READING

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Forward Markets and Contracts

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LEARNING OUTCOMES

Mastery	The candidate should be able to:
☐	**a.** explain how the value of a forward contract is determined at initiation, during the life of the contract, and at expiration;
☐	**b.** calculate and interpret the price and value of an equity forward contract, assuming dividends are paid either discretely or continuously;
☐	**c.** calculate and interpret the price and value of 1) a forward contract on a fixed-income security, 2) a forward rate agreement (FRA), and 3) a forward contract on a currency;
☐	**d.** evaluate credit risk in a forward contract, and explain how market value is a measure of exposure to a party in a forward contract.

INTRODUCTION

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OPTIONAL SEGMENT

Recall the definition of a forward contract: *A forward contract is an agreement between two parties in which one party, the buyer, agrees to buy from the other party, the seller, an underlying asset or other derivative, at a future date at a price established at the start of the contract.* Therefore, it is a commitment by two parties to engage in a transaction at a later date with the price set in advance. The buyer is often called the **long** and the seller is often called the **short**.[1] Although any two parties can agree on such a contract, in this book we are interested only in forward contracts that involve large corporations, financial institutions, nonprofit organizations, or governments.

As an example, a pension fund manager, anticipating the receipt of cash at a future date, might enter into a commitment to purchase a stock portfolio at a later date at a price agreed on today. By doing so, the manager's position is unaffected by any changes in the value of the stock portfolio between today and the date of the actual

1 The derivatives industry often uses nouns, verbs, adjectives, and adverbs as parts of speech other than what they are. Hence, words like *long* and *short* are used not as adjectives but as nouns.

investment in the stock portfolio. In this sense, the manager is hedged against an increase in stock prices until the cash is received and invested. The disadvantage of such a transaction is that the manager is also hedged against any decreases in stock prices. If stock prices fall between the time the commitment is established and the time the cash is received, the manager will regret having entered into the forward contract because the stock could have been acquired at a lower price. But that is the nature of a forward contract hedge: It locks in a price.

An important feature of a forward contract is that neither party pays any money at the start. The parties might require some collateral to minimize the risk of default, but we shall ignore this point. So keep in mind this very important aspect of forward contracts: *No money changes hands at the start.*

1.1 Delivery and Settlement of a Forward Contract

When a forward contract expires, there are two possible arrangements that can be used to settle the obligations of the parties. A deliverable forward contract stipulates that the long will pay the agreed-upon price to the short, who in turn will deliver the underlying asset to the long, a process called **delivery**. An alternative procedure, called **cash settlement**, permits the long and short to pay the net cash value of the position on the delivery date. For example, suppose two parties agree to a forward contract to deliver a zero-coupon bond at a price of \$98 per \$100 par. At the contract's expiration, suppose the underlying zero-coupon bond is selling at a price of \$98.25. The long is due to receive from the short an asset worth \$98.25, for which a payment to the short of \$98.00 is required. In a cash-settled forward contract, the short simply pays the long \$0.25. If the zero-coupon bond were selling for \$97.50, the long would pay the short \$0.50. Delivery of a zero-coupon bond is not a difficult thing to do, however, and cash-settled contracts are more commonly used in situations where delivery is impractical.[2] For example, if the underlying is the Russell 3000 Index, the short would have to deliver to the long a portfolio containing each of the Russell 3000 stocks proportionate to its weighting in the index. Consequently, cash settlement is much more practical. Cash-settled forward contracts are sometimes **nondeliverable forwards (NDFs)**, although this term is used predominately with respect to foreign exchange forwards.

1.2 Default Risk and Forward Contracts

An important characteristic of forward contracts is that they are subject to default. Regardless of whether the contract is for delivery or cash settlement, the potential exists for a party to default. In the zero-coupon bond example above, the long might be unable to pay the \$98 or the short might be unable to buy the zero-coupon bond and make delivery of the bond to the long. Generally speaking, however, forward contracts are structured so that only the party owing the greater amount can default. In other words, if the short is obligated to deliver a zero-coupon bond selling for more than \$98, then the long would not be obligated to make payment unless the short makes delivery. Likewise, in a cash settled contract, only one party—the one owing the greater amount—can default. We discuss the nature of this credit risk in the following section and in Section 5 after we have determined how to value forward contracts.

2 Be aware, however, that the choice of delivery or cash settlement is not an option available at expiration. It is negotiated between the parties at the start.

1.3 Termination of a Forward Contract

Let us note that a forward contract is nearly always constructed with the idea that the participants will hold on to their positions until the contract expires and either engage in delivery of the asset or settle the cash equivalent, as required in the specific contract. The possibility exists, however, that at least one of the participants might wish to terminate the position prior to expiration. For example, suppose a party goes long, meaning that she agrees to buy the asset at the expiration date at the price agreed on at the start, but she subsequently decides to terminate the contract before expiration. We shall assume that the contract calls for delivery rather than cash settlement at expiration.

To see the details of the contract termination, suppose it is part of the way through the life of the contract, and the long decides that she no longer wishes to buy the asset at expiration. She can then re-enter the market and create a new forward contract expiring at the same time as the original forward contract, taking the position of the seller instead. Because of price changes in the market during the period since the original contract was created, this new contract would likely have a different price at which she would have to commit to sell. She would then be long a contract to buy the asset at expiration at one price and short a contract to sell the asset at expiration at a different price. It should be apparent that she has no further exposure to the price of the asset.

For example, suppose she is long to buy at $40 and short to deliver at $42. Depending on the characteristics of the contract, one of several possibilities could occur at expiration. Everything could go as planned—the party holding the short position of the contract on which she is long at $40 delivers the asset to her, and she pays him $40. She then delivers the asset to the party who is long the contract on which she is short at $42. That party pays her $42. She nets $2. The transaction is over.

There is always a possibility that her counterparty on the long contract could default. She is still obligated to deliver the asset on the short contract, for which she will receive $42. But if her counterparty on the long contract defaults, she has to buy the asset in the market and could suffer a significant loss. There is also a possibility that the counterparty on her short contract could fail to pay her the $42. Of course, she would then not deliver the asset but would be exposed to the risk of changes in the asset's price. This type of problem illustrates the credit risk in a forward contract. We shall cover credit risk in more detail in Section 5 of this reading.

To avoid the credit risk, when she re-enters the market to go short the forward contract, she could contact the same counterparty with whom she engaged in the long forward contract. They could agree to cancel both contracts. Because she would be owed $2 at expiration, cancellation of the contract would result in the counterparty paying her the present value of $2. This termination or offset of the original forward position is clearly desirable for both counterparties because it eliminates the credit risk.[3] It is always possible, however, that she might receive a better price from another counterparty. If that price is sufficiently attractive and she does not perceive the credit risk to be too high, she may choose to deal with the other counterparty and leave the credit risk in the picture.

3 This statement is made under the assumption that the parties do not want the credit risk. Credit risk, like other risks, however, can be a risk that some parties want because of the potential for earning attractive returns by using their expertise in measuring the actual credit risk relative to the credit risk as perceived by the market. In addition, credit risk offers diversification benefits.

2 THE STRUCTURE OF GLOBAL FORWARD MARKETS

The global market for forward contracts is part of a vast network of financial institutions that make markets in these instruments as well as in other related derivatives, such as swaps and options. Some dealers specialize in certain markets and contracts, such as forward contracts on the euro or forward contracts on Japanese equity products. These dealers are mainly large global banking institutions, but many large non-banking institutions, such as Goldman Sachs and Merrill Lynch, are also big players in this market.

Dealers engage in transactions with two types of parties: end users and other dealers. An end user is typically a corporation, nonprofit organization, or government.[4] An end user is generally a party with a risk management problem that is searching for a dealer to provide it with a financial transaction to solve that problem. Although the problem could simply be that the party wants to take a position in anticipation of a market move, more commonly the end user has a risk it wants to reduce or eliminate.

As an example, Hoffman-LaRoche, the large Swiss pharmaceutical company, sells its products globally. Anticipating the receipt of a large amount of cash in US dollars and worried about a decrease in the value of the dollar relative to the Swiss franc, it could buy a forward contract to sell the dollar and buy Swiss francs. It might seek out a dealer such as UBS Warburg, the investment firm affiliated with the large Swiss bank UBS, or it might approach any of the other large multinational banks with which it does business. Or it might end up dealing with a non-bank entity, like Merrill Lynch. Assume that Hoffman-LaRoche enters into this contract with UBS Warburg. Hoffman-LaRoche is the end user; UBS Warburg is the dealer.

Transactions in forward contracts typically are conducted over the phone. Each dealer has a quote desk, whose phone number is well known to the major participants in the market. If a party wishes to conduct a transaction, it simply phones the dealer for a quote. The dealer stands ready to take either side of the transaction, quoting a bid and an ask price or rate. The bid is the price at which the dealer is willing to pay for the future purchase of the asset, and the ask is the price at which the dealer is willing to sell. When a dealer engages in a forward transaction, it has then taken on risk from the other party. For example, in the aforementioned transaction of Hoffman-LaRoche and UBS Warburg, by entering into the contract, UBS Warburg takes on a risk that Hoffman-LaRoche has eliminated. Specifically, UBS Warburg has now committed to buying dollars and selling Swiss francs at a future date. Thus, UBS Warburg is effectively long the dollar and stands to gain from a strengthening dollar/weakening Swiss franc. Typically dealers do not want to hold this exposure. Rather, they find another party to offset the exposure with another derivative or spot transaction. Thus, UBS Warburg is a wholesaler of risk—buying it, selling it, and trying to earn a profit off the spread between its buying price and selling price.

One might reasonably wonder why Hoffman-LaRoche could not avoid the cost of dealing with UBS Warburg. In some cases, it might be able to. It might be aware of another party with the exact opposite needs, but such a situation is rare. The market for financial products such as forward contracts is made up of wholesalers of risk management products who use their technical expertise, their vast network of contacts, and their access to critical financial market information to provide a more efficient means for end users to engage in such risk management transactions.

4 The US government does not transact in forward contracts or other derivatives, but some foreign governments and central banks do. Within the United States, however, some state and local governments do engage in forward contracts and other derivatives.

Dealers such as UBS Warburg lay off the risk they do not wish to assume by transacting with other dealers and potentially other end users. If they do this carefully, quickly, and at accurate prices, they can earn a profit from this market-making activity. One should not get the impression, however, that market making is a highly profitable activity. The competition is fierce, which keeps bid–ask spreads very low and makes it difficult to earn much money on a given transaction. Indeed, many market makers do not make much money on individual transactions—they typically make a small amount of money on each transaction and do a large number of transactions. They may even lose money on some standard transactions, hoping to make up losses on more-complicated, nonstandard transactions, which occur less frequently but have higher bid–ask spreads.

Risk magazine conducts annual surveys to identify the top dealers in various derivative products. Exhibit 1 presents the results of those surveys for two of the forward products we cover here, currency and interest rate forwards. Interest rate forwards are called forward rate agreements (FRAs). In the next section, we shall study the different types of forward contracts and note that there are some others not covered in the *Risk* surveys.

One of these surveys was sent to banks and investment banks that are active dealers in over-the-counter derivatives. The other survey was sent to end users. The tabulations are based on respondents' simple rankings of who they think are the best dealers. Although the identities of the specific dealer firms are not critical, it is interesting and helpful to be aware of the major players in these types of contracts. Most of the world's leading global financial institutions are listed, but many other big names are not. It is also interesting to observe that the perceptions of the users of these dealer firms' services differ somewhat from the dealers' self-perceptions. Be aware, however, that the rankings change, sometimes drastically, each year.

3 TYPES OF FORWARD CONTRACTS

In this section, we examine the types of forward contracts that fall within the scope of this reading. By the word "types," we mean the underlying asset groups on which these forward contracts are created. Because the CFA Program focuses on the asset management industry, our primary interest is in equity, interest rate and fixed-income, and currency forwards.

3.1 Equity Forwards

An **equity forward** is a contract calling for the purchase of an individual stock, a stock portfolio, or a stock index at a later date. For the most part, the differences in types of equity forward contracts are only slight, depending on whether the contract is on an individual stock, a portfolio of stocks, or a stock index.

Exhibit 1 *Risk* Magazine Surveys of Banks, Investment Banks, and Corporate End Users to Determine the Top Three Dealers in Currency and Interest Rate Forwards

	Respondents	
Currencies	**Banks and Investment Banks**	**Corporate End Users**
Currency Forwards		
$/€	UBS Warburg	Citigroup

(continued)

Exhibit 1 (Continued)

	Respondents	
Currencies	**Banks and Investment Banks**	**Corporate End Users**
	Deutsche Bank	Royal Bank of Scotland
	JP Morgan Chase	JP Morgan Chase/Bank of America
$/¥	UBS Warburg	Citigroup
	Citigroup	Bank of America
	JP Morgan Chase	JP Morgan Chase/UBS Warburg
$/£	UBS Warburg	Royal Bank of Scotland
	Royal Bank of Scotland	Citigroup
	Hong Kong Shanghai Banking Corporation	UBS Warburg
$/SF	UBS Warburg	UBS Warburg
	Credit Suisse First Boston	Citigroup
	BNP Paribas	Credit Suisse First Boston
Interest Rate Forwards (FRAs)		
$	JP Morgan Chase	JP Morgan Chase
	Bank of America	Royal Bank of Scotland
	Deutsche Bank	Bank of America
€	Deutsche Bank	Royal Bank of Scotland
	Intesa BCI	JP Morgan Chase
	Royal Bank of Scotland	Deutsche Bank
¥	Mizuho Securities	Citigroup
	JP Morgan Chase	Merrill Lynch
	BNP Paribas	Hong Kong Shanghai Banking Corporation
£	Royal Bank of Scotland	Royal Bank of Scotland
	Commerzbank	Bank of America/ING Barings
	Deutsche Bank	
SF	Credit Suisse First Boston	UBS Warburg
	UBS Warburg	Credit Suisse First Boston
	Deutsche Bank	Citigroup/ING Barings

Note: $ = US dollar, € = euro, ¥ = Japanese yen, £ = UK pound sterling, SF = Swiss franc.
Source: Risk, September 2002, pp. 30–67 for banks and investment banking dealer respondents, and June 2002, pp. 24–34 for end user respondents. The end user survey provides responses from corporations and asset managers. The above results are for corporate respondents only.

3.1.1 *Forward Contracts on Individual Stocks*

Consider an asset manager responsible for the portfolio of a high-net-worth individual. As is sometimes the case, such portfolios may be concentrated in a small number of stocks, sometimes stocks that have been in the family for years. In many cases, the individual may be part of the founding family of a particular company. Let us say that the stock is called Gregorian Industries, Inc., or GII, and the client is so heavily invested in this stock that her portfolio is not diversified. The client notifies the portfolio manager of her need for $2 million in cash in six months. This cash can be raised by selling 16,000 shares at the current price of $125 per share. Thus, the risk exposure concerns the market value of $2 million of stock. For whatever reason, it is considered best not to sell the stock any earlier than necessary. The portfolio manager realizes that a forward contract to sell GII in six months will accomplish the client's desired objective. The manager contacts a forward contract dealer and obtains a quote of $128.13 as the price at which a forward contract to sell the stock in six months could be constructed.[5] In other words, the portfolio manager could enter into a contract to sell the stock to the dealer in six months at $128.13. We assume that this contract is deliverable, meaning that when the sale is actually made, the shares will be delivered to the dealer. Assuming that the client has some flexibility in the amount of money needed, let us say that the contract is signed for the sale of 15,600 shares at $128.13, which will raise $1,998,828. Of course when the contract expires, the stock could be selling for any price. The client can gain or lose on the transaction. If the stock rises to a price above $128.13 during the six-month period, the client will still have to deliver the stock for $128.13. But if the price falls, the client will still get $128.13 per share for the stock.

3.1.2 *Forward Contracts on Stock Portfolios*

Because modern portfolio theory and good common sense dictate that investors should hold diversified portfolios, it is reasonable to assume that forward contracts on specific stock portfolios would be useful. Suppose a pension fund manager knows that in three months he will need to sell about $20 million of stock to make payments to retirees. The manager has analyzed the portfolio and determined the precise identities of the stocks he wants to sell and the number of shares of each that he would like to sell. Thus the manager has designated a specific subportfolio to be sold. The problem is that the prices of these stocks in three months are uncertain. The manager can, however, lock in the sale prices by entering into a forward contract to sell the portfolio. This can be done one of two ways.

