments use probe particles with very high energies. These are therefore relativistic particles of total energy:

$$E = \sqrt{p^2c^2 + m_0^2c^4} \qquad [1.44]$$

By definition, the *de Broglie wavelength* $\lambda = h/p$. Using relationship [1.44], the relativistic expression for the energy of the probe particles is obtained:

$$E = \sqrt{\frac{h^2c^2}{\lambda^2} + m_0^2c^4} \qquad [1.45]$$

For an electromagnetic probe, the experiment would require a photonic energy of more than 1,200 MeV. In photon scattering experiments, *X* photons with energies in the keV range are generally used. From a technical point of view, it proves very complex to create a photon beam with 1,200 MeV energy.

APPLICATION 1.9.– Show that the use of an electromagnetic probe would require energy of more than 1,200 MeV.

Given data: $c = 3.0 \times 10^8$ m · s $^{-1}$ and $h = 6.63 \times 10^{-34}$ J · s; 1 eV = 1.6×10^{-19} J.

ANSWER.– For the photon, the resting mass, $m_0 = 0$, expression [1.45] gives:

$$E = \frac{hc}{\lambda} \qquad [1.46]$$

NOTE.– (de Broglie wavelength of the order of 1 fm: $\lambda = 1$ fm).

$$E = 1.243 \times 10^{9} \text{ eV} = 1243 \text{ MeV}$$

The scattering of high-energy electrons (approximately 1 GeV) on nuclei was the first source of information on nuclear charge distribution. At these energies, electrons have a wavelength comparable to or less than the dimensions of the nucleus, and can thus probe the spatial distribution of protons within the nucleus [DEL 03]. As neutrons are neutral, neutron scattering experiments offer access to the mass distribution of the nucleus.

Let us thus consider a nucleus of *nuclear charge distribution density* $\rho(r)$. For the electron, the resting energy $m_0c^2 = 0.511$ MeV. For $\lambda = 1$ fm, expression [1.45] then shows that the energy of the electron beam to be used is equal to 1.24 GeV. However, the experiment reveals that electrons of energy between 100 and 1,000 MeV enables probing of the inside of the nuclei.

Using electrons as probe particles, the experiment shows that the density $\rho(r)$ presents saturation in a certain range of the nuclear volume and then gradually decreases at the surface of the nucleus. The study of several nuclei has established the form of $\rho(r)$, the simplest of which, accounting satisfactorily for the charge distribution in the nucleus assumed to be spherical, is that of Woods–Saxon, given by [CAR 79, DEL 03, STÖ 07, SEC 10, DUM 15, MAY 17]:

$$\rho(r) = \frac{\rho_0}{1 + \exp\left(\frac{r - R}{a}\right)} \qquad [1.47]$$

In relationship [1.47], $\rho_0 = \rho\,(0)$ designates the *saturation density*, the quantity, *a,* called the *diffusivity parameter* characterizes the extent of the nuclear surface, *r* is the distance measured in spherical coordinates from the center of the nucleus, and *R* is the radius of the nucleus assumed to be spherical.

The set of experimental results for fairly heavy nuclei ($A \geq 30$) agrees with the formula $R = 1.07 \times A^{1/3}$ fm and $a = 0.545$ fm. The variation of the nuclear charge distribution density $\rho\,(r)$ with the distance *r* is shown in Figure 1.17. At the center of the nucleus, the saturation density $\rho_0 \approx 0.14$ nucleons · fm $^{-3}$ (see application 1.10).

As shown in Figure 1.17, the nuclear charge distribution density is approximately constant inside the nucleus, regardless of the nucleus considered. This is due to the saturation property of short-range nuclear forces (see section 1.6.5).

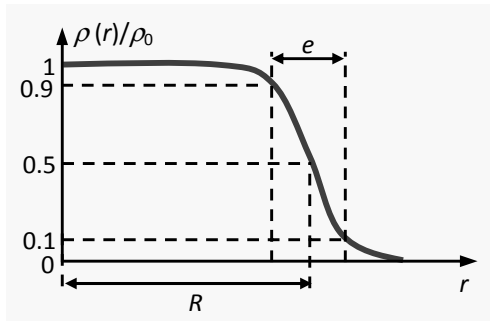


Figure 1.17. *Variation of the nuclear charge distribution density ρ (r); the nucleus radius $R = r_0 A^{1/3}$, e is the skin thickness and ρ_0 is the saturation density*

In addition, when $r \rightarrow R$, note the existence in the nucleus of a diffuse zone of approximately 2.2 fm, of almost constant value for all nuclei, in which the density gradually decreases. When $r > R$ the nuclear charge distribution density $\rho\,(r)$ gradually decreases at the surface and does not fall abruptly to zero. To translate this progressive decay, let us introduce the parameter, *e*, called *skin thickness* This parameter is the length for which the density $\rho\,(r)$ decreases from 90% to 10% of the saturation density ρ_0 (Figure 1.17). The skin thickness is linked to the diffusivity parameter, *a*, by the relationship (see application 1.11):

$$e = 4a\ln 3 \qquad [1.48]$$

For $a = 0.545$ fm, using [1.48] we find: $e = 2.4$ fm. If we make the approximation $a \approx 0.5$ fm, then we find $e \approx 2.2$ fm. This result shows that the length of the diffuse zone in the nucleus of approximately 2.2 fm corresponds to the skin thickness.

APPLICATION 1.10.– Show that the saturation density is worth approximately 0.14 nucleon · fm $^{-3}$.

Let $r_0 = 1.2$ fm.

ANSWER.– The saturation density corresponds to the nucleon density per unit volume. For a spherical core of volume V and mass number A we have:

$$\rho_0 = \frac{A}{V} = \frac{3}{4\pi r_0^3} \qquad [1.49]$$

NOTE.–

$$\rho_0 = \frac{3}{4\pi \times (1.2)^3} = 0.13815 \approx 0.14 \text{ nucleon} \cdot \text{fm}^{-3}$$

APPLICATION 1.11.– Demonstrate relationship [1.48].

ANSWER.– Let us use R_1 and R_9 to note the nuclear radii corresponding to densities $0.1\rho_0$ and $0.9\ \rho_0$, respectively. In this case the skin thickness:

$$e = R_1 - R_9 \qquad [1.50]$$

Using [1.47], we obtain:

$$\begin{cases} 0.1\rho_0 = \dfrac{\rho_0}{1+\exp\left(\dfrac{R_1 - R}{a}\right)} \\ 0.9\rho_0 = \dfrac{\rho_0}{1+\exp\left(\dfrac{R_9 - R}{a}\right)} \end{cases} \Rightarrow \begin{cases} R_1 - R = +a\ln 9 \\ R_9 - R = -a\ln 9 \end{cases}$$

Thus, according to [1.50]: $e = R_1 - R_9 = 2a\ln 9 = 2a\ln(3^2) \Rightarrow e = 4a\ln 3$.

David S. Saxon was an American physicist. He is famous for having proposed, in 1954 along with R. D. Woods (biography not mentioned in the available literature), what is known as the Woods–Saxon potential, thus describing the scattering of protons on heavy nuclei, such as platinum or nickel.

Box 1.9. *Saxon (1920–2005)*

1.5. Nomenclature of nuclides

1.5.1. *Isotopes, isobars, isotones*

In 1886, Crookes gave a speech to the British Association in which he referred to the concept of isotopes. He declared:

> When we say, for example, that the atomic mass of calcium is 40, we are actually saying that the majority of the calcium atoms have a real atomic mass of 40, that there are a small number of atoms which have a mass 39 or 41, others much smaller are 38 or 42, and so forth. [EVA 61]

Today, we know that calcium is a mixture of isotopes of mass numbers 40, 42, 43, 44, 46, and 48. Crookes' predictions were therefore correct; the mass numbers 38 and 39 corresponding to the isotopes of the element potassium. From an experimental perspective, Frederick Soddy [SOD 11, SOD 14] was the first to prove the existence of isotopes in radioactive substances. In addition, he highlighted in his work the existence of substances having the same mass number but different chemical properties, because of the difference in their nuclear charges. These substances were called isobars.

The nomenclature of nuclear species distinguishes between groups of nuclei that form three large families: isotopes, isobars and isotones.

Generally, a nucleus is characterized by its proton number, Z, and its nucleon number, A [SAK 16] is referred to as a nuclide. A nuclide of a chemical element, X, is symbolized by the notation ${}^{A}_{Z}X$, which we recall here. The neutron number, $N = A - Z$.