The manager can enter into a forward contract on each stock that he wants to sell. Alternatively, he can enter into a forward contract on the overall portfolio. The first way would be more costly, as each contract would incur administrative costs, whereas the second way would incur only one set of costs.[6] Assume that the manager chooses the second method. He provides a list of the stocks and number of shares of each he wishes to sell to the dealer and obtains a quote. The dealer gives him a quote of $20,200,000. So, in three months, the manager will sell the stock to the dealer and receive $20,200,000. The transaction can be structured to call for either actual delivery or cash settlement, but in either case, the client will effectively receive $20,200,000 for the stock.[7]

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5 In Section 4, we shall learn how to calculate forward prices such as this one.

6 Ignoring those costs, there would be no difference in doing forward contracts on individual stocks or a single forward contract on a portfolio. Because of the non-linearity of their payoffs, this is not true for options. A portfolio of options is not the same as an option on a portfolio, but a portfolio of forward contracts is the same as a forward contract on a portfolio, ignoring the aforementioned costs.

7 If, for example, the stock is worth $20,500,000 and the transaction calls for delivery, the manager will transfer the stocks to the dealer and receive $20,200,000. The client effectively takes an opportunity loss of $300,000. If the transaction is structured as a cash settlement, the client will pay the dealer $300,000. The client would then sell the stock in the market, receiving $20,500,000 and netting $20,200,000 after settling the forward contract with the dealer. Similarly, if the stock is selling for less than the amount guaranteed by the forward contract, the client will deliver the stock and receive $20,200,000 or, if the transaction is cash settled, the client will sell the stock in the market and receive a cash payment from the dealer, making the effective sale price still $20,200,000.

3.1.3 *Forward Contracts on Stock Indices*

Many equity forward contracts are based on a stock index. For example, consider a UK asset manager who wants to protect the value of her portfolio that is a Financial Times Stock Exchange 100 index fund, or who wants to eliminate a risk for which the FTSE 100 Index is a sufficiently accurate representation of the risk she wishes to eliminate. For example, the manager may be anticipating the sale of a number of UK blue chip shares at a future date. The manager could, as in our stock portfolio example, take a specific portfolio of stocks to a forward contract dealer and obtain a forward contract on that portfolio. She realizes, however, that a forward contract on a widely accepted benchmark would result in a better price quote, because the dealer can more easily hedge the risk with other transactions. Moreover, the manager is not even sure which stocks she will still be holding at the later date. She simply knows that she will sell a certain amount of stock at a later date and believes that the FTSE 100 is representative of the stock that she will sell. The manager is concerned with the systematic risk associated with the UK stock market, and accordingly, she decides that selling a forward contract on the FTSE 100 would be a good way to manage the risk.

Assume that the portfolio manager decides to protect £15,000,000 of stock. The dealer quotes a price of £6,000 on a forward contract covering £15,000,000. We assume that the contract will be cash settled because such index contracts are nearly always done that way. When the contract expiration date arrives, let us say that the index is at £5,925—a decrease of 1.25 percent from the forward price. Because the manager is short the contract and its price went down, the transaction makes money. But how much did it make on a notional principal of £15,000,000?

The index declined by 1.25 percent. Thus, the transaction should make 0.0125 × £15,000,000 = £187,500. In other words, the dealer would have to pay £187,500 in cash. If the portfolio were a FTSE 100 index fund, then it would be viewed as a portfolio initially worth £15,000,000 that declined by 1.25 percent, a loss of £187,500. The forward contract offsets this loss. Of course, in reality, the portfolio is not an index fund and such a hedge is not perfect, but as noted above, there are sometimes reasons for preferring that the forward contract be based on an index.

3.1.4 *The Effect of Dividends*

It is important to note the effect of dividends in equity forward contracts. Any equity portfolio nearly always has at least a few stocks that pay dividends, and it is inconceivable that any well-known equity index would not have some component stocks that pay dividends. Equity forward contracts typically have payoffs based only on the price of the equity, value of the portfolio, or level of the index. They do not ordinarily pay off any dividends paid by the component stocks. An exception, however, is that some equity forwards on stock indices are based on total return indices. For example, there are two versions of the well-known S&P 500 Index. One represents only the market value of the stocks. The other, called the S&P 500 Total Return Index, is structured so that daily dividends paid by the stocks are reinvested in additional units of the index, as though it were a portfolio. In this manner, the rate of return on the index, and the payoff of any forward contract based on it, reflects the payment and reinvestment of dividends into the underlying index. Although this feature might appear attractive, it is not necessarily of much importance in risk management problems. The variability of prices is so much greater than the variability of dividends that managing price risk is considered much more important than worrying about the uncertainty of dividends.

In summary, equity forwards can be based on individual stocks, specific stock portfolios, or stock indices. Moreover, these underlying equities often pay dividends, which can affect forward contracts on equities. Let us now look at bond and interest rate forward contracts.

3.2 Bond and Interest Rate Forward Contracts

Forward contracts on bonds are similar to forward contracts on interest rates, but the two are different instruments. Forward contracts on bonds, in fact, are no more difficult to understand than those on equities. Drawing on our experience of Section 3.1, we simply extend the notion of a forward contract on an individual stock, a specific stock portfolio, or a stock index to that of a forward contract on an individual bond, a specific bond portfolio, or a bond index.[8]

3.2.1 *Forward Contracts on Individual Bonds and Bond Portfolios*

Although a forward contract on a bond and one on a stock are similar, some basic differences nonetheless exist between the two. For example, the bond may pay a coupon, which corresponds somewhat to the dividend that a stock might pay. But unlike a stock, a bond matures, and a forward contract on a bond must expire prior to the bond's maturity date. In addition, bonds often have many special features such as calls and convertibility. Finally, we should note that unlike a stock, a bond carries the risk of default. A forward contract written on a bond must contain a provision to recognize how default is defined, what it means for the bond to default, and how default would affect the parties to the contract.

In addition to forward contracts on individual bonds, there are also forward contracts on portfolios of bonds as well as on bond indices. The technical distinctions between forward contracts on individual bonds and collections of bonds, however, are relatively minor.

The primary bonds for which we shall consider forward contracts are default-free zero-coupon bonds, typically called Treasury bills or T-bills in the United States, which serve as a proxy for the risk-free rate.[9] In a forward contract on a T-bill, one party agrees to buy the T-bill at a later date, prior to the bill's maturity, at a price agreed on today. T-bills are typically sold at a discount from par value and the price is quoted in terms of the discount rate. Thus, if a 180-day T-bill is selling at a discount of 4 percent, its price per $1 par will be $1 – 0.04(180/360) = $0.98. The use of 360 days is the convention in calculating the discount. So the bill will sell for $0.98. If purchased and held to maturity, it will pay off $1. This procedure means that the interest is deducted from the face value in advance, which is called **discount interest**.

The T-bill is usually traded by quoting the discount rate, not the price. It is understood that the discount rate can be easily converted to the price by the above procedure. A forward contract might be constructed that would call for delivery of a 90-day T-bill in 60 days. Such a contract might sell for $0.9895, which would imply a discount rate of 4.2 percent because $1 – 0.042(90/360) = $0.9895. Later in this reading, we shall see how forward prices of T-bills are derived.

In addition to forward contracts on zero-coupon bonds/T-bills, we shall consider forward contracts on default-free coupon-bearing bonds, also called Treasury bonds in the United States. These instruments pay interest, typically in semiannual installments, and can sell for more (less) than par value if the yield is lower (higher) than the coupon rate. Prices are typically quoted without the interest that has accrued since the last coupon date, but with a few exceptions, we shall always work with the

8 It may be useful to review Chapters 1 and 3 of *Fixed Income Analysis for the Chartered Financial Analyst Program* by Frank J. Fabozzi, New Hope, PA: Frank J. Fabozzi Associates (2000).

9 A government-issued zero-coupon bond is typically used as a proxy for a risk-free asset because it is assumed to be free of default risk. It can be purchased and held to maturity, thereby eliminating any market value risk, and it has no reinvestment risk because it has no coupons. If the bond is liquidated before maturity, however, some market value risk exists in addition to the risk associated with reinvesting the market price.

full price—that is, the price including accrued interest. Prices are often quoted by stating the yield. Forward contracts call for delivery of such a bond at a date prior to the bond's maturity, for which the long pays the short the agreed-upon price.

3.2.2 *Forward Contracts on Interest Rates: Forward Rate Agreements*

So far in Section 3.2 we have discussed forward contracts on actual fixed-income securities. Fixed-income security prices are driven by interest rates. A more common type of forward contract is the interest rate forward contract, more commonly called a **forward rate agreement (FRA)**. Before we can begin to understand FRAs, however, we must examine the instruments on which they are based.

There is a large global market for time deposits in various currencies issued by large creditworthy banks. This market is primarily centered in London but also exists elsewhere, though not in the United States. The primary time deposit instrument is called the **Eurodollar**, which is a dollar deposited outside the United States. Banks borrow dollars from other banks by issuing Eurodollar time deposits, which are essentially short-term unsecured loans. In London, the rate on such dollar loans is called the London Interbank Rate. Although there are rates for both borrowing and lending, in the financial markets the lending rate, called the **London interbank offered rate** (Libor), is more commonly used in derivative contracts. Libor is the rate at which London banks lend dollars to other London banks. Even though it represents a loan outside of the United States, Libor is considered to be the best representative rate on a dollar borrowed by a private, i.e., nongovernmental, high-quality borrower. It should be noted, however, that the London market includes many branches of banks from outside the United Kingdom, and these banks are also active participants in the Eurodollar market.

A Eurodollar time deposit is structured as follows. Let us say a London bank such as NatWest needs to borrow \$10 million for 30 days. It obtains a quote from the Royal Bank of Scotland for a rate of 5.25 percent. Thus, 30-day Libor is 5.25 percent. If NatWest takes the deal, it will owe $\$10,000,000 \times [1 + 0.0525\ (30/360)] = \$10,043,750$ in 30 days. Note that, like the Treasury bill market, the convention in the Eurodollar market is to prorate the quoted interest rate over 360 days. In contrast to the Treasury bill market, the interest is not deducted from the principal. Rather, it is added on to the face value, a procedure appropriately called **add-on interest**. The market for Eurodollar time deposits is quite large, and the rates on these instruments are assembled by a central organization and quoted in financial newspapers. The British Bankers Association publishes a semi-official Eurodollar rate, compiled from an average of the quotes of London banks.

The US dollar is not the only instrument for which such time deposits exist. Eurosterling, for example, trades in Tokyo, and Euroyen trades in London. You may be wondering about Euroeuro. Actually, there is no such entity as Euroeuro, at least not by that name. The Eurodollar instrument described here has nothing to do with the European currency known as the euro. Eurodollars, Euroyen, Eurosterling, etc. have been around longer than the euro currency and, despite the confusion, have retained their nomenclature. An analogous instrument does exist, however—a euro-denominated loan in which one bank borrows euros from another. Trading in euros and euro deposits occurs in most major world cities, and two similar rates on such euro deposits are commonly quoted. One, called EuroLibor, is compiled in London by the British Bankers Association, and the other, called Euribor, is compiled in Frankfurt and published by the European Central Bank. Euribor is more widely used and is the rate we shall refer to in this book.

Now let us return to the world of FRAs. FRAs are contracts in which the underlying is neither a bond nor a Eurodollar or Euribor deposit but simply an interest payment made in dollars, Euribor, or any other currency at a rate appropriate for that currency.

Our primary focus will be on dollar Libor and Euribor, so we shall henceforth adopt the terminology Libor to represent dollar Libor and Euribor to represent the euro deposit rate.

Because the mechanics of FRAs are the same for all currencies, for illustrative purposes we shall use Libor. Consider an FRA expiring in 90 days for which the underlying is 180-day Libor. Suppose the dealer quotes this instrument at a rate of 5.5 percent. Suppose the end user goes long and the dealer goes short. The end user is essentially long the rate and will benefit if rates increase. The dealer is essentially short the rate and will benefit if rates decrease. The contract covers a given notional principal, which we shall assume is $10 million.

The contract stipulates that at expiration, the parties identify the rate on new 180-day Libor time deposits. This rate is called 180-day Libor. It is, thus, the underlying rate on which the contract is based. Suppose that at expiration in 90 days, the rate on 180-day Libor is 6 percent. That 6 percent interest will be paid 180 days later. Therefore, the present value of a Eurodollar time deposit at that point in time would be

$$\frac{\$10{,}000{,}000}{1 + 0.06\left(\frac{180}{360}\right)}$$

At expiration, then, the end user, the party going long the FRA in our example, receives the following payment from the dealer, which is the party going short:

$$\$10{,}000{,}000\left[\frac{(0.06 - 0.055)\left(\frac{180}{360}\right)}{1 + 0.06\left(\frac{180}{360}\right)}\right] = \$24{,}272$$

If the underlying rate is less than 5.5 percent, the payment is calculated based on the difference between the 5.5 percent rate and the underlying rate and is paid by the long to the short. It is important to note that even though the contract expires in 90 days, the rate is on a 180-day Libor instrument; therefore, the rate calculation adjusts by the factor 180/360. The fact that 90 days have elapsed at expiration is not relevant to the calculation of the payoff.

Before presenting the general formula, let us review the calculations in the numerator and denominator. In the numerator, we see that the contract is obviously paying the difference between the actual rate that exists in the market on the contract expiration date and the agreed-upon rate, adjusted for the fact that the rate applies to a 180-day instrument, multiplied by the notional principal. The divisor appears because when Eurodollar rates are quoted in the market, they are based on the assumption that the rate applies to an instrument that accrues interest at that rate with the interest paid a certain number of days (here 180) later. When participants determine this rate in the London Eurodollar market, it is understood to apply to a Eurodollar time deposit that begins now and matures 180 days later. So the interest on an actual Eurodollar deposit would not be paid until 180 days later. Thus, it is necessary to adjust the FRA payoff to reflect the fact that the rate implies a payment that would occur 180 days later on a standard Eurodollar deposit. This adjustment is easily done by simply discounting the payment at the current Libor, which here is 6 percent, prorated over 180 days. These conventions are also followed in the market for FRAs with other underlying rates.

In general, the FRA payoff formula (from the perspective of the party going long) is

$$\text{Notional principal}\left[\frac{\left(\begin{array}{c}\text{Underlying rate at expiration}\\ -\text{ Forward contract rate}\end{array}\right)\left(\frac{\text{Days in underlying rate}}{360}\right)}{1 + \text{Undelrying rate at expiration}\left(\frac{\text{Days in underlying rate}}{360}\right)}\right]$$

where *forward contract rate* represents the rate the two parties agree will be paid and *days in underlying rate* refers to the number of days to maturity of the instrument on which the underlying rate is based.

One somewhat confusing feature of FRAs is the fact that they mature in a certain number of days and are based on a rate that applies to an instrument maturing in a certain number of days measured from the maturity of the FRA. Thus, there are two day figures associated with each contract. Our example was a 90-day contract on 180-day Libor. To avoid confusion, the FRA markets use a special type of terminology that converts the number of days to months. Specifically, our example FRA is referred to as a 3 × 9, reflecting the fact that the contract expires in three months and that six months later, or nine months from the contract initiation date, the interest is paid on the underlying Eurodollar time deposit on whose rate the contract is based.[10]

FRAs are available in the market for a variety of maturities that are considered somewhat standard. Exhibit 2 presents the most common maturities. Most dealers follow the convention that contracts should expire in a given number of exact months and should be on the most commonly traded Eurodollar rates such as 30-day Libor, 60-day Libor, 90-day Libor, 180-day Libor, and so on. If a party wants a contract expiring in 37 days on 122-day Libor, it would be considered an exception to the standard, but most dealers would be willing to make a market in such an instrument. Such nonstandard instruments are called *off the run*. Of course, FRAs are available in all of the leading currencies.

Exhibit 2 FRA Descriptive Notation and Interpretation

Notation	Contract Expires in	Underlying Rate
1 × 3	1 month	60-day Libor
1 × 4	1 month	90-day Libor
1 × 7	1 month	180-day Libor
3 × 6	3 months	90-day Libor
3 × 9	3 months	180-day Libor
6 × 12	6 months	180-day Libor
12 × 18	12 months	180-day Libor

Note: This list is not exhaustive and represents only the most commonly traded FRAs.

The FRA market is large, but not as large as the swaps market. It is important, however, to understand FRAs before trying to understand swaps. As we will show in the reading on swap markets and contracts, a swap is a special combination of FRAs. But let us now turn to another large forward market, the market for currency forwards.

3.3 Currency Forward Contracts

Spurred by the relaxation of government controls over the exchange rates of most major currencies in the early 1970s, a currency forward market developed and grew extremely large. Currency forwards are widely used by banks and corporations to manage foreign exchange risk. For example, suppose Microsoft has a European subsidiary that expects to send it €12 million in three months. When Microsoft receives the euros, it will then convert them to dollars. Thus, Microsoft is essentially long

10 The notation "3 × 9" is pronounced "three by nine."

euros because it will have to sell euros, or equivalently, it is short dollars because it will have to buy dollars. A currency forward contract is especially useful in this situation, because it enables Microsoft to lock in the rate at which it will sell euros and buy dollars in three months. It can do this by going short the forward contract, meaning that it goes short the euro and long the dollar. This arrangement serves to offset its otherwise long-euro, short-dollar position. In other words, it needs a forward contract to sell euros and buy dollars.

For example, say Microsoft goes to JP Morgan Chase and asks for a quote on a currency forward for €12 million in three months. JP Morgan Chase quotes a rate of \$0.925, which would enable Microsoft to sell euros and buy dollars at a rate of \$0.925 in three months. Under this contract, Microsoft would know it could convert its €12 million to 12,000,000 × \$0.925 = \$11,100,000. The contract would also stipulate whether it will settle in cash or will call for Microsoft to actually deliver the euros to the dealer and be paid \$11,100,000. This simplified example is a currency forward hedge.

Now let us say that three months later, the spot rate for euros is \$0.920. Microsoft is quite pleased that it locked in a rate of \$0.925. It simply delivers the euros and receives \$11,100,000 at an exchange rate of \$0.925.[11] Had rates risen, however, Microsoft would still have had to deliver the euros and accept a rate of \$0.925.

A few variations of currency forward contracts exist, but most of them are somewhat specialized and beyond the objectives of this reading. Let us now take a very brief look at a few other types of forward contracts.

3.4 Other Types of Forward Contracts

Although we focus primarily on the financial derivatives used by asset managers, we should mention here some of the other types. Commodity forwards—in which the underlying asset is oil, a precious metal, or some other commodity—are widely used. In addition, the derivatives industry has created forward contracts and other derivatives on various sources of energy (electricity, gas, etc.) and even weather, in which the underlying is a measure of the temperature or the amount of disaster damage from hurricanes, earthquakes, or tornados.

Many of these instruments are particularly difficult to understand, price, and trade. Nonetheless, through the use of derivatives and indirect investments, such as hedge funds, they can be useful for managing risk and investing in general. They are not, however, our focus.

In the examples and illustrations used above, we have made reference to certain prices. Determining appropriate prices and fair values of financial instruments is a central objective of much of the process of asset management. Accordingly, pricing and valuation occupies a major portion of the CFA Program. As such, we turn our attention to the pricing and valuation of forward contracts.

END OPTIONAL SEGMENT

4 PRICING AND VALUATION OF FORWARD CONTRACTS

Before getting into the actual mechanics of pricing and valuation, the astute reader might wonder whether we are being a bit redundant. Are pricing and valuation not the same thing?

11 Had the contract been structured to settle in cash, the dealer would have paid Microsoft 12,000,000 × (\$0.925 − \$0.920) = \$60,000. Microsoft would have converted the euros to dollars at the current spot exchange rate of \$0.920, receiving 12,000,000 × \$0.920 = \$11,040,000. Adding the \$60,000 payment from the dealer, Microsoft would have received \$11,100,000, an effective rate of \$0.925. risen, however, Microsoft would still have had to deliver the euros and accept a rate of \$0.925.

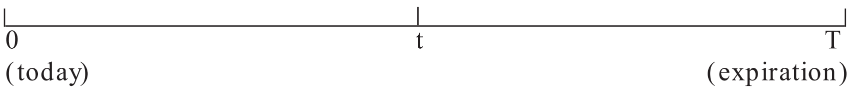
An equity analyst often finds that a stock is priced at more or less than its fair market value and uses this conclusion as the basis for a buy or sell recommendation.[12] In an efficient market, the price of a stock would always equal its value or the price would quickly converge to the value. Thus, for all practical purposes, pricing and valuation would be the same thing. In general, when we speak of the value and price of an *asset,* we are referring to what that asset is worth and what it sells for. With respect to certain *derivatives,* however, value and price take on slightly different meanings.

So let us begin by defining value: *Value is what you can sell something for or what you must pay to acquire something.* This applies to stocks, bonds, derivatives, and used cars.[13] Accordingly, *valuation is the process of determining the value of an asset or service.* Pricing is a related but different concept; let us explore what we mean by pricing a forward contract.

A forward contract price is the fixed price or rate at which the transaction scheduled to occur at expiration will take place. This price is agreed to on the contract initiation date and is commonly called the **forward price or forward rate**. Pricing means to determine the forward price or forward rate. Valuation, however, means to determine the amount of money that one would need to pay or would expect to receive to engage in the transaction. Alternatively, if one already held a position, valuation would mean to determine the amount of money one would either have to pay or expect to receive in order to get out of the position. Let us look at a generic example.

4.1 Generic Pricing and Valuation of a Forward Contract

Because derivative contracts have finite lives, it is important to carefully specify the time frame in which we are operating. We denote time in the following manner: Today is identified as time 0. The expiration date is time T. Time t is an arbitrary time between today and the expiration. Usually when we refer to "today," we are referring to the date on which the contract is created. Later we shall move forward to time t and time T, which will then be "today."



The price of the underlying asset in the spot market is denoted as S_0 at time 0, S_t at time t, and S_T at time T. The forward contract price, established when the contract is initiated at time 0, is F(0,T). This notation indicates that F(0,T) is the price of a forward contract initiated at time 0 and expiring at time T. The value of the forward contract is $V_0(0,T)$. This notation indicates that $V_0(0,T)$ is the value at time 0 of a forward contract initiated at time 0 and expiring at time T. In this reading, subscripts always indicate that we are at a specific point in time.

We have several objectives in this analysis. First, we want to determine the forward price F(0,T). We also want to determine the forward contract value today, denoted $V_0(0,T)$, the value at a point during the life of the contract such as time t, denoted $V_t(0,T)$, and the value at expiration, denoted $V_T(0,T)$. Valuation is somewhat easier to grasp from the perspective of the party holding the long position, so we shall take that point of view in this example. Once that value is determined, the value to the short is obtained by simply changing the sign.

12 From your study of equity analysis, you should recall that we often use the discounted cash flow model, sometimes combined with the capital asset pricing model, to determine the fair market value of a stock.
13 Be careful. You may think the "value" of a certain used car is $5,000, but if no one will give you that price, it can hardly be called the value.

If we are at expiration, we would observe the spot price as S_T. The long holds a position to buy the asset at the already agreed-upon price of F(0,T). Thus, the value of the forward contract at expiration should be obvious: $S_T - F(0,T)$. If the value at expiration does not equal this amount, then an arbitrage profit can be easily made. For example, suppose the forward price established at the initiation of the contract, F(0,T), is $20. Now at expiration, the spot price, S_T, is $23. The contract value must be $3. If it were more than $3, then the long would be able to sell the contract to someone for more than $3—someone would be paying the long more than $3 to obtain the obligation of buying a $23 asset for $20. Obviously, no one would do that. If the value were less than $3, the long would have to be willing to sell for less than $3 the obligation of buying a $23 asset for $20. Obviously, the long would not do that. Thus, we state that the value at expiration of a forward contract established at time 0 is

$$V_T(0,T) = S_T - F(0,T) \quad (1)$$

Note that the value of a forward contract can also be interpreted as its profit, the difference between what the long pays for the underlying asset, F(0,T), and what the long receives, the asset price S_T. Of course, we have still not explained how F(0,T) is determined, but the above equation gives the value of the contract at expiration, at which time F(0,T) would certainly be known because it was agreed on at the initiation date of the contract.

Now let us back up to the time when the contract was originated. Consider a contract that expires in one year. Suppose that the underlying asset is worth $100 and that the forward price is $108. We do not know if $108 is the correct forward price; we will simply try it and see.

Suppose we buy the asset for $100 and sell the forward contract for $108. We hold the position until expiration. We assume that there are no direct costs associated with buying or holding the asset, but we must recognize that we lose interest on the $100 tied up in the asset. Assume that the interest rate is 5 percent.

Recall that no money changes hands at the start with a forward contract. Consequently, the $100 invested in the asset is the full outlay. At the end of the year, the forward contract expires and we deliver the asset, receiving $108 for it—not bad at all. At a 5 percent interest rate, we lose only $5 in interest on the $100 tied up in the asset. We receive $108 for the asset regardless of its price at expiration. We can view $108 – $105 = $3 as a risk-free profit, which more than covered the cost. In fact, if we had also borrowed the $100 at 5 percent, we could have done this transaction without putting up any money of our own. We would have more than covered the interest on the borrowed funds and netted a $3 risk-free profit. This profit is essentially free money—there is no cost and no risk. Thus, it is an arbitrage profit. We would certainly want to execute any transaction that would generate an arbitrage profit.

In the market, the forces of arbitrage would then prevail. Other market participants would execute this transaction as well. Although it is possible that the spot price would bear some of the adjustment, in this reading we shall always let the derivative price make the full adjustment. Consequently, the derivative price would have to come down to $105.

If the forward price were below $105, we could also earn an arbitrage profit, although it would be a little more difficult because the asset would have to be sold short. Suppose the forward price is $103. If the asset were a financial asset, we could borrow it and sell it short. We would receive $100 for it and invest that $100 at the 5 percent rate. We would simultaneously buy a forward contract. At expiration, we would take delivery of the asset paying $103 and then deliver it to the party from whom we borrowed it. The short position is now covered, and we still have the $100 invested plus 5 percent interest on it. This transaction offers a clear arbitrage profit of $2. Again, the forces of arbitrage would cause other market participants to undertake the transaction, which would push the forward price up to $105.

If short selling is not permitted, too difficult, or too costly, a market participant who already owns the asset could sell it, invest the \$100 at 5 percent, and buy a forward contract. At expiration, he would pay \$103 and take delivery on the forward contract, which would return him to his original position of owning the asset. He would now, however, receive not only the stock but also 5 percent interest on \$100. Again, the forces of arbitrage would make this transaction attractive to other parties who held the asset, provided they could afford to part with it for the necessary period of time.[14]

Going back to the situation in which the forward contract price was \$103, an arbitrage profit could, however, be eliminated if the party going long the forward contract were required to pay some money up front. For example, suppose the party going long the forward contract paid the party going short \$1.9048. Then the party going long would lose \$1.9048 plus interest on this amount. Notice that \$1.9048 compounded at 5 percent interest equals precisely \$2, which not surprisingly is the amount of the arbitrage profit.

Thus, if the forward price were \$103, the value of the contract would be \$1.9048. With T = 1, this value equals

$$V_0(0,T) = V_0(0,1) = \$100 - \$103/1.05 = \$1.9048$$

Therefore, to enter into this contract at this forward price, one party must pay another. Because the value is positive, it must be paid by the party going long the forward contract to the party going short. Parties going long must pay positive values; parties going short pay negative values.[15]

If the forward price were \$108, the value would be

$$V_0(0,T) = \$100 - \$108/1.05 = -\$2.8571$$

In this case, the value is negative and would have to be paid from the short to the long. Doing so would eliminate the arbitrage profit that the short would have otherwise been able to make, given the forward price of \$108.

Arbitrage profits can be eliminated with an up-front payment from long to short or vice versa that is consistent with the forward price the parties select. The parties could simply negotiate a forward price, and any resulting market value could be paid from one party to the other. *It is customary, however, in the forward market for the initial value to be set to zero*. This convention eliminates the necessity of either party making a payment to the other and results in a direct and simple determination of the forward price. Specifically, setting $V_0(0,T) = 0$ and letting r represent the interest rate,

$$V_0(0,T) = S_0 - F(0,T)/(1 + r) = 0$$

which means that $F(0,T) = S_0(1 + r)$. In our example, $F(0,T) = \$100(1.05) = \$105$, which is the forward price that eliminates the arbitrage profit.

Our forward price formula can be interpreted as saying that the forward price is the spot price compounded at the risk-free interest rate. In our example, we had an annual interest rate of r and one year to expiration. With today being time 0 and expiration being time T, the time T − 0 = T is the number of years to expiration of the forward contract. Then we more generally write the forward price as

$$F(0,T) = S_0(1 + r)^T \quad (2)$$

Again, this result is consistent with the custom that no money changes hands at the start of a forward contract, meaning that the value of a forward contract at its start is zero.

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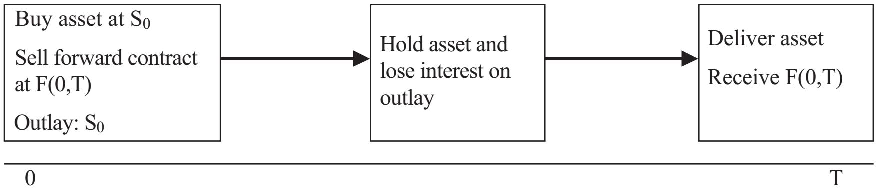
14 In other words, a party holding the asset must be willing to part with it for the length of time it would take for the forces of arbitrage to bring the price back in line, thereby allowing the party to capture the risk-free profit and return the party to its original state of holding the asset. The period of time required for the price to adjust should be very short if the market is relatively efficient.

15 For example, when a stock is purchased, its value, which is always positive, is paid from the long to the short. This is true for any asset.

Exhibit 3 summarizes the process of pricing a forward contract. At time 0, we buy the asset and sell a forward contract for a total outlay of the spot price of the asset.[16] Over the life of the contract, we hold the asset and forgo interest on the money. At expiration, we deliver the asset and receive the forward price for a payoff of F(0,T). The overall transaction is risk free and equivalent to investing the spot price of the asset in a risk-free bond that pays F(0,T) at time T. Therefore, the payoff at T must be the future value of the spot price invested at the risk-free rate. This equality can be true only if the forward price is the spot price compounded at the risk-free rate over the life of the asset.

Exhibit 3 Pricing a Forward Contract



The transaction is risk free and should be equivalent to investing S_0 dollars in a risk-free asset that pays F(0,T) at time T. Thus, the amount received at T must be the future value of the initial outlay invested at the risk-free rate. For this equality to hold, the forward price must be given as

$$F(0,T) = S_0(1 + r)^T$$

Example: The spot price is \$72.50, the risk-free rate is 8.25 percent, and the contract is for five years. The forward price would be

$$F(0,T) = F(0,5) = 72.50(1.0825)^5 = 107.76$$

A contract in which the initial value is intentionally set at a nonzero value is called an **off-market FRA**. In such a contract, the forward price is set arbitrarily in the process of negotiation between the two parties. Given the chosen forward price, the contract will have a nonzero value. As noted above, if the value is positive, the long pays that amount up front to the short. If it is negative, the short pays that amount up front to the long. Although off-market FRAs are not common, we shall use them in the reading on swap markets and contracts when studying swaps.

Now suppose we are at a time t, which is a point during the life of the contract. We may want to know the value of the forward contract for several reasons. For one, it makes good business sense to know the monetary value of an obligation to do something at a later date. Also, accounting rules require that a company mark its derivatives to their current market values and report the effects of those values in income statements and balance sheets. In addition, the market value can be used as a gauge of the credit exposure. Finally, the market value can be used to determine how much money one party can pay the other to terminate the contract.

Let us start by assuming that we established a long forward contract at time 0 at the price F(0,T). Of course, its value at time 0 was zero. But now it is time t, and we want to know its new value, $V_t(0,T)$. Let us consider what it means to hold the position of being long at time t a forward contract established at time 0 at the price F(0,T) and expiring at time T:

16 Remember that in a forward contract, neither party pays anything for the forward contract at the start.

We will have to pay F(0,T) dollars at T.

We will receive the underlying asset, which will be worth S_T, at T.