– *Isotopes* is the name given to nuclei with the same proton number but different neutron numbers, N. For example, ${}^{13}_{6}C$ ($N = 7$) and ${}^{14}_{6}C$ ($N = 8$);

– *isobars* refer to nuclei with the same mass number but different proton numbers. For example, ${}^{14}_{6}C$ and ${}^{14}_{7}N$;

– *isotones* refer to nuclei with the same neutron number but different proton numbers. For example ${}^{12}_{6}C$ ($N = 6$) and ${}^{13}_{7}N$ ($N = 6$).

Generally, for a family of given nuclei, the nuclide content is different. The most stable nuclei have the highest content, while unstable nuclei exist in the form of traces. Table 1.2 presents the contents of some isotopes. Today, 90 natural elements and 19 artificial elements are known, corresponding to approximately 1,500

different nuclides, approximately 350 of which are natural. Of the 1,500 nuclides identified, 256 are stable, the others spontaneously convert to more stable nuclides. Let us specify some of the isotopes of the first ten elements of the periodic table in addition to those of iron and uranium.

– Hydrogen (H) has three isotopes: ^{1}H, ^{2}H, and ^{3}H. The remaining isotopes from ^{4}H to ^{7}H have been synthesized in the laboratory. They are particularly unstable and have never been observed in nature.

– Helium (He) has eight known isotopes (^{2}He, ^{3}He, ^{4}He, ^{5}He, ^{6}He, ^{7}He and ^{8}He). However, only the two isotopes ^{3}He and ^{4}He are stable. All other radioisotopes have a short lifetime. The longest lifetime is that of the isotope ^{6}He with a half-life of 806.7 ms; the isotope ^{2}He is a hypothetical isotope that would be composed of two protons without any neutron, therefore called the "diproton".

– Lithium (Li) consists of two stable isotopes, ^{6}Li and ^{7}Li, the majority being the latter.

– Beryllium (Be) has twelve known isotopes, with mass numbers ranging from 5 to 16. Only the isotope ^{9}Be is stable. For this reason, beryllium is called a *monoisotopic element*. A monoisotopic element is an element with only one stable isotope. 26 monoisotopic elements can currently be enumerated. Fluorine-19, sodium-23, aluminum-27 and phosphorus-31 can be cited in addition to beryllium-9.

– Boron (B) has fourteen isotopes, of which only the two isotopes, ^{10}B and ^{11}B, are stable. All have a short lifetime. The isotope ^{8}B, with a half-life of 770 ms, has the longest lifetime.

– Carbon (C) has 15 isotopes, with mass numbers ranging from 8 to 22. Of these isotopes, only ^{12}C and ^{13}C are stable. The radioisotope with the longest lifetime is carbon 14, with a half-life of 5,730 years. This isotope is widely used for the dating of dead plant or animal species (see Chapter 1, Volume 2), and is the only radioisotope present in nature. It is formed by the reaction: $^{14}\text{N} + {}^{1}\text{n} \rightarrow {}^{14}\text{C} + {}^{1}\text{H}$. All other radioisotopes have a short lifetime. The isotope ^{11}C, with a half-life of 20.334 min, has the longest lifetime.

– Nitrogen (N) has 16 isotopes with mass numbers between 10 and 25, and a nuclear isomer, ^{11m}N, with a half-life of $6.90(80) \times 10^{-22}$ s (see the note at the end of this section for the definition of a nuclear isomer). Only the two isotopes ^{14}N and ^{15}N, which are present in nature, are stable.

– Oxygen (O) has 17 isotopes with mass numbers between 12 and 28, three of which, ^{16}O, ^{17}O, and ^{18}O, are stable. The radioisotope with the longest lifetime is ^{15}O, with a half-life of 122.24 s.

– Fluorine (F) is a monoisotopic element like beryllium. It consists of the only stable isotope, ^{19}F.

– Neon (Ne) has 19 isotopes with mass numbers ranging from 16 to 34, three of which, ^{20}Ne, ^{21}Ne and ^{22}Ne, are stable. The radioisotope with the longest lifetime is ^{24}Ne, with a half-life of 3.38 min.

Among the most significant heavy isotopes are those of iron and uranium. Indeed, of all known nuclides, the most stable is that of iron isotope 56. In addition, the uranium-235 isotope is widely used in nuclear power plants for the production of electrical energy through the fission reaction. The stability data for iron and uranium isotopes are as follows.

– Iron (Fe) has 28 isotopes with mass numbers ranging from 45 to 72, as well as six nuclear isomers [^{52m}Fe: 45.9 (6) s, ^{53m}Fe: 2.526 (24) min, ^{54m}Fe: 364 (7) ns, ^{61m}Fe: 250 (10) ns, ^{65m}Fe: 430 (130) ns and ^{67m}Fe: 64 (17) μs]; the values given in square brackets correspond to the half-lives of the nuclear isomers. Of these isotopes, only ^{54}Fe, ^{56}Fe, ^{57}Fe and ^{58}Fe are stable. The lightest isotope, ^{45}Fe, decays mainly (70%) by two-proton emission into ^{43}Cr. The isotope ^{56}Fe is the most stable of all nuclei (see section 1.6.4).

– Uranium (U) has 26 isotopes, with mass numbers ranging from 217 to 242. It also has seven nuclear isomers [^{234m}U: 33.5 (20) ms; ^{235m}U: ~ 26 min; ^{238m}U: 280 (6) ns; ^{239m1}U: >250 ns; ^{239m2}U: 780 (40) ns; ^{236m1}U: 100 (4) ns; ^{236m2}U: 120 (2) ns]. Uranium is a natural radioactive element and therefore has no stable isotope. The two isotopes ^{235}U and ^{238}U, present in appreciable quantities in the Earth's crust with their decay products, have long half-lives. ^{235}U is the only fissile isotope.

The molar percentages of the most significant isotopes of the first ten elements of the periodic table and those of iron and uranium are presented in Table 1.2.

NOTE.– generally, an excited nucleus normally returns to the lowest energy ground state after a fraction of a second. However, the nuclear transition to ground level can, in exceptional cases, be inhibited and significantly slowed down. The nucleus then remains in an abnormally long excited state. Such excited states are called *metastable states*. *Nuclear isomerism* is introduced to account for the existence of nuclei in different energy states. The term *nuclear isomers* describes the isomers of a nucleus that are all in a metastable state. The nuclear isomers are noted by attaching the letter "m" for "metastable" to the ^{A}X isotope being considered. Thus, a nuclear isotope is noted ^{Am}X. This is the case with the nucleus ^{11m}N of the isotope ^{11}N of nitrogen. A particularly significant example of a nuclear isomer in nuclear medicine is technetium^{99m}Tc (half-life 6.0058 hours) that remains several hours before returning to the normal state of technetium-99. Technetium-99m is obtained in hospitals from a precursor radioactive nucleus, molybdenum ^{99}Mo, generally produced in a reactor and is very useful in nuclear medicine (see Chapter 3, Volume 2).

Isotope	Percentage		Isotope	Percentage
${}^{1}_{1}H$	99.9885%		${}^{16}_{8}O$	99.757%
${}^{2}_{1}H$	0.00115%		${}^{17}_{8}O$	0.038%
${}^{3}_{1}H$	Traces		${}^{18}_{8}O$	0.205%
${}^{3}_{2}He$	0.000134%		${}^{19}_{9}F$	100%*
${}^{4}_{2}He$	99.999866%		${}^{20}_{10}Ne$	90.48%
${}^{6}_{3}Li$	7.59%		${}^{21}_{10}Ne$	0.27%
${}^{7}_{3}Li$	92.41%		${}^{22}_{10}Ne$	9.25%
${}^{9}_{4}Be$	100%*		${}^{57}_{26}Fe$	5.845%
${}^{10}_{5}B$	19.9%		${}^{56}_{26}Fe$	91.754%
${}^{11}_{5}B$	80.1%		${}^{57}_{26}Fe$	2.119%
${}^{12}_{6}C$	98.93%		${}^{58}_{26}Fe$	0.282%
${}^{13}_{6}C$	1.07%		${}^{60}_{26}Fe$	traces
${}^{14}_{6}C$	traces		${}^{234}_{92}U$	0.0050 to 0.0059%
${}^{14}_{7}N$	99.636%		${}^{235}_{92}U$	0.7198 to 0.7202%
${}^{15}_{7}N$	0.364%		${}^{238}_{92}U$	99.2739 to 99.2752%

Table 1.2. *Molar percentages of the most significant isotopes of the first ten elements of the periodic table, and those of iron and uranium. The asterisk (*) refers to the monoisotopic elements being the only stable elements*

With the discovery of isotopes, *mass spectrometry* saw the light of day for isotopic analysis. Thus, instruments called *mass spectrographs* were manufactured, which allowed the *separation of isotopes* of a given substance according to their masses [EVA 61, OLS 91, SAK 16]. The principle of a mass spectrograph involves subjecting the isotope ions of a given chemical to the action of electric and magnetic fields. The electric field makes it possible to accelerate the ions while the magnetic field enables their trajectories to be bent. In the magnetic field, the lower the ion's velocity, the more its trajectory is curved. In addition, the radius of curvature of the trajectory depends on the mass-to-charge ratio of the ion. The ions are then received on a photographic plate. Measuring the distance between the impact points of two ions on the plate enables them to be identified.