At least part of the value will clearly be the present value of a payment of F(0,T), or in other words, $-F(0,T)/(1 + r)^{T-t}$. The other part of the contract value comes from the fact that we have a claim on the asset's value at T. We do not know what S_T (the asset value at T) will be, but we do know that the market tells us its present value is S_t, the current asset price. *By definition, an asset's value today is the present value of its future value.*[17] Thus we can easily value our forward contract at time t during the life of the contract:

$$V_t(0,T) = S_t - F(0,T)/(1 + r)^{(T-t)} \quad (3)$$

Consider our earlier example in which we entered into a one-year forward contract to buy the asset at \$105. Now assume it is three months later and the price of the asset is \$102. With t = 0.25 and T = 1, the value of the contract would be

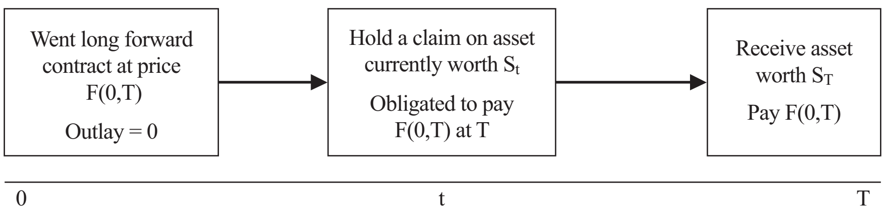
$$V_t(0,T) = V_{0.25}(0,1) = \$102 - \$105/(1.05)^{0.75} = \$0.7728$$

Again, why is this the value? The contract provides the long with a claim on the asset at expiration. That claim is currently worth the current asset value of \$102. That claim also obligates the long to pay \$105 at expiration, which has a present value of $\$105/(1.05)^{0.75}$ = \$101.2272. Thus, the long position has a value of \$102 – \$101.2272 = \$0.7728.

As noted above, this market value may well affect the income statement and balance sheet. In addition, it gives an idea of the contract's credit exposure, a topic we have touched on and will cover in more detail in Section 5. Finally, we noted earlier that a party could re-enter the market and offset the contract by paying the counterparty or having the counterparty pay him a cash amount. This cash amount is the market value as calculated here.[18]

Exhibit 4 summarizes how we value a forward contract. If we went long a forward contract at time 0 and we are now at time t prior to expiration, we hold a claim on the asset at expiration and are obligated to pay the forward price at expiration. The claim on the asset is worth its current price; the obligation to pay the forward price at expiration is worth the negative of its present value. Thus, the value of the forward contract is the current spot price minus the forward price discounted from expiration back to the present.

Exhibit 4 Valuing a Forward Contract



17 This statement is true for any type of asset or financial instrument. It always holds by definition.

18 If the market value is positive, the value of the asset exceeds the present value of what the long promises to pay. Thus, it makes sense that the short must pay the long. If the market value is negative, then the present value of what the long promises to pay exceeds the value of the asset. Then, it makes sense that the long must pay the short.

Exhibit 4 (Continued)

The value of the forward contract at t must be the value of what it will produce at T:

$$V_t(0,T) = S_t - F(0,T)/(1 + r)^{(T-t)}$$

Example: A two-year forward contract was established with a price of $62.25. Now, a year and a half later (t = 1.5), the spot price is $71.19 and the risk-free rate is 7 percent. The value of the forward contract is

$$V_t(0,T) = V_{1.5}(0,2) = 71.19 - 62.25/(1.07)^{0.5} = 11.01$$

Therefore, we have seen that the forward contract value is zero today: the asset price minus the present value of the forward price at a time prior to expiration, and the asset price minus the forward price at expiration. It may be helpful to note that in general, we can always say that *the forward contract value is the asset price minus the present value of the exercise price,* because given $V_t(0,T) = S_t - F(0,T)/(1 + r)^{(T-t)}$:

If t = 0, $V_t(0,T) = V_0(0,T) = S_0 - F(0,T)/(1 + r)^T = 0$ because $F(0,T) = S_0(1 + r)^T$
If t = T, $V_t(0,T) = V_T(0,T) = S_T - F(0,T)/(1 + r)^0 = S_T - F(0,T)$

The formulas for pricing and valuation of a forward contract are summarized in Exhibit 5.

Exhibit 5 Pricing and Valuation Formulas for a Forward Contract

Today = time 0

Arbitrary point during the contract's life = time t

Expiration = time T

Value of a forward contract at any time t:

$$V_t(0,T) = S_t - F(0,T)/(1 + r)^{(T-t)}$$

Value of a forward contract at expiration (t = T):

$$V_T(0,T) = S_T - F(0,T)$$

Value of a forward contract at initiation (t = 0):

$$V_0(0,T) = S_0 - F(0,T)/(1 + r)^T$$

Customarily, no money changes hands at initiation so V_0 (0,T) is set equal to zero. Thus,

$$F(0,T) = S_0(1 + r)^T$$

EXAMPLE 1

An investor holds title to an asset worth €125.72. To raise money for an unrelated purpose, the investor plans to sell the asset in nine months. The investor is concerned about uncertainty in the price of the asset at that time. The investor learns about the advantages of using forward contracts to manage this risk and enters into such a contract to sell the asset in nine months. The risk-free interest rate is 5.625 percent.

A Determine the appropriate price the investor could receive in nine months by means of the forward contract.

B Suppose the counterparty to the forward contract is willing to engage in such a contract at a forward price of €140. Explain what type of transaction the investor could execute to take advantage of the situation. Calculate the rate of return (annualized), and explain why the transaction is attractive.

C Suppose the forward contract is entered into at the price you computed in Part A. Two months later, the price of the asset is €118.875. The investor would like to evaluate her position with respect to any gain or loss accrued on the forward contract. Determine the market value of the forward contract at this point in time from the perspective of the investor in Part A.

D Determine the value of the forward contract at expiration assuming the contract is entered into at the price you computed in Part A and the price of the underlying asset is €123.50 at expiration. Explain how the investor did on the overall position of both the asset and the forward contract in terms of the rate of return.

Solution to A:

$$T = 9/12 = 0.75$$
$$S_0 = 125.72$$
$$r = 0.05625$$

$$F(0,T) = 125.72(1.05625)^{0.75} = 130.99$$

Solution to B:

As found in Part A, the forward contract should be selling at €130.99, but it is selling for €140. Consequently, it is overpriced—and an overpriced contract should be sold. Because the investor holds the asset, she will be hedged by selling the forward contract. Consequently, her asset, worth €125.72 when the forward contract is sold, will be delivered in nine months and she will receive €140 for it. The rate of return will be

$$\left(\frac{140}{125.72}\right) - 1 = 0.1136$$

This risk-free return of 11.36 percent for nine months is clearly in excess of the 5.625 percent annual rate. In fact, a rate of 11.36 percent for nine months annualizes to

$$(1.1136)^{12/9} - 1 = 0.1543$$

An annual risk-free rate of 15.43 percent is clearly preferred over the actual risk-free rate of 5.625 percent. The position is not only hedged but also earns an arbitrage profit.

Solution to C:

$$t = 2/12$$
$$T - t = 9/12 - 2/12 = 7/12$$
$$S_t = 118.875$$
$$F(0,T) = 130.99$$

$$V_t(0,T) = V_{2/12}(0,9/12) = 118.875 - 130.99(1.05625)^{7/12} = -8.0$$

The contract has a negative value. Note, however, that in this form, the answer applies to the holder of the long position. This investor is short. Thus, the value to the investor in this problem is positive 8.0.

Solution to D:

$$S_T = 123.50$$
$$V_T(0,T) = V_{9/12}(0,9/12) = 123.50 - 130.99 = -7.49$$

This amount is the value to the long. This investor is short, so the value is a positive 7.49. The investor incurred a loss on the asset of 125.72 – 123.50 = 2.22. Combined with the gain on the forward contract, the net gain is 7.49 – 2.22 = 5.27. A gain of 5.27 on an asset worth 125.72 when the transaction was initiated represents a return of 5.27/125.72 = 4.19 percent. When annualized, the rate of return equals

$$(1.0419)^{12/9} - 1 = 0.05625$$

It should come as no surprise that this number is the annual risk-free rate. The transaction was executed at the no-arbitrage forward price of €130.99. Thus, it would be impossible to earn a return higher or lower than the risk-free rate.

In our examples, there were no costs or cash flows associated with holding the underlying assets. In the specific examples below for equity derivatives, fixed-income and interest rate derivatives, and currency derivatives, we present cases in which cash flows on the underlying asset will slightly alter our results. We shall ignore any costs of holding assets. Such costs are primarily associated with commodities, an asset class we do not address in this reading.

4.2 Pricing and Valuation of Equity Forward Contracts

Equity forward contracts are priced and valued much like the generic contract described above, with one important additional feature. Many stocks pay dividends, and the effects of these dividends must be incorporated into the pricing and valuation process. Our concern is with the dividends that occur over the life of the forward contract, but not with those that may come after the contract ends. Following standard procedure, we assume that these dividends are known or are a constant percentage of the stock price.

We begin with the idea of a forward contract on either a single stock, a portfolio of stocks, or an index in which dividends are to be paid during the life of the contract. Using the time notation that today is time 0, expiration is time T, and there is an arbitrary time t during its life when we need to value the contract, assume that dividends can be paid at various times during the life of the contract between t and T.[19]

In the examples that follow, we shall calculate present and future values of this stream of dividends over the life of the forward contract. Given a series of these dividends of $D_1, D_2, \ldots, D_n$, whose values are known, that occur at times $t_1, t_2, \ldots, t_n$, the present value will be defined as PV(D,0,T) and computed as

$$PV(D,0,T) = \sum_{i=1}^{n} \frac{D_i}{(1+r)^{t_i}}$$

The future value will be defined as FV(D,0,T) and computed as

$$FV(D,0,T) = \sum_{i=1}^{n} D_i (1+r)^{T-t_i}$$

Recall that the forward price is established by eliminating any opportunity to arbitrage from establishing a forward contract without making any cash outlay today, as is customary with forward contracts. We found that the forward price is the spot price compounded at the risk-free interest rate. To include dividends, we adjust our formula slightly to

$$F(0,T) = [S_0 - PV(D,0,T)](1+r)^T \quad \textbf{(4)}$$

In other words, we simply subtract the present value of the dividends from the stock price. Note that the dividends reduce the forward price, a reflection of the fact that holders of long positions in forward contracts do not benefit from dividends in comparison to holders of long positions in the underlying stock.

For example, consider a stock priced at \$40, which pays a dividend of \$3 in 50 days. The risk-free rate is 6 percent. A forward contract expiring in six months (T = 0.5) would have a price of

$$F(0,T) = F(0,0.5) = \left[\$40 - \$3/(1.06)^{50/365}\right](1.06)^{0.5} = \$38.12$$

If the stock had more than one dividend, we would simply subtract the present value of all dividends over the life of the contract from the stock price, as in the following example.

The risk-free rate is 4 percent. The forward contract expires in 300 days and is on a stock currently priced at \$35, which pays quarterly dividends according to the following schedule:

Days to Ex-Dividend Date	Dividend ($)
10	0.45
102	0.45
193	0.45
283	0.45

19 Given the way dividends are typically paid, the right to the dividend leaves the stock on the ex-dividend date, which is prior to the payment date. To precisely incorporate this feature, either the dividend payment date should be the ex-dividend date or the dividend should be the present value at the ex-dividend date of the dividend to be paid at a later date. We shall ignore this point here and assume that it would be taken care of in practice.

The present value of the dividends is found as follows:

$$\text{PV(D,0,T)} = \$0.45/(1.04)^{10/365} + \$0.45/(1.04)^{102/365} + \$0.45/(1.04)^{193/365} + \$0.45/(1.04)^{283/365} = \$1.77$$

The time to expiration is T = 300/365. Therefore, the forward price equals

$$\text{F(0,T)} = \text{F(0,300/365)} = (\$35 - \$1.77)(1.04)^{300/365} = \$34.32$$

Another approach to incorporating the dividends is to use the future value of the dividends. With this forward contract expiring in 300 days, the first dividend is reinvested for 290 days, the second for 198 days, the third for 107 days, and the fourth for 17 days. Thus,

$$\text{FV(D,0,T)} = \$0.45(1.04)^{290/365} + \$0.45(1.04)^{198/365} + \$0.45(1.04)^{107/365} + \$0.45(1.04)^{17/365} = \$1.83$$

To obtain the forward price, we compound the stock value to expiration and subtract the future value of the dividends. Thus, the forward price would be

$$\text{F(0,T)} = S_0(1 + r)^T - \text{FV(D,0,T)} \quad \textbf{(5)}$$

This formula will give the same answer as the one using the present value of the dividends, as shown below:

$$\text{F(0,300/365)} = \$35(1.04)^{300/365} - \$1.83 = \$34.32$$

An alternative way to incorporate dividends is to express them as a fixed percentage of the stock price. The more common version of this formulation is to assume that the stock, portfolio, or index pays dividends continuously at a rate of δ^c. By specifying the dividends in this manner, we are allowing the dividends to be uncertain and completely determined by the stock price at the time the dividends are being paid. In this case, the stock is constantly paying a dividend at the rate δ^c. In the reading on futures markets and contracts, we will again discuss how to incorporate dividends.

Because we pay dividends continuously, for consistency we must also compound the interest continuously. The continuously compounded equivalent of the discrete risk-free rate r will be denoted r^c and is found as $r^c = \ln(1 + r)$.[20] The future value of \$1 at time T is $\exp(r^cT)$. Then the forward price is given as

$$F(0,T) = \left(S_0 e^{-\delta^c T}\right) e^{r^c T} \quad \textbf{(6)}$$

The term in parentheses, the stock price discounted at the dividend yield rate, is equivalent to the stock price minus the present value of the dividends. This value is then compounded at the risk-free rate over the life of the contract, just as we have done in the other versions.

Some people attach significance to whether the forward price is higher than the spot price. It is important to note that the forward price should not be interpreted as a forecast of the future price of the underlying. This misperception is common. If the forward price is higher than the spot price, it merely indicates that the effect of the risk-free rate is greater than the effect of the dividends. In fact, such is usually the case with equity forwards. Interest rates are usually greater than dividend yields.

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20 The notation "ln" stands for natural logarithm. A logarithm is the power to which its base must be raised to equal a given number. The base of the natural logarithm system is *e*, approximately 2.71828. With an interest rate of r = 0.06, we would have $r^c = \ln(1.06) = 0.058$. Then $e^{0.058} = 1.06$ is called the exponential function and often written as exp(0.058) = 1.06. The future value factor is thus $\exp(r^c)$. The present value factor is $1/\exp(r^c)$ or $\exp(-r^c)$. If the period is more or less than one year, we also multiply the rate by the number of years or fraction of a year—that is, $\exp(-r^cT)$ or $\exp(r^cT)$.

As an example, consider a forward contract on France's CAC 40 Index. The index is at 5475, the continuously compounded dividend yield is 1.5 percent, and the continuously compounded risk-free interest rate is 4.625 percent. The contract life is two years. With T = 2, the contract price is, therefore,

$$F(0,T) = F(0,2) = \left(5475 \times e^{-0.015(2)}\right)e^{0.04625(2)} = 5828.11$$

This specification involving a continuous dividend yield is commonly used when the underlying is a portfolio or stock index. If a single stock in the portfolio pays a dividend, then the portfolio or index can be viewed as paying a dividend. Given the diversity of dividend policies and ex-dividend dates, such an assumption is usually considered a reasonable approximation for stock portfolios or stock indices, but the assumption is not as appropriate for individual stocks. No general agreement exists on the most appropriate approach, and you must become comfortable with all of them. To obtain the appropriate forward price, the most important point to remember is that one way or another, the analysis must incorporate the dividend component of the stock price, portfolio value, or index level. If the contract is not trading at the correct price, then it is mispriced and arbitrage, as described in the generic forward contract pricing section, will force an alignment between the market forward price and the theoretical forward price.

Recall that the value of a forward contract is the asset price minus the forward price discounted back from the expiration date. Regardless of how the dividend is specified or even whether the underlying stock, portfolio, or index pays dividends, the valuation formulas for a forward contract on a stock differ only in that the stock price is adjusted by removing the present value of the remaining dividends:

$$V_t(0,T) = S_t - PV(D,t,T) - F(0,T)/(1+r)^{(T-t)} \quad (7)$$

where we now note that the dividends are only those paid after time t. If we are using continuous compounding,

$$V_t(0,T) = S_t e^{-\delta^c(T-t)} - F(0,T)e^{-r^c(T-t)} \quad (8)$$

At the contract initiation date, t = 0 and $V_0(0,T)$ is set to zero because no cash changes hands. At expiration, t = T and no dividends remain, so the valuation formula reduces to $S_T - F(0,T)$.

The formulas for pricing and valuation of equity forward contracts are summarized in Exhibit 6.