Frederick Soddy was a British radiochemist. In 1902 he established, together with Rutherford, the exponential law of radioactive decay. Soddy was awarded the Nobel Prize in Chemistry 1921 for his contributions on radioactive substances and for his research on the nature of isotopes.

Box 1.10. *Soddy (1877-1956)*

1.5.2. *Mirror nuclei, Magic nuclei*

Nuclei known as *mirror nuclei* are used to study neutron/proton symmetry. These are pairs of nuclei with the same mass number, A, and where the proton number, Z, in one equals the neutron number, N, in the other. Thus, for mirror nuclei, the relationship $Z - N = \pm 1$ is confirmed. For example, ${}^{7}_{3}Li$ $(Z - N = -1)$ and ${}^{7}_{4}Be$ $(Z - N = +1)$.

In addition, there exist particularly stable nuclei called "magic nuclei". For a magic nucleus, the number of protons, Z, or neutrons, N, corresponds to complete *nucleon shells* (see Chapter 2, section 2.3). For such a nucleus, Z or N is thus a magic number. The magic numbers are: 2, 8, 20, 28, 50, 82 and 126. For example, ${}^{4}_{2}He$ $(Z = 2; N = 2)$, ${}^{40}_{20}Ca$ $(Z = 20; N = 20)$, ${}^{208}_{82}Pb$ $(Z = 82; N = 126)$. Note that these nuclei are also said to be *doubly-magic nuclei* since Z and N are all magic numbers.

APPLICATION 1.12.– Determine the isospin of mirror nuclei. Conclude.

ANSWER.– For mirror nuclei, $Z - N = \pm 1 \Rightarrow N - Z = \pm 1$. Using [1.29], we obtain: $T_z = \pm 1/2$. The isospin of mirror nuclei is therefore equal to $T = 1/2$.

CONCLUSION.– Mirror nuclei have the same isospin.

1.6. Nucleus stability

1.6.1. *Atomic mass unit*

The *mass-energy equivalence* relationship reflects the fact that mass is a form of energy. It is expressed by the relationship $E = mc^2$ with m the relativistic mass of the particle, c is velocity of light in a vacuum; $E_0 = m_0c^2$ is called the *mass energy*, or the *resting energy*, of the particle of *resting mass*, m_0.

As shown in Table 1.1, the mass of a nucleon is of the order of 10^{-27} kg. This mass, expressed in kg, is therefore very low. To have an appreciable mass, let us introduce a unit (more convenient than kg for calculations), called the atomic mass unit, noted *u*. By definition, the *atomic mass unit* is a twelfth of the mass of the carbon isotope ^{12}C, i.e. [SAK 16]:

$$1\text{u} = \frac{1}{12}\, m\,(^{12}\text{C}) \qquad [1.51]$$

Yet, $m\,(^{12}\text{C}) = M\,(\text{C})/N_\text{A} = 12/N_\text{A}$ with N_A Avogadro's number. Hence [1.51] is written:

$$1\text{u} = \frac{1}{N_A}\ (\text{g}) \qquad [1.52]$$

Knowing that $N_\text{A} = 6.02214179 \times 10^{23}$ mol $^{-1}$, then numerically:

$$1\text{u} = 1.660538783 \times 10^{-27}\ \text{kg} \qquad [1.53]$$

As result [1.53] indicates, the value of the atomic unit expressed in kg is of the order of the mass of a nucleon. Another much more convenient unit of mass is then introduced: MeV/c^2.

Applying the mass-energy equivalence relationship gives:

$$E = mc^2 = \text{u}c^2 \qquad [1.54]$$

Given that $c = 299792458\ \text{m} \cdot \text{s}^{-1}$ and $1\ \text{eV} = 1.6021794 \times 10^{-19}$ J, using [1.54] and value [1.53], we obtain:

$$E = \frac{1.660538783 \times 10^{-27} \times (299792458)^2}{1.6021794 \times 10^{-19}} = 931.4923\ \text{MeV} \qquad [1.55]$$

Hence:

$$1\text{u} = 931.5\ \text{MeV}/c^2 \qquad [1.56]$$

Table 1.3 summarizes the nucleon and electron masses.

Particles	Mass in atomic unit (u)	Mass in MeV/c^2
Electron	$5.485\,799\,094\,3 \times 10^{-4}$	0.510999910
Neutron	1.00866491597	939.565346
Proton	1.00727646677	938.272013

Table 1.3. *Electron and nucleon masses in u and MeV/c^2*

1.6.2. *Segrè diagram, nuclear energy surface*

To study the stability of the nuclei, it is convenient to represent each isotope with a point on a diagram (Z, N), with abscissa Z and ordinate N. The resulting graph (Figure 1.18) is called the *Segrè diagram*.

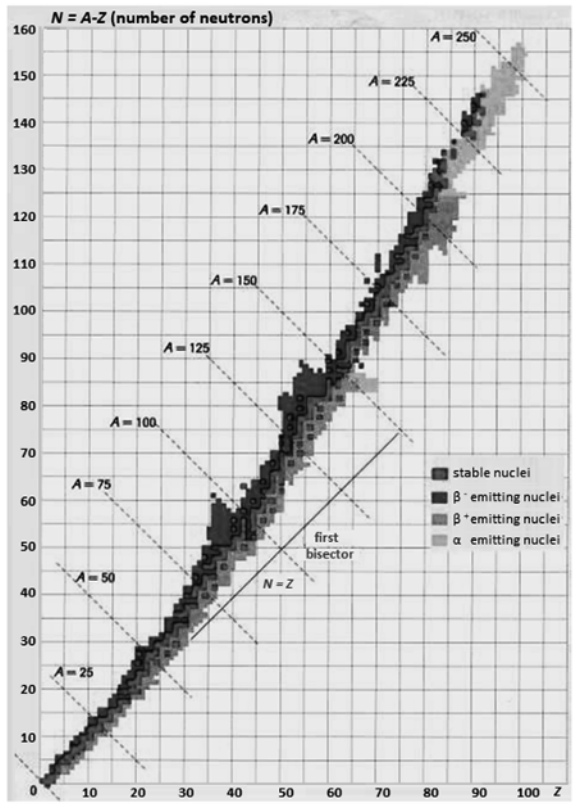


Figure 1.18. *Segrè diagram, the nuclear energy surface indicated in red, grouping together the stable nuclei for which Z = N*

This diagram reveals the existence of four nuclide distribution zones according to the values of Z and N:

– a red area, called the *nuclear energy surface*, or *valley of stability,* where the most stable nuclei are distributed; for $Z < 30$, note that the stable nuclei are located near the first $N = Z$ bisector;

– an orange area occupied by unstable heavy nuclei that decay by α radioactivity;

– a blue area occupied by nuclei presenting excess neutrons compared to stable nuclei with the same mass number, A; they decay by β^- radioactivity;

– a green area occupied by nuclei presenting excess protons compared to stable nuclei with the same mass number, A; they decay by β^+ radioactivity.

The modes of α and β decay are discussed in Chapter 3.

Emilio Gino Segrè was an Italian-American physicist. In 1936 he discovered technetium, the first artificial element, then astatine in 1940 and later plutonium-239, whose fissionable character he demonstrated (^{239}Pu would be used in the first atomic bomb [FER 64]). In 1945, Segrè established a map of nuclides to describe several of their properties graphically, marking them on a system of N/Z axes. He shared the Nobel Prize in Physics 1959 with American physicist Owen Chamberlain (1920-2006) for their discovery of the antiproton (negative proton).

Box 1.11. *Segrè (1905–1989)*

1.6.3. *Mass defect, binding energy*

Let us compare, as an illustrative example, the sum of the proton and neutron masses separated at infinity and the rest to that of *deuteron* formed by the bound system {neutron-proton}. Using Table 1.3, we can derive the sum of the mass, m_p, of the proton and m_n of the neutron: $m_p + m_n = 2.015941$ u. Knowing that the mass of deuteron, $m_d = 2.013553$ u, we can thus see that: $(m_p + m_n) > m_d$. The deuteron nucleus lacks one mass:

$$m = (m_p + m_n) - m_d = 0.002385 \text{ u} > 0 \qquad [1.57]$$

The missing mass [1.57] of the nucleus of deuteron is called a *mass defect*, counted as positive. By applying the mass-energy equivalence relationship, the missing mass is not lost. It is converted into *binding energy* thus ensuring the cohesion of the deuteron.

Generally, the mass defect of a nucleus noted Δm is the difference between the mass of nucleons (protons and neutrons), separated at rest (infinity), and the mass of the nucleus formed at rest (Figure 1.19).

By definition, for a nucleus of mass $M(A, Z)$, mass number A, proton number Z and neutron number $N = (A - Z)$:

$$\Delta m = [Zm_p + (A - Z)m_n - M(A, Z)] \quad [1.58]$$

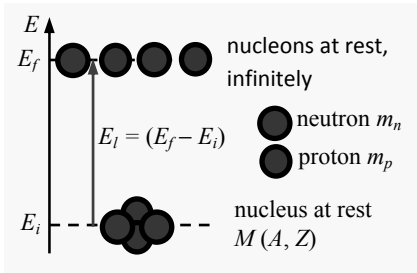


Figure 1.19. *Energy E_l necessary to separate, to infinity, the nucleons of a nucleus (example here: helium-4 nucleus)*

In addition, the binding energy, E_l, is the energy that must be provided to separate, at rest, to infinity, the nucleons of a nucleus as shown in Figure 1.18. Let us express the *principle of energy conservation* when separating nucleons:

– initial state (nucleus at rest): $E_i = M(A, Z)\, c^2$; [1.59]

– final state (nucleons at rest): $E_f = ZM_p c^2 + (A - Z)\, m_n c^2$.

Using [1.59], the energy variation is then written:

$$\Delta E = E_f - E_i = [Zm_p c^2 + (A - Z)\, m_n c^2] - M(A, Z)\, c^2$$

This relationship enables us to derive the expression for the binding energy, $E_l = \Delta E$, i.e.:

$$E_l = \{[Zm_p c^2 + (A - Z)\, m_n c^2] - M(A, Z)\}\, c^2 \quad [1.60]$$

Taking into account the mass defect [1.58] gives:

$$E_l = \Delta m c^2 \quad [1.61]$$

Relationship [1.61] reflects the mass-energy equivalence in the case of a nucleus. As the mass defect, Δm, is positive, then according to [1.61], the binding energy, E_l, is also positive. This sign is characteristic of a bound system such as the nucleus. Thus, for an unbound system, $\Delta m < 0 \Rightarrow E_l < 0$.

APPLICATION 1.13.– Determine the binding energy of deuteron, the α particle, the iron isotope ^{56}Fe (Z = 26; M_{Fe} = 55.9206 u), and the uranium-238 isotope, ^{238}U (Z = 92; M_U = 238.0003 u). Conclude.

We will use the data given in Table 1.3. In addition: m_α = 4.0015 u; 1u = 931.5 MeV/c^2.

ANSWER.–

– For deuteron, using [1.57] we obtain:

$$E_l\,(^2\text{H}) = \Delta m c^2 = 0.002385\ \text{u}c^2 = 0.002385 \times 931.5 = 2.22\ \text{MeV}$$

– For the α particle (helium nucleus):

$$E_l\,(^4\text{He}) = 2 \times [m_p + m_n]\, c^2 - m_\alpha c^2$$

NOTE.–

$$E_l\,(^4\text{He}) = (2 \times 2.015941 - 4.0015) \times 931.5 = 28.30\ \text{MeV}$$

– For the iron-56 nucleus and uranium, we obtain, as above:

$$E_l\,(^{56}\text{Fe}) = (26 \times 1.00727646677 + 30 \times 1.00866491597 - 55.9206) \times 931.5 = 492.331\ \text{MeV}$$

$$E_l\,(^{238}\text{U}) = (92 \times 1.00727646677 + 146 \times 1.00866491597 - 238.0003) \times 931.5 = 1801.719\ \text{MeV}$$

In summary:

$$E_l\,(^2\text{H}) = 2.22\ \text{MeV}; \qquad E_l\,(^4\text{He}) = 28.30\ \text{MeV} \qquad [1.62a]$$

$$E_l\,(^{56}\text{Fe}) = 492.331\ \text{MeV}; \; E_l\,(^{238}\text{U}) = 1801.719\ \text{MeV} \qquad [1.62b]$$

CONCLUSION.– the more nucleons there are in the nucleus, the greater the binding energy.

1.6.4. *Binding energy per nucleon, Aston curve*

As we found in application 1.13, the binding energy of a nucleus rises with the increase in the number of nucleons. However, experience shows that a nucleus is not more stable the higher its binding energy is. For this reason, binding energy is not a good indicator for comparing nucleus stability. To achieve this, let us define the *binding energy per nucleon*. We thus show that a nucleus is not more stable the higher its binding energy (E/A) is. The binding energy per nucleon is of the order of 8 MeV.

APPLICATION 1.14.– Verify by calculating which of the nuclei of iron-56 and uranium-238 is most stable.

ANSWER.– Using [1.62b], we obtain:

$$E_l\,(^{56}\text{Fe})/A = 492.331/56 = 8.79 \text{ MeV/nucleon} \qquad [1.63a]$$

$$E_l\,(^{238}\text{U})/\,A = 1801.719/238 = 7.57 \text{ MeV/nucleon} \qquad [1.63b]$$

$E_l\,(^{56}\text{Fe})/A > E_l\,(^{238}\text{U})/\,A$: the nucleus of iron-56 is more stable than that of uranium-238.

In the general case, let us look at the variation of the curve – $E_l/\text{A} = f\,(A)$. A valley-shape curve is obtained whose bottom is occupied by the nucleus of iron-56 with binding energy per nucleon equaling 8.8 MeV (Figure 1.20) in accordance with result [1.63a]. This graph, called the *Aston curve* shows that the nucleon of iron-56 is the most stable of all nuclei.

The shape of the Aston curve calls for four main observations:

– light nuclei such as deuteron ^{2}H and triton ^{3}H can fuse to give heavier stable nuclei located in the valley;

– the curve presents something of a plateau for $50 < A < 100$ with a minimum for $A = 56$. These nuclides correspond to the most stable nuclei;

– for $A > 100$, the curve increases slightly; the corresponding heavy nuclei are less stable;

– very heavy nuclei for $A > 200$ may undergo fission reactions to give lighter, more stable nuclei.

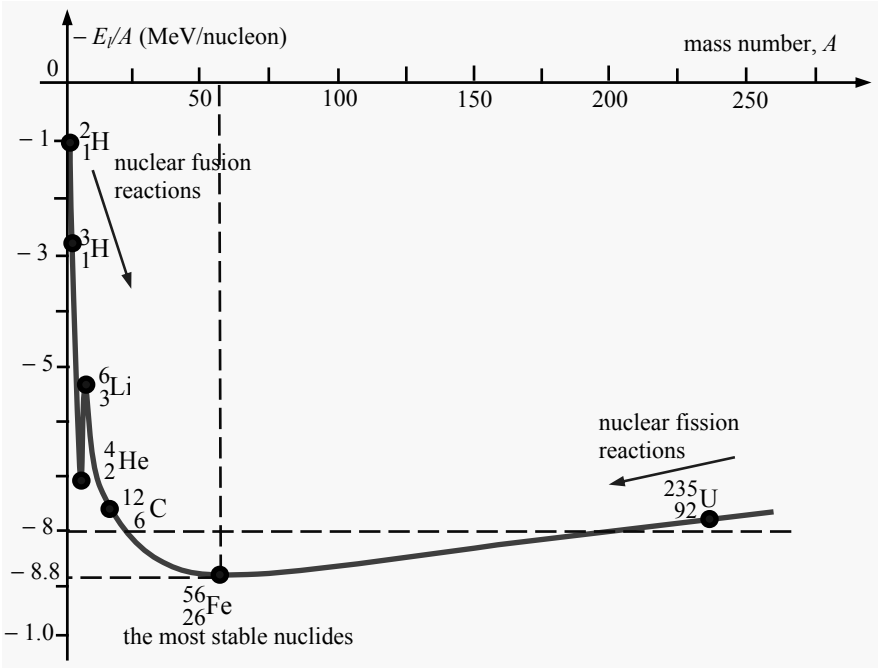


Figure 1.20. *Aston curve*

Thermonuclear fusion and nuclear fission reactions are nuclear reactions that are induced, i.e., not spontaneous. These reactions are therefore not studied in this book. However, as an illustration, let us give an example of a fusion reaction and another example of a fission reaction to account for the fact that the products of these induced reactions are more stable than the starting nuclides.