Exhibit 6 Pricing and Valuation Formulas for Equity Forward Contracts

Forward price = (Stock price – Present value of dividends over life of contract) × $(1 + r)^T$

or (Stock price) × $(1 + r)^T$ – Future value of dividends over life of contract

Discrete dividends over the life of the contract:

$$F(0,T) = [S_0 - PV(D,0,T)](1+r)^T \text{ or } S_0(1+r)^T - FV(D,0,T)$$

Continuous dividends at the rate δ^c:

$$F(0,T) = \left(S_0 e^{-\delta^c T}\right)e^{r^c T}$$

Value of forward contract:

$$V_t(0,T) = S_t - PV(D,t,T) - F(0,T)/(1+r)^{(T-t)}$$

Exhibit 6 (Continued)

or

$$V_t(0,T) = S_t e^{-\delta^c (T-t)} - F(0,T) e^{-r^c (T-t)}$$

EXAMPLE 2

An asset manager anticipates the receipt of funds in 200 days, which he will use to purchase a particular stock. The stock he has in mind is currently selling for \$62.50 and will pay a \$0.75 dividend in 50 days and another \$0.75 dividend in 140 days. The risk-free rate is 4.2 percent. The manager decides to commit to a future purchase of the stock by going long a forward contract on the stock.

A At what price would the manager commit to purchase the stock in 200 days through a forward contract?

B Suppose the manager enters into the contract at the price you found in Part A. Now, 75 days later, the stock price is \$55.75. Determine the value of the forward contract at this point.

C It is now the expiration day, and the stock price is \$58.50. Determine the value of the forward contract at this time.

S_0 = \$62.50
T = 200/365
D_1 = \$0.75, t_1 = 50/365
D_2 = \$0.75, t_2 = 140/365
r = 0.042

Solution to A:

First find the present value of the dividends:

$$\$0.75/(1.042)^{50/365} + \$0.75(1.042)^{140/365} = \$1.48$$

Then find the forward price:

$$F(0,T) = F(0,200/365) = (\$62.50 - \$1.48)(1.042)^{200/365} = \$62.41$$

Solution to B:

We must now find the present value of the dividends 75 days after the contract begins. The first dividend has already been paid, so it is not relevant. Because only one remains, the second dividend is now the "first" dividend. It will be paid in 65 days. Thus, $t_1 - t = 65/365$. The present value of this dividend is \$0.75/$(1.042)^{65/365}$ = \$0.74. The other information is

t = 75/365
T − t = (200 − 75)/365 = 125/365
S_t = \$55.75

The value of the contract is, therefore,

$$V_t(0,T) = V_{75/365}(0,200/365) = (\$55.75 - \$0.74) - \$62.41/(1.042)^{125/365} = -\$6.53$$

Thus, the contract has a negative value.

Solution to C:

S_T = \$58.50

The value of the contract is

$V_{200/365}(0,200/365) = V_T(0,T) = \$58.50 - \$62.41 = -\$3.91$

Thus, the contract expires with a value of negative \$3.91.

4.3 Pricing and Valuation of Fixed-Income and Interest Rate Forward Contracts

Forward contracts on fixed-income securities are priced and valued in a virtually identical manner to their equity counterparts. We can use the above formulas if S_t represents the bond price at time t and D_i represents a coupon paid at time t_i. We denote B^c as a coupon bond and then use notation to draw attention to those coupons that must be included in the forward contract pricing calculations. We will let $B_t^c(T + Y)$ represent the bond price at time t, T is the expiration date of the forward contract, Y is the remaining maturity of the bond on the forward contract expiration, and (T + Y) is the time to maturity of the bond at the time the forward contract is initiated. Consider a bond with n coupons to occur before its maturity date. Converting our formula for a forward contract on a stock into that for a forward contract on a bond and letting CI be the coupon interest over a specified period of time, we have a forward price of

$$F(0,T) = \left[B_0^c(T + Y) - PV(CI,0,T)\right](1 + r)^T \quad \textbf{(9)}$$

where PV(CI,0,T) is the present value of the coupon interest over the life of the forward contract. Alternatively, the forward price can be obtained as

$$F(0,T) = \left[B_0^c(T + Y)\right](1 + r)^T - FV(CI,0,T) \quad \textbf{(10)}$$

where FV(CI,0,T) is the future value of the coupon interest over the life of the forward contract.

The value of the forward contract at time t would be

$$V_t(0,T) = B_t^c(T + Y) - PV(CI,t,T) - F(0,T)/(1 + r)^{(T-t)} \quad \textbf{(11)}$$

at time t; note that the relevant coupons are only those remaining as of time t until expiration of the forward contract. As in the case for stock, this formula will reduce to the appropriate values at time 0 and at expiration. For example, at expiration, no coupons would remain, t = T, and $V_T(0,T) = B_T^C(T + Y) - F(0,T)$. At time t = 0, the contract is being initiated and has a zero value, which leads to the formula for F(0,T) above.

Consider a bond with semiannual coupons. The bond has a current maturity of 583 days and pays four coupons, each six months apart. The next coupon occurs in 37 days, followed by coupons in 219 days, 401 days, and 583 days, at which time the principal is repaid. Suppose that the bond price, which includes accrued interest, is \$984.45 for a \$1,000 par, 4 percent coupon bond. The coupon rate implies that each coupon is \$20. The risk-free interest rate is 5.75 percent. Assume that the forward contract expires in 310 days. Thus, T = 310, T + Y = 583, and Y = 273, meaning that the bond has 273 days remaining after the forward contract expires. Note that only the first two coupons occur during the life of the forward contract.

The present value of the coupons is

$\$20/(1.0575)^{37/365} + \$20/(1.0575)^{219/365} = \$39.23$

The forward price if the contract is initiated now is

$$F(0,T) = (\$984.45 - \$39.23)(1.0575)^{310/365} = \$991.18$$

Thus, we assume that we shall be able to enter into this contract to buy the bond in 310 days at the price of $991.18.

Now assume it is 15 days later and the new bond price is $973.14. Let the risk-free interest rate now be 6.75 percent. The present value of the remaining coupons is

$$\$20/(1.0675)^{22/365} + \$20/(1.0675)^{204/365} = \$39.20$$

The value of the forward contract is thus

$$\$973.14 - \$39.20 - \$991.19/(1.0675)^{295/365} = -\$6.28$$

The contract has gone from a zero value at the start to a negative value, primarily as a result of the decrease in the price of the underlying bond.

Exhibit 7 Pricing and Valuation Formulas for Fixed-Income Forward Contracts

Forward price = (Bond price – Present value of coupons over life of contract)$(1 + r)^T$ or (Bond price)$(1 + r)^T$ – Future value of coupons over life of contract

Price of forward contract on bond with coupons CI:

$$F(0,T) = \left[B_0^c(T+Y) - PV(CI,0,T)\right](1+r)^T$$

$$\text{or } \left[B_0^c(T+Y)\right](1+r)^T - FV(CI,0,T)$$

Value of forward contract on bond with coupons CI:

$$V_t(0,T) = B_t^c(T+Y) - PV(CI,t,T) - F(0,T)\big/(1+r)^{(T-t)}$$

If the bond is a zero-coupon bond/T-bill, we can perform the same analysis as above, but we simply let the coupons equal zero.

Exhibit 7 summarizes the formulas for the pricing and valuation of forward contracts on fixed-income securities.

EXAMPLE 3

An investor purchased a bond when it was originally issued with a maturity of five years. The bond pays semiannual coupons of $50. It is now 150 days into the life of the bond. The investor wants to sell the bond the day after its fourth coupon. The first coupon occurs 181 days after issue, the second 365 days, the third 547 days, and the fourth 730 days. At this point (150 days into the life of the bond), the price is $1,010.25. The bond prices quoted here include accrued interest.

A At what price could the owner enter into a forward contract to sell the bond on the day after its fourth coupon? Note that the owner would receive that fourth coupon. The risk-free rate is currently 8 percent.

B Now move forward 365 days. The new risk-free interest rate is 7 percent and the new price of the bond is $1,025.375. The counterparty to the forward contract believes that it has received a gain on the position. Determine the value of the forward contract and the gain or loss to the counterparty at this time. Note that we have now introduced a new risk-free rate, because interest rates can obviously change over the life of the bond and any calculations of the forward contract value must reflect this fact. The new risk-free rate is used instead of the old rate in the valuation formula.

Solution to A:

First we must find the present value of the four coupons over the life of the forward contract. At the 150th day of the life of the bond, the coupons occur 31 days from now, 215 days from now, 397 days from now, and 580 days from now. Keep in mind that we need consider only the first four coupons because the owner will sell the bond on the day after the fourth coupon. The present value of the coupons is

$$\$50/(1.08)^{31/365} + \$50/(1.08)^{215/365} + \$50/(1.08)^{397/365} + \$50/(1.08)^{580/365} = \$187.69$$

Because we want the forward contract to expire one day after the fourth coupon, it expires in 731 − 150 = 581 days. Thus, T = 581/365.

$$F(0,T) = F(0,581/365) = (\$1{,}010.25 - \$187.69)(1.08)^{581/365} = \$929.76$$

Solution to B:

It is now 365 days later—the 515th day of the bond's life. There are two coupons to go, one occurring in 547 − 515 = 32 days and the other in 730 − 515 = 215 days. The present value of the coupons is now

$$\$50/(1.07)^{32/365} + \$50/(1.07)^{215/365} = \$97.75$$

To address the value of the forward contract and the gain or loss to the counterparty, note that 731 − 515 = 216 days now remain until the contract's expiration. Because the bondholder would sell the forward contract to hedge the future sale price of the bond, the bondholder's counterparty to the forward contract would hold a long position. The value of the forward contract is the current spot price minus the present value of the coupons minus the present value of the forward price:

$$\$1{,}025.375 - \$97.75 - \$929.76/(1.07)^{216/365} = \$34.36$$

Because the contract was initiated with a zero value at the start and the counterparty is long the contract, the value of $34.36 represents a gain to the counterparty.

Now let us look at the pricing and valuation of FRAs. Previously we used the notations t and T to represent the time to a given date. The expressions t or T were, respectively, the number of days to time point t or T, each divided by 365. In the FRA market, contracts are created with specific day counts. We will use the letter h to refer to the day on which the FRA expires and the letter g to refer to an arbitrary day prior to expiration. Consider the time line shown below. We shall initiate an FRA on day 0. The FRA expires on day h. The rate underlying the FRA is the rate on an m-day Eurodollar deposit. Thus, there are h days from today until the FRA expiration and h + m days until the maturity date of the Eurodollar instrument on which the FRA rate is based. The date indicated by g will simply be a date during the life of the FRA at which we want to determine a value for the FRA.

Now let us specify some notation. We let $L_i(j)$ represent the rate on a j-day Libor deposit on an arbitrary day i, which falls somewhere in the above period from 0 to h, inclusive. Remember that this instrument is a j-day loan from one bank to another. For example, the bank borrowing \$1 on day i for j days will pay back the amount

$$\$1\left[1 + L_i(j)\left(\frac{j}{360}\right)\right]$$

in j days.

The rate for m-day Libor on day h, $L_h(m)$, will determine the payoff of the FRA. We denote the fixed rate on the FRA as FRA(0,h,m), which stands for the rate on an FRA established on day 0, expiring on day h, and based on m-day Libor. We shall use a \$1 notional principal for the FRA, which means that at expiration its payoff is

$$\frac{\left[L_h(m) - \text{FRA}(0,h,m)\right]\left(\frac{m}{360}\right)}{1 + L_h(m)\left(\frac{m}{360}\right)} \tag{12}$$

The numerator is the difference between the underlying Libor on the expiration day and the rate agreed on when the contract was initiated, multiplied by the adjustment factor m/360. Both of these rates are annual rates applied to a Eurodollar deposit of m days; hence, multiplying by m/360 is necessary. The denominator discounts the payoff by the m-day Libor in effect at the time of the payoff. As noted earlier, this adjustment is necessary because the rates in the numerator apply to Eurodollar deposits created on day h and paying off m days later. If the notional principal is anything other than \$1, we also must multiply the above payoff by the notional principal to determine the actual payoff.

To derive the formula for pricing an FRA, a specific arbitrage transaction involving Eurodollars and FRAs is required. We omit the details of this somewhat complex transaction, but the end result is that the FRA rate is given by the following formula:

$$\text{FRA}(0,h,m) = \left[\frac{1 + L_0(h+m)\left(\frac{h+m}{360}\right)}{1 + L_0(h)\left(\frac{h}{360}\right)} - 1\right]\left(\frac{360}{m}\right) \tag{13}$$

This somewhat awkward-looking formula is actually just the formula for a Libor forward rate, given the interest payment conventions in the FRA market. The numerator is the future value of a Eurodollar deposit of h + m days. The denominator is the future value of a shorter-term Eurodollar deposit of h days. This ratio is 1 plus a rate; subtracting 1 and multiplying by 360/m annualizes the rate.[21]

Consider a 3 × 9 FRA. This instrument expires in 90 days and is based on 180-day Libor. Thus, the Eurodollar deposit on which the underlying rate is based begins in 90 days and matures in 270 days. Because we are on day 0, h = 90, m = 180, and h + m = 270. Let the current rates be

$L_0(h) = L_0(90) = 0.056$

$L_0(h + m) = L_0(270) = 0.06$

21 To compare with the traditional method of calculating a forward rate, consider a two-year rate of 10 percent and a one-year rate of 9 percent. The forward rate is $[(1.10)^2/(1.09)] - 1 = 0.1101$. The numerator is the future value of the longer-term bond, and the denominator is the future value of the shorter-term bond. The ratio is 1 plus the rate. We do not need to annualize in this example, because the forward rate is on a one-year bond.

In other words, the 90-day rate is 5.6 percent, and the 270-day rate is 6 percent. With h = 90 and m = 180, using our formula for the FRA rate, we obtain

$$\text{FRA}(0,h,m) = \text{FRA}(0,90,180) = \left[\frac{1 + 0.06\left(\frac{270}{360}\right)}{1 + 0.056\left(\frac{90}{360}\right)} - 1\right]\left(\frac{360}{180}\right) = 0.0611$$

So to enter into an FRA on day 0, the rate would be 6.11 percent.[22]

As noted, the initial outlay for entering the forward contract is zero. Thus, the initial value is zero. Later during the life of the contract, its value will rise above or fall below zero. Now let us determine the value of an FRA during its life. Specifically, we use the notation $V_g(0,h,m)$ to represent the value of an FRA on day g, which was established on day 0, expires on day h, and is based on m-day Libor. Omitting the derivation, the approximate value of the FRA will be

$$V_g(0,h,m) = \frac{1}{1 + L_g(h-g)\left(\frac{h-g}{360}\right)} - \frac{1 + \text{FRA}(0,h,m)\left(\frac{m}{360}\right)}{1 + L_g(h+m-g)\left(\frac{h+m-g}{360}\right)} \tag{14}$$

This formula looks complicated, but the ideas behind it are actually quite simple. Recall that we are at day g. The first term on the right-hand side is the present value of $1 received at day h. The second term is the present value of 1 plus the FRA rate to be received on day h + m, the maturity date of the underlying Eurodollar time deposit.

Assume that we go long the FRA, and it is 25 days later. We need to assign a value to the FRA. First note that g = 25, h − g = 90 − 25 = 65, and h + m − g = 90 + 180 − 25 = 245. In other words, we are 25 days into the contract, 65 days remain until expiration, and 245 days remain until the maturity of the Eurodollar deposit on which the underlying Libor is based. First we need information about the new term structure. Let

$L_g(h - g) = L_{25}(65) = 0.059$

$L_g(h + m - g) = L_{25}(245) = 0.065$

We now use the formula for the value of the FRA to obtain

$$V_g(0,h,m) = V_{25}(0,90,180) = \frac{1}{1 + 0.059\left(\frac{65}{360}\right)} - \frac{1 + 0.0611\left(\frac{180}{360}\right)}{1 + 0.065\left(\frac{245}{360}\right)} = 0.0026$$

Thus, we went long this FRA on day 0. Then 25 days later, the term structure changes to the rates used here and the FRA has a value of $0.0026 per $1 notional principal. If the notional principal is any amount other than $1, we multiply the notional principal by $0.0026 to obtain the full market value of the FRA.

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22 It is worthwhile to point out again that this rate is the forward rate in the Libor term structure.