– *Fusion of deuteron* ^{2}H and triton ^{3}H:

$$^{2}_{1}H + {}^{3}_{1}H \rightarrow {}^{4}_{2}He + {}^{1}_{0}n \qquad [1.64]$$

– *Fission of uranium-235* under the impact of a neutron, *n*:

$$^{1}_{0}n + {}^{235}_{92}U \rightarrow {}^{94}_{38}Sr + {}^{140}_{54}Xe + 2\,{}^{1}_{0}n + \gamma \qquad [1.65]$$

The nucleus of helium ^{4}He obtained after the fusion reaction [1.64] is much more stable than deuteron ^{2}H and triton ^{3}H, in accordance with their positions on the Aston curve (Figure 1.20). Likewise, products of fission ^{94}Sr (strontium-94) and ^{140}Xe

(xenon-140) are more stable than the original ^{235}U uranium nucleus (see application 1.15).

APPLICATION 1.15.– Compare the stability of the nuclei of strontium-54, uranium-235 and xenon-140 using a horizontal axis. We will classify the nuclides in order of increasing stability. In addition, we will use the data given in Table 1.3.

Additional data: M (^{235}U) = 235.04392 u; M (^{94}Sr) = 93.91536 u; M (140 Xe) = 139.91879 u.

ANSWER.– Simply compare the binding energy per nucleon of each nuclide. Using [1.60], we obtain:

– E_l (^{94}Sr) = (38 × 1.00727646677 + 56 × 1.00866491597 – 93.91536) × 931.5 = 788.404 MeV;

– E_l (^{140}Xe) = (54 × 1.00727646677 + 86 × 1.00866491597 – 139.91536) × 931.5 = 1138.993 MeV;

– E_l (^{235}U) = (92 × 1.00727646677 + 143 × 1.00866491597 – 235.04392) × 931.5 = 1736.873 MeV.

The binding energies per nucleon are thus equal to:

$$E_l\,(^{94}Sr)/A = 788.404/94 = 8.4 \text{ MeV/nucleon} \quad [1.66a]$$

$$E_l\,(^{140}Xe)/A = 1138.993/140 = 8.1 \text{ MeV/nucleon} \quad [1.66b]$$

$$E_l\,(^{235}U)/A = 1736.873/235 = 7.4 \text{ MeV/nucleon} \quad [1.66c]$$

Considering results [1.66], the nuclides considered are classified in order of increasing binding energy per nucleon (Figure 1.21).

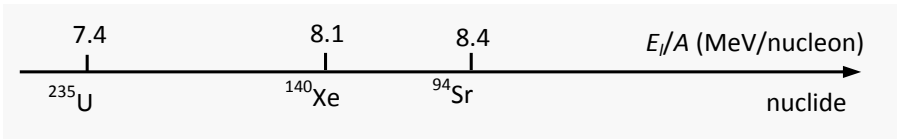


Figure 1.21. *Comparison of the stability of the nuclei of uranium-238, strontium-54 and xenon-140. The products of the fission reaction [1.65] are much more stable than the starting uranium-238*

Francis William Aston was a British chemist and physicist. He is famous for having developed the Aston spectrograph in 1919. The curve for locating the most stable nuclides in the valley of stability is named in his honor. He was awarded the Nobel Prize in Chemistry 1922 for his discovery, using his mass spectrometer, of isotopes of a large number of non-radioactive elements and for his formulation of the whole-number rule.

Box 1.12. *Aston (1877-1945)*

1.6.5. *Separation energy of a nucleon*

The experimental study of the variation in the separation energy of a nucleon from a nucleus provides an idea of its cohesion. By definition, the *separation energy of a nucleon* is the energy required to remove a nucleon from the nucleus. Nucleon separation energies are analogous to the first-ionization potential of neutral atoms (extraction of an electron).

For a nucleus of mass $M(Z, N)$, an isotope of mass $M(Z, N-1)$ is obtained if a neutron is removed, and an isotone of mass $M(Z-1, N)$ if a proton is removed. By convention, $S_n(Z, N)$ is used to designate the *separation energy of a neutron* and $S_p(Z, N)$ designates the *separation energy of a proton*. These energies are given by the following respective expressions:

$$S_n(Z, N) = [M(Z, N-1) + m_n - M(Z, N)]c^2 \qquad [1.67a]$$

$$S_p(Z, N) = [M(Z-1, N) + m_p - M(Z, N)]c^2 \qquad [1.67b]$$

According to the binding energies, E_l, relationships [1.67] are written:

$$S_n(Z, N) = E_l(Z, N) - E_l(Z, N-1) \qquad [1.68a]$$

$$S_p(Z, N) = E_l(Z, N) - E_l(Z-1, N) \qquad [1.68b]$$

APPLICATION 1.16.– Demonstrate relationships [1.48].

ANSWER.– Let us consider the binding energy definition relationship [1.60] by noting the mass $M(A, Z)$ of the nucleus with $M(Z, N)$. Knowing that $N = A - Z$, we obtain:

$$E_l(Z, N) = \{[ZM_p + Nm_n] - \mathrm{M}(Z, N)\}c^2 \qquad [1.69]$$

For the isotope of mass $M(Z, N-1)$ corresponding to the removal of a neutron, the binding energy is calculated from relationship [1.69]. Thus:

$$E_l(Z, N-1) = \{[Zm_p + (N-1)\, m_n] - M(Z, N-1)\}c^2 \qquad [1.70a]$$

For the isotone of mass $M(Z-1, N)$ corresponding to the removal of a proton, we obtain:

$$E_l(Z-1, N) = \{[(Z-1)\, m_p + Nm_n] - M(Z-1, N)\}c^2 \qquad [1.70b]$$

By subtracting [1.69] from [1.70], we find, respectively:

$$E_l(Z, N) - E_l(Z, N-1) = \{M(Z, N-1) + m_n - M(Z, N)\}c^2 \qquad [1.71a]$$

$$E_l(Z, N) - E_l(Z-1, N) = \{M(Z-1, N) + m_p - M(Z, N)\}c^2 \qquad [1.71b]$$

By comparing relationships [1.67] and [1.71], relationships [1.68] are found.

As shown in equations [1.68], the separation energy of a neutron represents the increase in the binding energy, $E_l(Z, N-1)$ when a neutron is added to the smallest isotope $(Z, N-1)$, giving the isotope (Z, N) (equation [1.68a]). Similarly, the separation energy of a proton represents the increase in the binding energy, $E_l(Z-1, N)$ when a proton is added to the smallest isotone $(Z-1, N)$, giving the isotone (Z, N) (equation [1.68b]). For these reasons, the separation energies of a neutron, $S_n(Z, N)$ or a proton, $S_p(Z, N)$ are also referred to, respectively, as the *last neutron separation energy* (i.e. the last neutron added) or *last proton separation energy* (the last proton added).

APPLICATION 1.17.– Determine the separation energy of the last neutron for the ${}^{7}_{3}Li$ nucleus and the separation energy of the last proton for the ${}^{16}_{8}O$ nucleus. We will start by writing the nucleon extraction equations for the two cases considered.

Given data: m_n = 1.0087 u; m_p = 1.0073 u; $M(3.4)$ = 7.0143 u; $M(3.3)$ = 6.0135 u; $M(8.8)$ = 15.9905 u;

$M(7.8)$ = 14.9963 u; 1u = 931.5 MeV/c^2.

ANSWER.– For the extraction equations of a lithium-7 neutron and an oxygen-16 proton, we obtain:

– extraction of a lithium-7 neutron, the lithium-6 isotope is obtained:

$${}^{7}_{3}Li \xrightarrow{extraction} {}^{6}_{3}Li + {}^{1}_{0}n \qquad [1.72a]$$

– extraction of an oxygen-16 proton, the nitrogen-15 isotone is obtained:

$$^{16}_{8}O \xrightarrow{extraction} {}^{15}_{7}N + {}^{1}_{1}H \qquad [1.72b]$$

Using [1.67], we obtain:

$$S_n\,(3.4) = [M\,(3.3) + m_n - M\,(3.4)]c^2 \qquad [1.73a]$$

$$S_p\,(8.8) = [M\,(7.8) + m_p - M\,(8.8)]c^2 \qquad [1.73b]$$

Numerically, we thus obtain:

$$S_n\,(3.4) = [6.0135 + 1.0087 - 7.0143] \times 931.5 = 7.359 \text{ MeV} \approx 7.4 \text{ MeV}$$

$$S_p\,(3.4) = [14.9963 + 1.0073 - 15.9905] \times 931.5 = 2.203 \text{ MeV} \approx 12.2 \text{ MeV}$$

1.6.6. *Nuclear forces*

In principle, nuclei should be unstable due to Coulomb repulsion between protons. *Nuclear forces* are manifested at a very short range equaling around 2×10^{-13} cm). At this distance, they are attractive and thus mask the forces of Coulomb repulsion between protons of the same nucleus. This explains the stability of the nuclei. Nevertheless, for distances under than 5×10^{-14} cm, nuclear forces are repellent as if there is an impenetrable hard core of nucleons within the nucleus. As we saw in section 1.3.4, the introduction of isospin is based on the fundamental assumption that *pp*, *pn* or *nn* nuclear interaction is the same owing to the charge independence of nuclear forces. The cohesion of the nuclei is explained by the existence of so-called nuclear forces. These forces are attractive and independent of the electrical charge

1.7. Exercises

EXERCISE 1.1.– Lifetime of the Rutherford planetary atom

Let us account for the instability of the planetary atomic model according to Rutherford. For this purpose, let us consider the electron of the hydrogen atom in motion around the proton.

a) Show that the electron is subjected to a central acceleration.

b) According to the conceptions of classical electrodynamics, any charged particle animated by accelerated motion emits electromagnetic waves. The *loss of energy per unit time* (radiated power) is expressed by the relationship:

$$-\frac{dE}{dt}=\frac{2}{3}\frac{e^2}{4\pi\varepsilon_0 c^3}a^2 \qquad [1.74]$$

In [1.74], a designates the acceleration of the particle.

i) Is Rutherford's atomic model then stable according to the conceptions of classical electrodynamics? Substantiate the answer.

ii) Show that the radius, r, of the electron orbit confirms the differential equation:

$$\frac{d}{dr}\left(\frac{1}{r}\right)=\frac{A}{r^4} \qquad [1.75]$$

with A a constant that will be explained below.

iii) Considering the fall of the electron from its initial position, $r = a_0$, express the duration, Δt, of the falling motion of the electron on the proton. Deduce therefrom the lifetime of the hydrogen atom.

Given datum: *Bohr radius,* a_0= 52.9 pm.

EXERCISE 1.2.– Discovery of isotopes

In 1886, Goldstein discovered positively charged rays that he named "*canal rays*". Sir J.J. Thomson showed in 1912, using the device presented in Figure 1.22, that these "canal rays" were composed of ions of two neon isotopes.

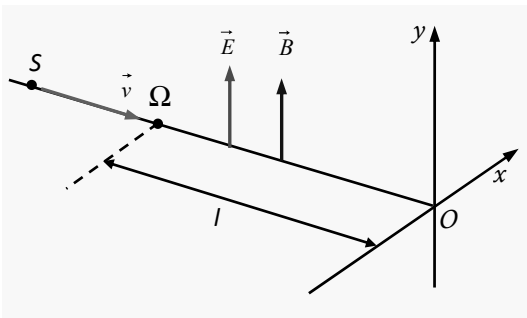


Figure 1.22. *Simplified Thomson device for identifying the canal rays discovered by Goldstein*

A beam consisting of positive ions moving along the *SO* axis, and different velocity vectors, $\vec{v}$, is subjected to the simultaneous actions of an electric field, $\vec{E}$, and a magnetic field, $\vec{B}$, which are parallel and facing the same way (in the *Oy* axis direction). Let $l = \Omega O$ be the width of the portion of space on which both fields are active.

a) Express the vertical deviation, y, of an ion of charge q, velocity v and mass m, subjected to the action of the electric field, $\vec{E}$, alone.

b) Similarly, express the horizontal deviation, x, of the same ion, subjected to the action of the magnetic field, $\vec{B}$, alone. We will assume that the deviation angle is small.

c) Under the conditions of the experiment, the x and y coordinates of the particle when the $\vec{E}$ and $\vec{B}$ fields are started up simultaneously are virtually equal to those obtained when they act alone. Establish the Cartesian equation of the trace formed on the screen by the impacts of ions of the same charge and mass but of different velocities. What shape is this trace?

d) What happens if the beam consists of ions of different mass charges, q/m?

e) In the case where the ion source is neon, two arcs of parabola can be observed on the screen, the extensions of which are tangent at *O* to the *Ox* axis.

i) Why are the parabolas not drawn up to point *O*?

ii) On the first parabola, note the Cartesian coordinates of a point, M_1: x_1 = 12.0 mm; y_1 = 4.5 mm. On the second parabola, the coordinates of a point M_2 are noted: x_2 = 10.9 mm; y_2 = 4.1 mm. Show that these are the traces of the two isotopes, ${}^{20}_{10}Ne^{+}$ and ${}^{22}_{10}Ne^{+}$, of neon.

Given data:

– *atomic molecular mass of* ${}^{20}_{10}Ne^{+}$: 20×10^{-3} kg · mol $^{-1}$;

– *atomic molecular mass of* ${}^{22}_{10}Ne^{+}$: 22×10^{-3} kg · mol $^{-1}$.

Eugen Goldstein was a German physicist. He is known as one of the first to study discharge tubes and is famous for discovering anode rays, or canal rays. The positive ions produced in the discharge flow from the anode to the cathode in the opposite direction to the cathode rays. These ions, which arrive at one of the holes (canals) of the cathode, cannot reach it, because of their inertia. They thus form a "ray", the cross-section of which is that of the hole. In 1886, Goldstein called these positive rays "canal rays" because they appeared to pass through a canal.

Box 1.13. *Goldstein (1850–1930)*

EXERCISE 1.3.– Mass spectrograph

An ionization chamber (*IC*) produces $^{79}Br^-$ and $^{81}Br^-$ isotope ions of charge q and respective masses, m_1 and m_2. These ions penetrate with negligible velocity at O into a vacuum chamber (A) where they are accelerated by a constant voltage, U_0.

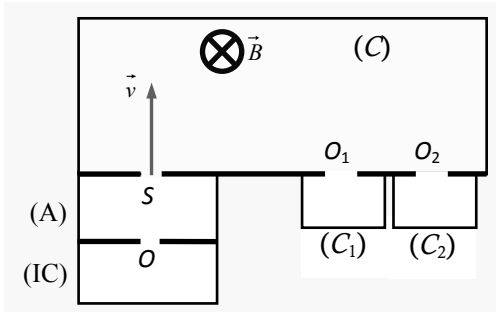


Figure 1.23. *Simplified diagram of a mass spectrograph*

They then exit at S with velocity $\vec{v}$ to penetrate into a magnetic deflector (C) where there is a uniform magnetic field $\vec{B}$ perpendicular to the plane of the figure. The two types of ions are then collected in the collectors (C_1) and (C_2), where they penetrate from orifices O_1 and O_2, respectively (Figure 1.23). All orifices are assumed to be point-like.

a) Express the input velocity, *v,* of an ion of mass m and charge q in the magnetic deflector according to m, q and U_0.

b) Show that the trajectory of an ion in the magnetic deflector is planar and circular. What, then, is the nature of the motion?

c) Identify by name the ion collected in the collector (C_1). Now plot the trajectories of the ions up to the two collectors.

d) Express the distance $D = O_1O_2$. Perform the numerical application.

e) Within one minute, the collectors (C_1) and (C_2) receive, respectively, the following amounts of electricity: $Q_1 = -\ 6.60 \times 10^{-8}$ C and $Q_2 = -\ 1.95 \times 10^{-8}$ C. Determine the composition of the isotope ion mixture.

Given numerical data:

– *absolute value of the ion charge*: $|q| = 1.6 \times 10^{-19}$ C;

– *applied voltage*: $|U_0| = 4$ kV;

– *ion masses*: $m_1 = 1.310\,4 \cdot 10^{-25}$ kg; $m_2 = 1.343\,6 \times 10^{-25}$ kg;

– *magnetic field value*: $B = 0.10$ T.