Exhibit 8 Pricing and Valuation Formulas for Interest Rate Forward Contracts (FRAs)

Forward price (rate):

$$FRA(0,h,m) = \left[\frac{1 + L_0(h+m)\left(\frac{h+m}{360}\right)}{1 + L_0(h)\left(\frac{h}{360}\right)} - 1\right]\left(\frac{360}{m}\right)$$

Value of FRA on day g:

$$V_g(0,h,m) = \frac{1}{1 + L_g(h-g)\left(\frac{h-g}{360}\right)} - \frac{1 + FRA(0,h,m)\left(\frac{m}{360}\right)}{1 + L_g(h+m-g)\left(\frac{h+m-g}{360}\right)}$$

We summarize the FRA formulas in Exhibit 8. We have now looked at the pricing and valuation of equity, fixed-income, and interest rate forward contracts. One of the most widely used types of forward contracts is the currency forward. The pricing and valuation of currency forwards is remarkably similar to that of equity forwards.

EXAMPLE 4

A corporate treasurer needs to hedge the risk of the interest rate on a future transaction. The risk is associated with the rate on 180-day Euribor in 30 days. The relevant term structure of Euribor is given as follows:

30-day Euribor	5.75%
210-day Euribor	6.15%

A State the terminology used to identify the FRA in which the manager is interested.

B Determine the rate that the company would get on an FRA expiring in 30 days on 180-day Euribor.

C Suppose the manager went long this FRA. Now, 20 days later, interest rates have moved significantly downward to the following:

10-day Euribor	5.45%
190-day Euribor	5.95%

The manager would like to know where the company stands on this FRA transaction. Determine the market value of the FRA for a €20 million notional principal.

D On the expiration day, 180-day Euribor is 5.72 percent. Determine the payment made to or by the company to settle the FRA contract.

Solution to A:

This transaction would be identified as a 1 × 7 FRA.

Solution to B:

Here the notation would be h = 30, m = 180, h + m = 210. Then

$$\text{FRA}(0,h,m) = \text{FRA}(0,30,180) = \left[\frac{1 + 0.0615\left(\frac{210}{360}\right)}{1 + 0.0575\left(\frac{30}{360}\right)} - 1\right]\left(\frac{360}{180}\right) = 0.0619$$

Solution to C:

Here g = 20, h − g = 30 − 20 = 10, h + m − g = 30 + 180 − 20 = 190. The value of the FRA for a €1 notional principal would be

$$V_g(0,h,m) = V_{20}(0,30,180) = \frac{1}{1 + 0.0545\left(\frac{10}{360}\right)} - \frac{1 + 0.0619\left(\frac{180}{360}\right)}{1 + 0.0595\left(\frac{190}{360}\right)} = -0.0011$$

Thus, for a notional principal of €20 million, the value would be €20,000,000(−0.0011) = −€22,000.

Solution to D:

At expiration, the payoff is

$$\frac{\left[L_h(m) - \text{FRA}(0,h,m)\right]\left(\frac{m}{360}\right)}{1 + L_h(m)\left(\frac{m}{360}\right)} = \frac{(0.0572 - 0.0619)\left(\frac{180}{360}\right)}{1 + 0.0572\left(\frac{180}{360}\right)} = -0.0023$$

For a notional principal of €20 million, the payoff would then be €20,000,000(−0.0023) = −€46,000. Thus, €46,000 would be paid by the company, because it is long and the final rate was lower than the FRA rate.

4.4 Pricing and Valuation of Currency Forward Contracts

Foreign currency derivative transactions as well as spot transactions must be handled with care. The exchange rate can be quoted in terms of units of the domestic currency per unit of foreign currency, or units of the foreign currency per unit of the domestic currency. In this reading, we shall always quote exchange rates in terms of units of the domestic currency per unit of the foreign currency, which is also called a direct quote. This approach is in keeping with the way in which other underlying assets are quoted. For example, from the perspective of a US investor, a stock that sells for $50 is quoted in units of the domestic currency per unit (share) of stock. Likewise, if the euro exchange rate is quoted as $0.90, then the euro sells for $0.90 per unit, which is one euro. Alternatively, we could quote that $1 sells for 1/$0.90 = €1.1111—that is, €1.1111 per $1; in this case, units of foreign currency per one unit of domestic currency from the perspective of a US investor. In fact, this type of quote is commonly used and is called an indirect quote. Taking that approach, however, we would quote the stock price as 1/$50 = 0.02 shares per $1, a very unusual and awkward way to quote a stock price.

By taking the approach of quoting prices in terms of units of the domestic currency per unit of foreign currency, we facilitate a comparison of currencies and their derivatives with equities and their derivatives—a topic we have already covered. For example, we have previously discussed the case of a stock selling for S_0, which represents units of the domestic currency per share of stock. Likewise, we shall treat the currency as having an exchange rate of S_0, meaning that it is selling for S_0. We also need the foreign interest rate, denoted as r^f, and the domestic interest rate, denoted as r.[23]

Consider the following transactions executed today (time 0), assuming a contract expiration date of T:

> *Take $S_0/(1 + r^f)^T$ units of the domestic currency and convert it to $1/(1 + r^f)^T$ units of the foreign currency.*[24]
>
> *Sell a forward contract to deliver one unit of the foreign currency at the rate F(0,T) expiring at time T.*
>
> *Hold the position until time T. The $1/(1 + r^f)^T$ units of foreign currency will accrue interest at the rate r^f and grow to one unit of the currency at T as follows:*

$$\left(\frac{1}{1 + r^f}\right)^T \left(1 + r^f\right)^T = 1$$

Thus, at expiration we shall have one unit of the foreign currency, which is then delivered to the holder of the long forward contract, who pays the amount F(0,T). This amount was known at the start of the transaction. Because the risk has been hedged away, the exchange rate at expiration is irrelevant. Hence, this transaction is risk-free. Accordingly, the present value of F(0,T), found by discounting at the domestic risk-free interest rate, must equal the initial outlay of $S_0/(1 + r^f)^T$. Setting these amounts equal and solving for F(0,T) gives

$$F(0,T) = \left[\frac{S_0}{\left(1 + r^f\right)^T}\right](1 + r)^T \quad \textbf{(15)}$$

The term in brackets is the spot exchange rate discounted by the foreign interest rate. This term is then compounded at the domestic interest rate to the expiration day.[25]

Recall that in pricing equity forwards, we always reduced the stock price by the present value of the dividends and then compounded the resulting value to the expiration date. We can view currencies in the same way. The stock makes cash payments that happen to be called dividends; the currency makes cash payments that happen to be called interest. Although the time pattern of how a stock pays dividends is quite different from the time pattern of how interest accrues, the general idea is the same. After reducing the spot price or rate by any cash flows over the life of the contract, the resulting value is then compounded at the risk-free rate to the expiration day.

The formula we have obtained here is simply a variation of the formula used for other types of forward contracts. In international financial markets, however, this formula has acquired its own name: **interest rate parity** (sometimes called covered interest rate parity). It expresses the equivalence, or parity, of spot and forward exchange rates, after adjusting for differences in the interest rates in the two countries. One implication of interest rate parity is that the forward rate will exceed (be less than)

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23 We do not use a superscript "d" for the domestic rate, because in all previous examples we have used r to denote the interest rate in the home country of the investor.

24 In other words, if one unit of the foreign currency costs S_0, then $S_0/(1 + r^f)^T$ units of the domestic currency would, therefore, buy $1/(1 + r^f)^T$ units of the foreign currency.

25 It is also common to see the above Equation 15 written inversely, with the spot rate divided by the domestic interest factor and compounded by the foreign interest factor. This variation would be appropriate if the spot and forward rates were quoted in terms of units of the foreign currency per unit of domestic currency (indirect quotes). As we mentioned earlier, however, it is easier to think of a currency as just another asset, which naturally should have its price quoted in units of the domestic currency per unit of the asset or foreign currency.

the spot rate if the domestic interest rate exceeds (is less than) the foreign interest rate. With a direct quote, if the forward rate exceeds (is less than) the spot rate, the foreign currency is said to be selling at a premium (discount). One should not, on the basis of this information, conclude that a currency selling at a premium is expected to increase or one selling at a discount is expected to decrease. A forward premium or discount is merely an implication of the relationship between interest rates in the two countries. More information would be required to make any assumptions about the outlook for the exchange rate.

If the forward rate in the market does not equal the forward rate given by interest rate parity, then an arbitrage transaction can be executed. Indeed, a similar relationship is true for any of the forward rates we have studied. In the foreign exchange markets, however, this arbitrage transaction has its own name: **covered interest arbitrage**. If the forward rate in the market is higher than the rate given by interest rate parity, then the forward rate is too high. When the price of an asset or derivative is too high, it should be sold. Thus, a trader would 1) sell the forward contract at the market rate, 2) buy $1/(1 + r^f)^T$ units of the foreign currency, 3) hold the position, earning interest on the currency, and 4) at maturity of the forward contract deliver the currency and be paid the forward rate. This arbitrage transaction would earn a return in excess of the domestic risk-free rate without any risk. If the forward rate is less than the rate given by the formula, the trader does the opposite, selling the foreign currency and buying a forward contract, in a similar manner. The combined actions of many traders undertaking this transaction will bring the forward price in the market in line with the forward price given by the model.

In Equation 15, both interest rates were annual rates with discrete compounding. In dealing with equities, we sometimes assume that the dividend payments are made continuously. Similarly, we could also assume that interest is compounded continuously. If that is the case, let r^{fc} be the continuously compounded foreign interest rate, defined as $r^{fc} = \ln(1 + r^f)$, and as before, let r^c be the continuously compounded domestic interest rate. Then the forward price is given by the same formula, with appropriately adjusted symbols, as we obtained when working with equity derivatives:

$$F(0,T) = \left(S_0 e^{-r^{fc}T}\right) e^{r^c T} \tag{16}$$

Now consider how we might value a foreign currency forward contract at some point in time during its life. In fact, we already know how: We simply apply to foreign currency forward contracts what we know about the valuation of equity forwards during the contract's life. Recall that the value of an equity forward is the stock price minus the present value of the dividends over the remaining life of the contract minus the present value of the forward price over the remaining life of the contract. An analogous formula for a currency forward gives us

$$V_t(0,T) = \frac{S_t}{\left(1 + r^f\right)^{(T-t)}} - \frac{F(0,T)}{(1 + r)^{(T-t)}} \tag{17}$$

In other words, we take the current exchange rate at time t, S_t, discount it by the foreign interest rate over the remaining life of the contract, and subtract the forward price discounted by the domestic interest rate over the remaining life of the contract. Under the assumption that we are using continuous compounding and discounting, the formula would be

$$V_t(0,T) = \left(S_t e^{-r^{fc}(T-t)}\right) - F(0,T) e^{-r^c(T-t)} \tag{18}$$

For example, suppose the domestic currency is the US dollar and the foreign currency is the Swiss franc. Let the spot exchange rate be \$0.5987, the US interest rate be 5.5 percent, and the Swiss interest rate be 4.75 percent. We assume these interest rates are fixed and will not change over the life of the forward contract. We also assume that these rates are based on annual compounding and are not quoted as Libor-type rates. Thus, we compound using formulas like $(1 + r)^T$, where T is the number of years and r is the annual rate.[26]

Assuming the forward contract has a maturity of 180 days, we have T = 180/365. Using the above formula for the forward rate, we find that the forward price should be

$$F(0,T) = F(0,180/365) = \left[\frac{\$0.5987}{(1.0475)^{180/365}}\right](1.055)^{180/365} = \$0.6008$$

Thus, if we entered into a forward contract, it would call for us to purchase (if long) or sell (if short) one Swiss franc in 180 days at a price of $0.6008.

Suppose we go long this forward contract. It is now 40 days later, or 140 days until expiration. The spot rate is now $0.65. As assumed above, the interest rates are fixed. With t = 40/365 and T − t = 140/365, the value of our long position is

$$V_t(0,T) = V_{40/365}(0,180/365) = \frac{\$0.6500}{(1.0475)^{140/365}} - \frac{\$0.6008}{(1.055)^{140/365}} = \$0.0499$$

So the contract value is $0.0499 per Swiss franc. If the notional principal were more than one Swiss franc, we would simply multiply the notional principal by $0.0499.

If we were working with continuously compounded rates, we would have r^c = ln(1.055) = 0.0535 and r^{fc} = ln(1.0475) = 0.0464. Then the forward price would be $F(0,T) = F(0,180/365) = (0.5987e^{-0.0464(180/365)})e^{0.0535(180/365)} = 0.6008$, and the value 40 days later would be $V_{40/365}(0,180/365) = 0.65e^{-0.0464(140/365)} - 0.6008e^{-0.0535(140/365)}$ = 0.0499. These are the same results we obtained working with discrete rates.

Exhibit 9 summarizes the formulas for pricing and valuation of currency forward contracts.

Exhibit 9 Pricing and Valuation Formulas for Currency Forward Contracts

Forward price (rate) = (Spot price discounted by foreign interest rate) compounded at domestic interest rate:

$$\text{Discrete interest: } F(0,T) = \left[\frac{S_0}{(1 + r^f)^T}\right](1 + r)^T$$

$$\text{Continuous interest: } F(0,T) = \left(S_0 e^{-r^{fc}T}\right)e^{r^cT}$$

Value of forward contract:

$$\text{Discrete interest: } V_t(0,T) = \left[\frac{S_t}{(1 + r^f)^{(T-t)}}\right] - \frac{F(0,T)}{(1 + r)^{(T-t)}}$$

$$\text{Continuous interest: } V_t(0,T) = \left[S_t e^{-r^{fc}(T-t)}\right] - F(0,T)e^{-r^c(T-t)}$$

Note: The exchange rate is quoted in units of domestic currency per unit of foreign currency.

26 If these were Libor-style rates, the interest would be calculated using the factor 1 + [Rate(Days/360)].

EXAMPLE 5

The spot rate for British pounds is \$1.76. The US risk-free rate is 5.1 percent, and the UK risk-free rate is 6.2 percent; both are compounded annually. One-year forward contracts are currently quoted at a rate of \$1.75.

A Identify a strategy with which a trader can earn a profit at no risk by engaging in a forward contract, regardless of her view of the pound's likely movements. Carefully describe the transactions the trader would make. Show the rate of return that would be earned from this transaction. Assume the trader's domestic currency is US dollars.

B Suppose the trader simply shorts the forward contract. It is now one month later. Assume interest rates are the same, but the spot rate is now \$1.72. What is the gain or loss to the counterparty on the trade?

C At expiration, the pound is at \$1.69. What is the value of the forward contract to the short at expiration?

Solution to A:

The following information is given:

$$S_0 = \$1.76$$
$$r = 0.051$$
$$r^f = 0.062$$
$$T = 1.0$$

The forward price should be

$$F(0,T) = \left(\frac{\$1.76}{1.062}\right)(1.051) = \$1.7418$$

With the forward contract selling at \$1.75, it is slightly overpriced. Thus, the trader should be able to buy the currency and sell a forward contract to earn a return in excess of the risk-free rate at no risk. The specific transactions are as follows:

- Take \$1.76/(1.062) = \$1.6573. Use it to buy 1/1.062 = £0.9416.
- Sell a forward contract to deliver £1.00 in one year at the price of \$1.75.
- Hold the position for one year, collecting interest at the UK risk-free rate of 6.2 percent. The £0.9416 will grow to (0.9416)(1.062) = £1.00.
- At expiration, deliver the pound and receive \$1.75. This is a return of

$$\frac{1.75}{1.6573} - 1 = 0.0559$$

A risk-free return of 5.59 percent is better than the US risk-free rate of 5.1 percent, a result of the fact that the forward contract is overpriced.

Solution to B:

We now need the value of the forward contract to the counterparty, who went long at \$1.75. The inputs are

$$t = 1/12$$
$$S_t = \$1.72$$
$$T - t = 11/12$$
$$F(0,T) = \$1.75$$

The value of the forward contract to the long is

$$V_t(0,T) = \frac{1.72}{(1.062)^{11/12}} - \frac{1.75}{(1.051)^{11/12}} = -0.0443$$

which is a loss of \$0.0443 to the long and a gain of \$0.0443 to the short.

Solution to C:

The pound is worth \$1.69 at expiration. Thus, the value to the long is

$$V_T(0,T) = 1.69 - 1.75 = -0.06$$

and the value to the short is +\$0.06. Note the minus sign in the equation $V_T(0,T) = -0.06$. The value to the long is always the spot value at expiration minus the original forward price. The short will be required to deliver the foreign currency and receive \$1.75, which is \$0.06 more than market value of the pound. The contract's value to the short is thus \$0.06, which is the negative of its value to the long.

We have now seen how to determine the price and value of equity, fixed-income and interest rate, and currency forward contracts. We observed that the price is determined such that no arbitrage opportunities exist for either the long or the short. We have found that the value of a forward contract is the amount we would pay or receive to enter or exit the contract. Because no money changes hands up front, the value of a forward contract when initiated is zero. The value at expiration is determined by the difference between the spot price or rate at expiration and the forward contract price or rate. The value prior to expiration can also be determined and is the present value of the claim at expiration.