EXERCISE 1.4.– Separation of the uranium isotopes, ^{235}U and ^{238}U

Natural uranium essentially contains two isotopes of different masses: uranium-235 and uranium-238.

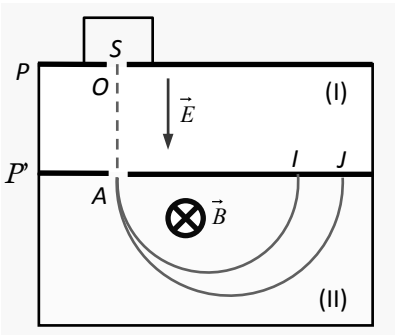


Figure 1.24. *Uranium isotope trajectories in a magnetic deflector*

One of the historical processes for separating these isotopes is based on the difference between the radii of curvature of the trajectories of ionized atoms in motion in a uniform magnetic field using a mass spectrograph. First, the neutral atoms are injected into an ion source where they each lose one electron. Thus, the ions formed in this way each carry the same electrical charge, $q = +e$. They exit the ion source with negligible velocity, then they are accelerated from O to A by a constant electrical voltage, $U_0 = V_P - V_{P'}$, applied between the two metal plates, P and P' (Figure 1.24). The ions then penetrate at A with a velocity of direction perpendicular to the plates in a region where there is a uniform magnetic field, $\vec{B}$, perpendicular to the direction of the electric field, $\vec{E}$, created between the two plates.

a) The ions of an isotope i ($i = 1, 2$) penetrate into the magnetic deflector with a velocity of v_I. Express values v_1 and v_2 of the respective velocities of the uranium 235 and 238 ions according to the charge, the mass of the corresponding ion and the accelerating voltage. Calculate v_1 and v_2.

b) Beyond point A, the ions travel with circular trajectories located in the plane of Figure 1.23. R_1 designates the radius of trajectory AI and R_2 the radius of

trajectory *AJ*. Calculate R_1 and R_2 and express the distance $l = IJ$ between the two traces *I* and *J* of the isotope ions. Calculate *l*.

c) The ion current from the source corresponds to an intensity of 10 μA. Knowing that natural uranium contains, in number of atoms, 0.7% light isotope (the only fissile), what is the mass, in μg, of this isotope collected in 24 hrs?

Given data:

– $B = 0.10$ T; $U_0 = 4$ kV; $e = 1.6 \times 10^{-19}$ C;

– *atomic mass unit* (u): 1 u = 1.66×10^{-27} kg;

– *Avogadro's number*: $N_A = 6.02 \times 10^{23}$ mol $^{-1}$;

– *mass of the uranium*-235 *ion*: $m_1 = 235$ u;

– *mass of the uranium*-238 *ion*: $m_2 = 238$ u.

1.8. Solutions to exercises

SOLUTION 1.1.– Lifetime of the Rutherford planetary atom

a) *Nature of electron acceleration*

In the Frenet frame $(M, \vec{\tau}, \vec{n})$, a particle in circular motion with the velocity $\vec{v}$ is subjected to the acceleration given by the relationship:

$$\vec{a} = \frac{d\vec{v}}{dt}\vec{\tau} + \frac{v^2}{\rho}\vec{n} \qquad [1.76]$$

In relation [1.76], ρ is the radius of curvature of the trajectory.

The motion of the electron of the hydrogen atom is uniform, therefore v = const. By writing $\rho = R$, we obtain, according to [1.76]:

$$\vec{a} = \vec{a}_n = \frac{v^2}{R}\vec{n} \qquad [1.77]$$

b) *Planetary atomic model*

i) Stability

According to the conceptions of classical electrodynamics, the electron of the hydrogen atom subjected to central or centripetal acceleration [1.77] radiates energy.

This loss of energy through radiation induces a falling motion in the electron that eventually falls on the nucleus. Yet this does not happen because the hydrogen atom exists as a stable bound system: the Rutherford atomic model is therefore unstable.

ii) Demonstration

The total mechanical energy of the bound system {proton – electron} constituting the hydrogen atom is given by the relationship:

$$E(r) = -k\frac{e^2}{2r} \qquad [1.78]$$

Furthermore, the electric force between the proton and the electron being a central force in $1/r^2$, the kinetic energy $E_c(r) = -E(r)$, thus, by using:

$$E_c(r) = \frac{1}{2}mv^2 = k\frac{e^2}{2r} \qquad [1.79]$$

By deriving v from [1.79], we obtain the relationship:

$$v^4 = k^2\frac{e^4}{m^2r^2} \qquad [1.80]$$

Given that acceleration $a = a_n = v^2/r$, the radiated power [1.74] is written, taking into account [1.80]:

$$-\frac{dE(r)}{dt} = \frac{2}{3}\times\frac{ke^2}{c^3}\frac{v^4}{r^2} = \frac{2}{3}\times\frac{k^3e^6}{m^2c^3}\times\frac{1}{r^4} \qquad [1.81]$$

If we then derive [1.78] with respect to time and then take into account [1.81], we obtain:

$$\frac{dE(r)}{dt} = -\frac{ke^2}{2}\frac{d}{dt}\left(\frac{1}{r}\right) = \frac{2}{3}\times\frac{k^3e^6}{m^2c^3}\times\frac{1}{r^4}$$

Which then gives:

$$\frac{d}{dt}\left(\frac{1}{r}\right) = \frac{A}{r^4},\ A = \frac{4}{3}\frac{k^2e^4}{m^2c^3} \qquad [1.82]$$

iii) Lifetime of the hydrogen atom

At instant $t_0 = 0$, the electron is at the distance $r = a_0$ from the proton. When it falls on the proton at instant t_c, then $r = 0$. By integrating [1.82] between the limits t_0, t_c and a_0 and 0, we obtain, using the expression of A, according to [1.82]:

$$\Delta t = A(t_c - t_0) = \frac{a_0^3}{3} \Rightarrow \Delta t = \frac{m^2 c^3 a_0^3}{4k^2 e^4} \qquad [1.83]$$

NOTE.–

$$\Delta t = \frac{(9.1\times10^{-31})^2 \times (3\times10^8)^3 \times (5.29\times10^{-11})^3}{4\times(9\times10^9)^2 \times (1.6\times10^{-19})^4} = 1.559\times10^{-11} \approx 1.6\times10^{-11} \text{ s.}$$

SOLUTION 1.2.– Discovery of isotopes

a) *Expression for vertical deviation*

The electric field acts alone. Drawing on the *theorem of the center of inertia*, we obtain:

$$\vec{F} = m\vec{a} = q\vec{E} \Rightarrow \begin{cases} \dfrac{d^2x}{dt^2} = 0 \\ \dfrac{d^2y}{dt^2} = \dfrac{qE}{m} \end{cases} \Rightarrow \begin{cases} x = vt \\ y = \dfrac{1}{2}\dfrac{qE}{m}t^2 \end{cases} \qquad [1.84]$$

On the screen , $x = l \Rightarrow t = l/v$. We obtain, according to [1.84]:

$$y = \frac{1}{2}\frac{qEl^2}{mv^2} \qquad [1.85]$$

b) Expression for horizontal deviation

The magnetic field acts alone. Likewise we obtain:

$$\vec{F} = m\vec{a} = q\vec{v} \wedge \vec{B} \Rightarrow \begin{cases} \dfrac{d^2y}{dt^2} = 0 \\ \dfrac{d^2x}{dt^2} = \dfrac{qvB}{m} \end{cases} \Rightarrow \begin{cases} y = vt \\ x = \dfrac{1}{2}\dfrac{qvB}{m}t^2 \end{cases} \qquad [1.86]$$

On the screen , $y = l \Rightarrow t = l/v$. Using [1.86], we obtain:

$$y = \frac{1}{2}\frac{qEl^2}{mv^2} \qquad [1.87]$$

c) Cartesian equation, the nature of the trace formed on the screen

Using [1.85] and [1.87] we obtain:

$$y = 2\frac{m}{q}\frac{E}{mB^2l^2}x^2 \qquad [1.88]$$

The trace is a parabola of vertex O.

d) Case of an ion beam of different mass charges

For ions of different mass/charge ratios, q/m, separate parabolas are obtained on the screen: the isotopes are then separated.

e) Observation on the screen

i) Explanation

Since the velocities of the ions are low, they are deflected before arriving at O. The impact points of ions of the same charge and mass but of different velocities are then distributed over two arcs of parabola whose extensions are tangent at O to the Ox axis.

ii) Isotope identification

For an ion (i) we obtain, according to [1.88]:

$$\frac{q}{m_i} = 2\frac{E}{B^2l^2}\frac{x_i^2}{y_i} \qquad [1.89]$$

Using [1.89], we then obtain:

$$\frac{m_2}{m_1} = \frac{x_1^2}{x_2^2}\frac{y_2}{y_1} \qquad [1.90]$$

Experimental results: in M_1: $x_1 = 12.0$ mm; $y_1 = 4.5$ mm; in M_2: $x_2 = 10.9$ mm; $y_2 = 4.1$ mm. We then find, numerically, according to [1.90]:

$$\begin{cases} \dfrac{m_2}{m_1} = \dfrac{12^2}{10.9^2}\dfrac{4.1}{4.5} = 1.104 \\ \dfrac{m_2}{m_1} = \dfrac{A_2 u}{A_1 u} = \dfrac{22}{20} = 1.100 \end{cases} \qquad [1.91]$$

The results [1.91] show that the isotopes are those of neon, $^{20}Ne^+$ and $^{22}Ne^+$.

SOLUTION 1.3.– Mass spectrograph

a) Expression for input velocity

Let us apply the *theorem of kinetic energy*. We obtain:

$$\frac{1}{2}mv^2 = |q|U_0 \Rightarrow v = \sqrt{\frac{2|q|U_0}{m}} \qquad [1.92]$$

b) Demonstration

– Nature of the trajectory

Let us consider a portion of the trajectory of an ion of charge q in the Frenet frame (Figure 1.25).

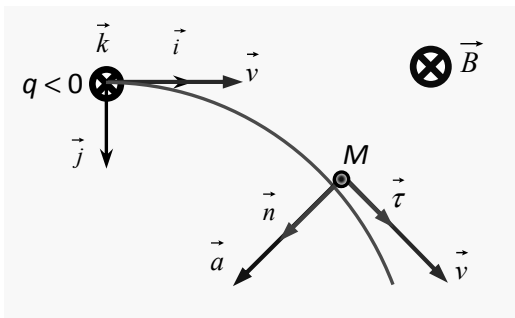


Figure 1.25. *Portion of the trajectory of a particle of charge q in motion in a magnetic deflector*

Let us determine the power developed by the *Lorentz force* acting on the particle. Taking into account the *properties of the vector product*, we obtain:

$$P = \vec{F} \cdot \vec{v} = (q\vec{v} \wedge \vec{B}) \cdot \vec{v} = 0 \qquad [1.93]$$

Using [1.93], the kinetic energy theorem gives:

$$\Delta E_c = W(\vec{F}) = P\Delta t = 0 \Rightarrow v = \text{constant} \qquad [1.94]$$

The motion of an ion in the magnetic deflector is therefore uniform according to [1.94]. Let us show that the trajectory described by the ion is planar.

Let us determine the scalar product $\vec{a} \cdot \vec{k}$ in the Frenet frame. Knowing that $\vec{B} = B \cdot \vec{k}$ (see Figure 1.25), we obtain, by deriving the acceleration from the first of relationships [1.86]:

$$\vec{a} \cdot \vec{k} = \left(\frac{q}{m} \vec{v} \wedge \vec{B} \right) \cdot \vec{k} = 0 \qquad [1.95]$$

Knowing that $a_z = 0$ according to [1.95] and that at instant $t = 0$, the particle is at the origin point, O ($z_0 = 0$, $v_{0Z} = 0$), then the time equation of the motion along Oz is written:

$$z(t) = \frac{1}{2} a_z t^2 + v_{0z} t + z_0 = 0 \qquad [1.96]$$

The z coordinate = 0, the particle motion occurs in the xOy plane, orthogonal to the magnetic field; the trajectory described by the particle is therefore planar.

– *Nature of the motion*

Using expression [1.77] for the acceleration and the first of relationships [1.86], we obtain:

$$R = \frac{mv}{|q|B} = \text{Constant} \qquad [1.97]$$

The velocity [1.94] and the radius of curvature [1.97] are constant: the motion of an ion in the magnetic deflector is uniform circular motion.

c) Identification of the ion collected in the collector (C_1)

According to [1.97], the ion collected in the collector (C_1) has a trajectory with the smallest radius and therefore with the smallest mass, $m = A$u: it is $^{79}Br^-$ and $^{81}Br^-$. The shape of the trajectories of the two ions is shown in Figure 1.26.

d) Expression of the distance, D, numerical application

The distance $D = O_1O_2 = 2(R_2 - R_1)$. Thus, using [1.97] and [1.92]:

$$D = \frac{2}{eB}(m_2v_2 - m_1v_1) = \frac{2}{B}\sqrt{\frac{2|U_0|}{e}}\left(\sqrt{m_2} - \sqrt{m_1}\right) \qquad [1.98]$$

NOTE.–

$$D = \frac{2}{0.1}\sqrt{\frac{2\times4\times10^3}{1.6\times10^{-19}}}\left(\sqrt{1.3436\times10^{-25}} - \sqrt{1.3104\times10^{-25}}\right) = 0.0204 \text{ m}$$

Thus: $D = 20$ mm

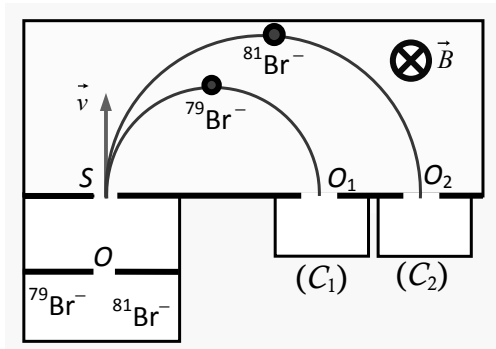


Figure 1.26. *Circular trajectories of isotope ions in the magnetic deflector*

e) Composition of the isotope ion mixture

In one minute, the collectors (C_1) and (C_2) receive the amounts of electricity, Q_1 and Q_2, respectively. The total charge, $Q = |Q_1| + |Q_2|$. This then gives:

$Q_1 = -6.60 \times 10^{-8}$ C; $Q_2 = -1.95 \times 10^{-8}$ C $\Rightarrow Q = 8.55 \times 10^{-8}$ C.

$^{79}Br^-$: $x_1 = |Q_1|/Q = 77.19\%$; $^{80}Br^-$: $x_2 = |Q_2|/Q = 22.81\%$

SOLUTION 1.4.– Separation of the uranium isotopes, ^{235}U and ^{238}U.

a) Velocity expressions and values

Drawing on the theorem of kinetic energy, we obtain:

$$m_2 v_2^2 = m_2 v_2^2 = 2qU_0 \Rightarrow v_i = \sqrt{\frac{2qU_0}{m_i}} = \sqrt{\frac{2eU_0}{A_i u}} \qquad [1.99]$$

NOTE.– ^{235}U: $v_1 = 5.73 \times 10^4$ m · s $^{-1}$; ^{238}U: $v_2 = 5.69 \times 10^4$ m · s $^{-1}$

b) Radius values: expression and value of the length between traces

– Radius value

The radius of curvature of the trajectory of an ion is given by [1.97]. Taking account of [1.99], for an ion, *i*, we obtain:

$$R = \frac{m_i v_i}{|q|B} = \frac{1}{B}\sqrt{\frac{2U_0 A_i u}{e}} \qquad [1.100]$$

NOTE.– $^{235}U^+$: $R_1 = 1.397$ m; $^{238}U^+$: $R_1 = 1.406$ m.

– Value of the length between traces

The value of *l* is given by the expression [1.98], which we write in the form:

$$l = \frac{2}{B}\sqrt{\frac{2U_0 u}{e}}\left(\sqrt{A_2} - \sqrt{A_1}\right) \qquad [1.101]$$

NOTE.– $l \approx 18$ mm.

c) Mass of uranium-235 collected

Let *Q* be the amount of electricity that has circulated. Using *N* to designate the number of particles (here uranium-235 isotope ions), *n* for the number of moles of ions and N_A for Avogadro's number, the following relationships are obtained:

$$\begin{cases} Q = Ne = It \\ n = \dfrac{N}{N_A} = \dfrac{m}{A} \Rightarrow N = \dfrac{mN_A}{A} = \dfrac{It}{e} \end{cases} \quad [1.102]$$

Knowing that the ^{235}U isotope contributes 0.7% to the ion current, using [1.102], we obtain a mass:

$$m_1 = 0.7\,\% \frac{ItA_1}{eN_A} \quad [1.103]$$

NOTE.– $m_1 \approx 15$ µg.