Determining the value of a forward contract is important for several reasons. One, however, is particularly important: Forward contracts contain the very real possibility that one of the parties might default. By knowing the market value, one can determine the amount of money at risk if a counterparty defaults. Let us now look at how credit risk enters into a forward contract.

CREDIT RISK AND FORWARD CONTRACTS 5

To illustrate how credit risk affects a forward contract, consider the currency forward contract example we just finished in the previous section. It concerns a contract that expires in 180 days in which the long will pay a forward rate of \$0.6008 for each Swiss franc to be received at expiration. Assume that the contract covers 10 million Swiss francs. Let us look at the problem from the point of view of the holder of the long position and the credit risk faced by this party.

Assume it is the contract expiration day and the spot rate for Swiss francs is \$0.62. The long is due to receive 10 million Swiss francs and pay \$0.6008 per Swiss franc, or \$6,008,000 in total. Now suppose that perhaps because of bankruptcy or insolvency, the short cannot come up with the \$6,200,000 that it would take to purchase the Swiss francs on the open market at the prevailing spot rate.[27] In order to obtain the Swiss francs, the long would have to buy them in the open market. Doing so would incur

27 Even if the short already holds the Swiss franc, she might be declaring bankruptcy or otherwise unable to pay debts such that the forward contract claim is combined with the claims of all of the short's other creditors.

an additional cost of \$6,200,000 – \$6,008,000 = \$192,000, which can be viewed as the credit risk at the point of expiration when the spot rate is \$0.62. Not surprisingly, this amount is also the market value of the contract at this point.

This risk is an immediate risk faced at expiration. Prior to expiration, the long faces a potential risk that the short will default. If the long wanted to gauge the potential exposure, he would calculate the current market value. In the example we used in which the long is now 40 days into the life of the contract, the market value to the long is \$0.0499 per Swiss franc. Hence, the long's exposure would be 10,000,000(\$0.0499) = \$499,000. Although no payments are due at this point, \$499,000 is the market value of the claim on the payment at expiration. Using an estimate of the probability that the short would default, the long can gauge the expected credit loss from the transaction by multiplying that probability by \$499,000.

The market value of a forward contract reflects the current value of the claim at expiration, given existing market conditions. If the Swiss franc rises significantly, the market value will increase along with it, thereby exposing the long to the potential for even greater losses. Many participants in derivatives markets estimate this potential loss by running simulations that attempt to reflect the potential market value of the contract along with the probability of the counterparty defaulting.

We have viewed credit risk from the viewpoint of the long, but what about the short's perspective? In the case in which we went to expiration and the short owed the long the greater amount, the short faces no credit risk. In the case prior to expiration in which the contract's market value was positive, the value of the future claim was greater to the long than to the short. Hence, the short still did not face any credit risk.

The short would face credit risk, however, if circumstances were such that the value of the transaction were negative to the long, which would make the value to the short positive. In that case, the scenario discussed previously in this section would apply from the short's perspective.

There are various methods of managing the credit risk of various types of derivatives transactions. At this point, however, it will be helpful to specifically examine one particular method. Let us go back to the long currency forward contract that had a market value of \$499,000. As it stands at this time, the holder of the long position has a claim on the holder of the short position that is worth \$499,000. Suppose the two parties had agreed when they entered into the transaction that in 40 days, the party owing the greater amount to the other would pay the amount owed and the contract would be repriced at the new forward rate. Now on the 40th day, the short would pay the long \$499,000. Recalling that the US interest rate was 5.5 percent and the Swiss interest rate was 4.75 percent, the contract, which now has 140 days to go (T = 140/365), would then be repriced to the rate

$$F(0,T) = F(0,140/365) = \left[\frac{\$0.65}{(1.0475)^{140/365}}\right](1.055)^{140/365} = \$0.6518$$

In other words, from this point, the contract has a new rate of \$0.6518. The long now agrees to pay \$0.6518 for the currency from the short in 140 days.

What the two parties have done is called **marking to market**. They have settled up the amount owed and marked the contract to its current market rate. If the parties agree in advance, a forward contract can be marked to market at whatever dates the parties feel are appropriate. Marking to market keeps one party from becoming too deeply indebted to the other without paying up. At the dates when the contract is marked to market, the parties restructure the contract so that it remains in force but with an updated price.

Forward contracts and swaps are sometimes marked to market to mitigate credit risk. In the reading on futures markets and contracts, we will note that a distinguishing characteristic of futures contracts is that they are marked to market every day. In essence, they are forward contracts that are marked to market and repriced daily to reduce the credit risk.

THE ROLE OF FORWARD MARKETS

6

In this reading we have discussed many aspects of forward contracts and forward markets. We will conclude the reading with a brief discussion of the role that these markets play in our financial system. Although forward, futures, options, and swap markets serve similar purposes in our society, each market is unique. Otherwise, these markets would consolidate.

Forward markets may well be the least understood of the various derivative markets. In contrast to their cousins, futures contracts, forward contracts are a far less visible segment of the financial markets. Both forwards and futures serve a similar purpose: They provide a means in which a party can commit to the future purchase or sale of an asset at an agreed-upon price, without the necessity of paying any cash until the asset is actually purchased or sold. In contrast to futures contracts, forward contracts are private transactions, permitting the ultimate in customization. As long as a counterparty can be found, a party can structure the contract completely to its liking. Futures contracts are standardized and may not have the exact terms required by the party. In addition, futures contracts, with their daily marking to market, produce interim cash flows that can lead to imperfections in a hedge transaction designed not to hedge interim events but to hedge a specific event at a target horizon date. Forward markets also provide secrecy and have only a light degree of regulation. In general, forward markets serve a specialized clientele, specifically large corporations and institutions with specific target dates, underlying assets, and risks that they wish to take or reduce by committing to a transaction without paying cash at the start.

As the reading on swap markets and contracts will make clear, however, forward contracts are just miniature versions of swaps. A swap can be viewed as a series of forward contracts. Swaps are much more widely used than forward contracts, suggesting that parties that have specific risk management needs typically require the equivalent of a series of forward contracts. A swap contract consolidates a series of forward contracts into a single instrument at lower cost.

Forward contracts are the building blocks for constructing and understanding both swaps and futures. Swaps and futures are more widely used and better known, but forward contracts play a valuable role in helping us understand swaps and futures. Moreover, as noted, for some parties, forward contracts serve specific needs not met by other derivatives.

In the reading on futures markets and contracts, we shall demonstrate how similar futures contracts are to forward contracts, but the differences are important, and some of their benefits to society are slightly different and less obvious than those of forwards.

SUMMARY

OPTIONAL SEGMENT

- The holder of a long forward contract (the "long") is obligated to take delivery of the underlying asset and pay the forward price at expiration. The holder of a short forward contract (the "short") is obligated to deliver the underlying asset and accept payment of the forward price at expiration.
- At expiration, a forward contract can be terminated by having the short make delivery of asset to the long or having the long and short exchange the equivalent cash value. If the asset is worth more (less) than the forward price, the short (long) pays the long (short) the cash difference between the market price or rate and the price or rate agreed on in the contract.
- A party can terminate a forward contract prior to expiration by entering into an opposite transaction with the same or a different counterparty. It is possible to leave both the original and new transactions in place, thereby leaving both transactions subject to credit risk, or to have the two transactions cancel each other. In the latter case, the party owing the greater amount pays the market value to the other party, resulting in the elimination of the remaining credit risk. This elimination can be achieved, however, only if the counterparty to the second transaction is the same counterparty as in the first.
- A dealer is a financial institution that makes a market in forward contracts and other derivatives. A dealer stands ready to take either side of a transaction. An end user is a party that comes to a dealer needing a transaction, usually for the purpose of managing a particular risk.
- Equity forward contracts can be written on individual stocks, specific stock portfolios, or stock indices. Equity forward contract prices and values must take into account the fact that the underlying stock, portfolio, or index could pay dividends.
- Forward contracts on bonds can be based on zero-coupon bonds or on coupon bonds, as well as portfolios or indices based on zero-coupon bonds or coupon bonds. Zero-coupon bonds pay their return by discounting the face value, often using a 360-day year assumption. Forward contracts on bonds must expire before the bond's maturity. In addition, a forward contract on a bond can be affected by special features of bonds, such as callability and convertibility.
- Eurodollar time deposits are dollar loans made by one bank to another. Although the term "Eurodollars" refers to dollar-denominated loans, similar loans exist in other currencies. Eurodollar deposits accrue interest by adding it on to the principal, using a 360-day year assumption. The primary Eurodollar rate is called Libor.
- Libor stands for London Interbank Offer Rate, the rate at which London banks are willing to lend to other London banks. Euribor is the rate on a euro time deposit, a loan made by banks to other banks in Frankfurt in which the currency is the euro.
- An FRA is a forward contract in which one party, the long, agrees to pay a fixed interest payment at a future date and receive an interest payment at a rate to be determined at expiration. FRAs are described by a special notation. For example, a 3 × 6 FRA expires in three months; the underlying is a Eurodollar deposit that begins in three months and ends three months later, or six months from now.

- The payment of an FRA at expiration is based on the net difference between the underlying rate and the agreed-upon rate, adjusted by the notional principal and the number of days in the instrument on which the underlying rate is based. The payoff is also discounted, however, to reflect the fact that the underlying rate on which the instrument is based assumes that payment will occur at a later date.
- A currency forward contract is a commitment for one party, the long, to buy a currency at a fixed price from the other party, the short, at a specific date. The contract can be settled by actual delivery, or the two parties can choose to settle in cash on the expiration day.

END OPTIONAL SEGMENT

- A forward contract is priced by assuming that the underlying asset is purchased, a forward contract is sold, and the position is held to expiration. Because the sale price of the asset is locked in as the forward price, the transaction is risk free and should earn the risk-free rate. The forward price is then obtained as the price that guarantees a return of the risk-free rate. If the forward price is too high or too low, an arbitrage profit in the form of a return in excess of the risk-free rate can be earned. The combined effects of all investors executing arbitrage transactions will force the forward price to converge to its arbitrage-free level.
- The value of a forward contract is determined by the fact that a long forward contract is a claim on the underlying asset and a commitment to pay the forward price at expiration. The value of a forward contract is, therefore, the current price of the asset less the present value of the forward price at expiration. Because no money changes hands at the start, the value of the forward contract today is zero. The value of a forward contract at expiration is the price of the underlying asset minus the forward price.
- Valuation of a forward contract is important because 1) it makes good business sense to know the values of future commitments, 2) accounting rules require that forward contracts be accounted for in income statements and balance sheets, 3) the value gives a good measure of the credit exposure, and 4) the value can be used to determine the amount of money one party would have to pay another party to terminate a position.
- An off-market forward contract is established with a nonzero value at the start. The contract will, therefore, have a positive or negative value and require a cash payment at the start. A positive value is paid by the long to the short; a negative value is paid by the short to the long. In an off-market forward contract, the forward price will not equal the price of the underlying asset compounded at the risk-free rate but rather will be set in the process of negotiation between the two parties.
- An equity forward contract is priced by taking the stock price, subtracting the present value of the dividends over the life of the contract, and then compounding this amount at the risk-free rate to the expiration date of the contract. The present value of the dividends can be found by assuming the dividends are risk-free and calculating their present value using the risk-free rate of interest. Or one can assume that dividends are paid at a constant continuously compounded rate and then discount the stock price by the exponential function using the continuously compounded dividend rate. Alternatively, an equity forward can be priced by compounding the stock price to the expiration date and then subtracting the future value of the dividends at the expiration date. The value of an equity forward contract is the stock price minus the present value of the dividends minus the present value of the forward price that will be paid at expiration.

- To price a fixed-income forward contract, take the bond price, subtract the present value of the coupons over the life of the contract, and compound this amount at the risk-free rate to the expiration date of the contract. The value of a fixed-income forward contract is the bond price minus the present value of the coupons minus the present value of the forward price that will be paid at expiration.
- The price of an FRA, which is actually a rate, is simply the forward rate embedded in the term structure of the FRA's underlying rate. The value of an FRA based on a Eurodollar deposit is the present value of $1 to be received at expiration minus the present value of $1 plus the FRA rate to be received at the maturity date of the Eurodollar deposit on which the FRA is based, with appropriate (days/360) adjustments.
- The price, which is actually an exchange rate, of a forward contract on a currency is the spot rate discounted at the foreign interest rate over the life of the contract and then compounded at the domestic interest rate to the expiration date of the contract. The value of a currency forward contract is the spot rate discounted at the foreign interest rate over the life of the contract minus the present value of the forward rate at expiration.
- Credit risk in a forward contract arises when the counterparty that owes the greater amount is unable to pay at expiration or declares bankruptcy prior to expiration. The market value of a forward contract is a measure of the net amount one party owes the other. Only one party, the one owing the lesser amount, faces credit risk at any given time. Because the market value can change from positive to negative, however, the other party has the potential for facing credit risk at a later date. Counterparties occasionally mark forward contracts to market, with one party paying the other the current market value; they then reprice the contract to the current market price or rate.
- Forward markets play an important role in society, providing a means by which a select clientele of parties can engage in customized, private, unregulated transactions that commit them to buying or selling an asset at a later date at an agreed-upon price without paying any cash at the start. Forward contracts also are a simplified version of both futures and swaps and, therefore, form a basis for understanding these other derivatives.

PRACTICE PROBLEMS

1 Consider a security that sells for $1,000 today. A forward contract on this security that expires in one year is currently priced at $1,100. The annual rate of interest is 6.75 percent. Assume that this is an off-market forward contract.

A Calculate the value of the forward contract today (at inception) $V_0(0,T)$.

B Indicate whether payment is made by the long to the short or vice versa.

2 Assume that you own a security currently worth $500. You plan to sell it in two months. To hedge against a possible decline in price during the next two months, you enter into a forward contract to sell the security in two months. The risk-free rate is 3.5 percent.

A Calculate the forward price on this contract.

B Suppose the dealer offers to enter into a forward contract at $498. Indicate how you could earn an arbitrage profit.

C After one month, the security sells for $490. Calculate the gain or loss to your position.

3 Consider an asset currently worth $100. An investor plans to sell it in one year and is concerned that the price may have fallen significantly by then. To hedge this risk, the investor enters into a forward contract to sell the asset in one year. Assume that the risk-free rate is 5 percent.

A Calculate the appropriate price at which this investor can contract to sell the asset in one year.

B Three months into the contract, the price of the asset is $90. Calculate the gain or loss that has accrued to the forward contract.

C Assume that five months into the contract, the price of the asset is $107. Calculate the gain or loss on the forward contract.

D Suppose that at expiration, the price of the asset is $98. Calculate the value of the forward contract at expiration. Also indicate the overall gain or loss to the investor on the whole transaction.

E Now calculate the value of the forward contract at expiration assuming that at expiration, the price of the asset is $110. Indicate the overall gain or loss to the investor on the whole transaction. Is this amount more or less than the overall gain or loss from Part D?

4 A security is currently worth $225. An investor plans to purchase this asset in one year and is concerned that the price may have risen by then. To hedge this risk, the investor enters into a forward contract to buy the asset in one year. Assume that the risk-free rate is 4.75 percent.

A Calculate the appropriate price at which this investor can contract to buy the asset in one year.

B Four months into the contract, the price of the asset is $250. Calculate the gain or loss that has accrued to the forward contract.

C Assume that eight months into the contract, the price of the asset is $200. Calculate the gain or loss on the forward contract.

D Suppose that at expiration, the price of the asset is $190. Calculate the value of the forward contract at expiration. Also indicate the overall gain or loss to the investor on the whole transaction.

E Now calculate the value of the forward contract at expiration assuming that at expiration, the price of the asset is $240. Indicate the overall gain or loss to the investor on the whole transaction. Is this amount more or less than the overall gain or loss from Part D?

5 Assume that a security is currently priced at $200. The risk-free rate is 5 percent.

A A dealer offers you a contract in which the forward price of the security with delivery in three months is $205. Explain the transactions you would undertake to take advantage of the situation.

B Suppose the dealer were to offer you a contract in which the forward price of the security with delivery in three months is $198. How would you take advantage of the situation?

6 Assume that you own a dividend-paying stock currently worth $150. You plan to sell the stock in 250 days. In order to hedge against a possible price decline, you wish to take a short position in a forward contract that expires in 250 days. The risk-free rate is 5.25 percent. Over the next 250 days, the stock will pay dividends according to the following schedule:

Days to Next Dividend	Dividends per Share ($)
30	1.25
120	1.25
210	1.25

A Calculate the forward price of a contract established today and expiring in 250 days.

B It is now 100 days since you entered the forward contract. The stock price is $115. Calculate the value of the forward contract at this point.

C At expiration, the price of the stock is $130. Calculate the value of the forward contract at expiration.

7 A portfolio manager expects to purchase a portfolio of stocks in 90 days. In order to hedge against a potential price increase over the next 90 days, she decides to take a long position on a 90-day forward contract on the S&P 500 stock index. The index is currently at 1145. The continuously compounded dividend yield is 1.75 percent. The discrete risk-free rate is 4.25 percent.

A Calculate the no-arbitrage forward price on this contract.

B It is now 28 days since the portfolio manager entered the forward contract. The index value is at 1225. Calculate the value of the forward contract 28 days into the contract.

C At expiration, the index value is 1235. Calculate the value of the forward contract.

8 An investor purchased a newly issued bond with a maturity of 10 years 200 days ago. The bond carries a coupon rate of 8 percent paid semiannually and has a face value of $1,000. The price of the bond with accrued interest is currently $1,146.92. The investor plans to sell the bond 365 days from now. The schedule of coupon payments over the first two years, from the date of purchase, is as follows:

Coupon	Days after Purchase	Amount ($)
First	181	40
Second	365	40
Third	547	40
Fourth	730	40

A Should the investor enter into a long or short forward contract to hedge his risk exposure? Calculate the no-arbitrage price at which the investor should enter the forward contract. Assume that the risk-free rate is 6 percent.

B The forward contract is now 180 days old. Interest rates have fallen sharply, and the risk-free rate is 4 percent. The price of the bond with accrued interest is now $1,302.26. Determine the value of the forward contract now and indicate whether the investor has accrued a gain or loss on his position.

9 A corporate treasurer wishes to hedge against an increase in future borrowing costs due to a possible rise in short-term interest rates. She proposes to hedge against this risk by entering into a long 6 × 12 FRA. The current term structure for Libor is as follows:

Term (Days)	Interest Rate (%)
30	5.10
90	5.25
180	5.70
360	5.95

A Indicate when this 6 × 12 FRA expires and identify which term of the Libor this FRA is based on.

B Calculate the rate the treasurer would receive on a 6 × 12 FRA.

Suppose the treasurer went long this FRA. Now, 45 days later, interest rates have risen and the Libor term structure is as follows:

Term (Days)	Interest Rate (%)
135	5.90
315	6.15

C Calculate the market value of this FRA based on a notional principal of $10,000,000.

D At expiration, the 180-day Libor is 6.25 percent. Calculate the payoff on the FRA. Does the treasurer receive a payment or make a payment to the dealer?

10 A financial manager needs to hedge against a possible decrease in short-term interest rates. He decides to hedge his risk exposure by going short on an FRA that expires in 90 days and is based on 90-day Libor. The current term structure for Libor is as follows:

Term (Days)	Interest Rate (%)
30	5.83
90	6.00
180	6.14
360	6.51

A Identify the type of FRA used by the financial manager using the appropriate terminology.

B Calculate the rate the manager would receive on this FRA.

It is now 30 days since the manager took a short position in the FRA. Interest rates have shifted down, and the new term structure for Libor is as follows:

Term (Days)	Interest Rate (%)
60	5.50
150	5.62

C Calculate the market value of this FRA based on a notional principal of \$15,000,000.

11 Consider a US-based company that exports goods to Switzerland. The US company expects to receive payment on a shipment of goods in three months. Because the payment will be in Swiss francs, the US company wants to hedge against a decline in the value of the Swiss franc over the next three months. The US risk-free rate is 2 percent, and the Swiss risk-free rate is 5 percent. Assume that interest rates are expected to remain fixed over the next six months. The current spot rate is \$0.5974.

A Indicate whether the US company should use a long or short forward contract to hedge currency risk.

B Calculate the no-arbitrage price at which the US company could enter into a forward contract that expires in three months.

C It is now 30 days since the US company entered into the forward contract. The spot rate is \$0.55. Interest rates are the same as before. Calculate the value of the US company's forward position.

12 The euro currently trades at \$1.0231. The dollar risk-free rate is 4 percent, and the euro risk-free rate is 5 percent. Six-month forward contracts are quoted at a rate of \$1.0225. Indicate how you might earn a risk-free profit by engaging in a forward contract. Clearly outline the steps you undertake to earn this risk-free profit.

13 Suppose that you are a US-based importer of goods from the United Kingdom. You expect the value of the pound to increase against the US dollar over the next 30 days. You will be making payment on a shipment of imported goods in 30 days and want to hedge your currency exposure. The US risk-free rate is 5.5 percent, and the UK risk-free rate is 4.5 percent. These rates are expected to remain unchanged over the next month. The current spot rate is \$1.50.

A Indicate whether you should use a long or short forward contract to hedge the currency risk.

B Calculate the no-arbitrage price at which you could enter into a forward contract that expires in 30 days.

C Move forward 10 days. The spot rate is \$1.53. Interest rates are unchanged. Calculate the value of your forward position.

14 Consider the following: The US risk-free rate is 6 percent, the Swiss risk-free rate is 4 percent, and the spot exchange rate between the United States and Switzerland is \$0.6667.

A Calculate the continuously compounded US and Swiss risk-free rates.

B Calculate the price at which you could enter into a forward contract that expires in 90 days.

C Calculate the value of the forward position 25 days into the contract. Assume that the spot rate is \$0.65.

15 The Japanese yen currently trades at \$0.00812. The US risk-free rate is 4.5 percent, and the Japanese risk-free rate is 2.0 percent. Three-month forward contracts on the yen are quoted at \$0.00813. Indicate how you might earn a risk-free profit by engaging in a forward contract. Outline your transactions.

SOLUTIONS

1 A S_0 = \$1,000

F(0,T) = \$1,100

T = 1

V_0(0,T) = \$1,000 – \$1,100/(1.0675) = –\$30.44

B Because the value is negative, the payment is made by the short to the long.

2 A S_0 = \$500

T = 2/12 = 0.1667

r = 0.035

F(0,T) = \$500 × $(1.035)^{0.1667}$ = \$502.88

B Sell the security for \$500 and invest at 3.5 percent for two months. At the end of two months, you will have \$502.88. Enter into a forward contract now to buy the security at \$498 in two months.

Arbitrage profit = \$502.88 – \$498 = \$4.88

C S_t = \$490

t = 1/12 = 0.0833

T = 2/12 = 0.1667

T – t = 0.0834

r = 0.035

V_t(0,T) = \$490.00 – \$502.88/$(1.035)^{0.0834}$ = –\$11.44. This represents a gain to the short position.

3 A S_0 = \$100

T = 1

r = 0.05

F(0,T) = \$100(1.05) = \$105

B S_t = \$90

t = 3/12 = 0.25

T = 1

T – t = 0.75

r = 0.05

V_t(0,T) = \$90 – \$105/$(1.05)^{0.75}$ = –\$11.23

The investor is short so this represents a gain.

C S_t = \$107

t = 5/12 = 0.4167

T = 1

T – t = 0.5834

r = 0.05

V_t(0,T) = \$107 – \$105/$(1.05)^{0.5834}$ = \$4.95

The investor is short, so this represents a loss to the short position.

D S_t = \$98

F(0,T) = \$105

V_T(0,T) = \$98 – \$105 = –\$7

Gain to short position = \$7

Loss on asset = –\$2 (based on \$100 – \$98)

Net gain = \$5

This represents a return of 5 percent on an asset worth \$100, the same as the risk-free rate.

E S_t = \$110

F(0,T) = \$105

$V_T(0,T)$ = 110 – 105 = \$5

Loss to short position = –\$5

Gain on asset = \$10 (based on \$110 – \$100)

Net gain = \$5

This represents a return of 5 percent on an asset worth \$100, the same as the risk-free rate. The overall gain on the transaction is the same as in Part D because the forward contract was executed at the no-arbitrage price of \$105.

4 A S_0 = \$225

T = 1

r = 0.0475

F(0,T) = \$225(1.0475) = \$235.69

B S_t = \$250

t = 4/12 = 0.3333

T = 1

T – t = 0.6667

r = 0.0475

$V_t(0,T) = \$250.00 - \$235.69/(1.0475)^{0.6667} = \$21.49$

The investor is long, so a positive value represents a gain.

C S_t = \$200

t = 8/12 = 0.6667

T = 1

T – t = 0.3333

r = 0.0475

$V_t(0,T) = \$200.00 - \$235.69/(1.0475)^{0.3333} = -\$32.07$

The investor is long, so this represents a loss to the long position.

D S_t = \$190

F(0,T) = \$235.69

$V_T(0,T)$ = \$190.00 – \$235.69 = –\$45.69

Loss to long position = –\$45.69

Gain on asset = \$35.00 (based on \$225 – \$190)

Net loss = –\$10.69

E S_t = \$240

F(0,T) = \$235.69

$V_T(0,T)$ = \$240.00 – \$235.69 = \$4.31

Gain to long position = \$4.31

Loss on asset = –\$15.00 (based on \$240 – \$225)

Net loss = –$10.69

This loss is the same as the loss in Part D. In fact, the loss would be the same for any other price as well, because the forward contract was executed at the no-arbitrage price of $235.69. The loss of $10.69 is the risk-free rate of 4.75 percent applied to the initial asset price of $225.

5 **A** The no-arbitrage forward price is $F(0,T) = \$200(1.05)^{3/12} = \$202.45$.

Because the forward contract offered by the dealer is overpriced, sell the forward contract and buy the security now. Doing so will yield an arbitrage profit of $2.55.

Borrow $200 and buy security. At the end of three months, repay	$202.45
At the end of three months, deliver the security for	$205.00
Arbitrage profit	$2.55

B At a price of $198.00, the contract offered by the dealer is underpriced relative to the no-arbitrage forward price of $202.45. Enter into a forward contract to buy in three months at $198.00. Short the stock now, and invest the proceeds. Doing so will yield an arbitrage profit of $4.45.

Short security for $200 and invest proceeds for three months	$202.45
At the end of three months, buy the security for	$198.00
Arbitrage profit	$4.45

6 **A** $S_0 = \$150$

$T = 250/365$

$r = 0.0525$

$PV(D,0,T) = \$1.25/(1.0525)^{30/365} + \$1.25/(1.0525)^{120/365} + \$1.25/(1.0525)^{210/365} = \$3.69$

$F(0,T) = (\$150.00 - \$3.69)(1.0525)^{250/365} = \$151.53$

B $S_t = \$115$

$F(0,T) = \$151.53$

$t = 100/365$

$T = 250/365$

$T - t = 150/365$

$r = 0.0525$

After 100 days, two dividends remain: the first one in 20 days, and the second one in 110 days.

$PV(D,t,T) = \$1.25/(1.0525)^{20/365} + \$1.25/(1.0525)^{110/365} = \$2.48$

$V_t(0,T) = \$115.00 - \$2.48 - \$151.53/(1.0525)^{150/365} = -\$35.86$

A negative value is a gain to the short.

C $S_T = \$130$

$F(0,T) = \$151.53$

$V_T(0,T) = \$130.00 - \$151.53 = -\$21.53$

The contract expires with a value of negative $21.53, a gain to the short.

7 **A** $S_0 = \$1{,}145$

$T = 90/365 = 0.2466$

$r = 0.0425$

$r^c = \ln(1 + 0.0425) = 0.0416$

$\delta^c = 0.0175$

$F(0,T) = (\$1{,}145 \times e^{-0.0175(0.2466)})(e^{0.0416(0.2466)}) = \$1{,}151.83$

B $S_t = \$1{,}225$

$T = 90/365 = 0.2466$

$t = 28/365 = 0.0767$

$T - t = 0.1699$

$r = 0.0425$

$r^c = \ln(1 + 0.0425) = 0.0416$

$\delta^c = 0.0175$

$V_t(0,T) = (\$1{,}225 \times e^{-0.0175(0.1699)}) - (1151.83e^{-0.0416(0.1699)}) = \$77.65$

This is a gain to the long position.

C $S_T = \$1{,}235$

$F(0,T) = \$1{,}151.83$

$V_T(0,T) = \$1{,}235.00 - \$1{,}151.83 = \$83.17$

The contract expires with a value of $83.17, a gain to the long.

8 **A** The investor should enter into a short forward contract, locking in the price at which he can sell the bond in 365 days.

$B_0^c(T + Y) = \$1{,}146.92$

$T = 365/365 = 1$

$r = 0.06$

Between now (i.e., 200 days since the original purchase) and the next 365 days, the investor will receive two coupons, the first 165 days from now and the second 347 days from now.

$PV(CI,0,T) = \$40/(1.06)^{165/365} + \$40/(1.06)^{347/365} = \$76.80$

$F(0,T) = (\$1{,}146.92 - \$76.81)(1.06)^{365/365} = \$1{,}134.32$

B $B_t^c(T + Y) = \$1{,}302.26$

$F(0,T) = \$1{,}134.32$

$t = 180/365$

$T = 365/365$

$T - t = 185/365$

$r = 0.04$

We are now on the 380th day of the bond's life. One more coupon payment remains until the expiration of the forward contract. The coupon payment is in 547 − 380 = 167 days.

$PV(CI,0,T) = \$40/(1.04)^{167/365} = \$39.29$

$V_t(0,T) = \$1{,}302.26 - \$39.29 - \$1{,}134.32/(1.04)^{185/365} = \$150.98$

A positive value is a loss to the short position.

9 **A** A 6 × 12 FRA expires in 180 days and is based on 180-day Libor.

B $h = 180$

$m = 180$

$h + m = 360$

$L_0(h + m) = 0.0595$

$L_0(h) = 0.057$

$$\text{FRA}(0,\text{h},\text{m}) = \left[\frac{1 + 0.0595\left(\frac{360}{360}\right)}{1 + 0.0570\left(\frac{180}{360}\right)} - 1\right]\left(\frac{360}{180}\right) = 0.0603$$

C h = 180

m = 180

g = 45

h − g = 135

h + m − g = 315

L_{45}(h − g) = 0.0590

L_{45}(h + m − g) = 0.0615

$$V_t(0,\text{h},\text{m}) = \frac{1}{1 + 0.0590\left(\frac{135}{360}\right)} - \frac{1 + 0.0603\left(\frac{180}{360}\right)}{1 + 0.0615\left(\frac{315}{360}\right)} = 0.00081$$

For \$10,000,000 notional principal, the value of the FRA would be = 0.00081 × 10,000,000 = \$8,100.

D h = 180

m = 180

L_{180}(m) = 0.0625

$$\text{At expiration, the payoff is } = \frac{(0.0625 - 0.0603)\left(\frac{180}{360}\right)}{1 + 0.0625\left(\frac{180}{360}\right)} = 0.001067$$

Based on a notional principal of \$10,000,000, the corporation, which is long, will receive \$10,000,000 × 0.001067 = \$10,670 from the dealer.

10 A A 3 × 6 FRA expires in 90 days and is based on 90-day Libor.

B h = 90

m = 90

h + m = 180

L_0(h) = 0.06

L_0(h + m) = 0.0614

$$\text{FRA}(0,\text{h},\text{m}) = \left[\frac{1 + 0.0614\left(\frac{180}{360}\right)}{1 + 0.06\left(\frac{90}{360}\right)} - 1\right]\left(\frac{360}{90}\right) = 0.0619$$

C h = 90

m = 90

g = 30

h − g = 60

h + m − g = 150

L_{30}(h − g) = 0.055

L_{30}(h + m − g) = 0.0562

$$V_t(0,h,m) = \frac{1}{1 + 0.055\left(\frac{60}{360}\right)} - \frac{1 + 0.0619\left(\frac{90}{360}\right)}{1 + 0.0562\left(\frac{150}{360}\right)}$$

$$= -0.001323$$

For \$15,000,000 notional principal, the value of the FRA would be = −0.001323 × 15,000,000 = −\$19,845. Because the manager is short, this represents a gain to his company.

11 A The risk to the US company is that the value of the Swiss franc will decline and it will receive fewer US dollars on conversion. To hedge this risk, the company should enter into a contract to sell Swiss francs forward.

B $S_0 = \$0.5974$

$T = 90/365$

$r = 0.02$

$r^f = 0.05$

$$F(0,T) = \left[\frac{0.5974}{(1.05)^{90/365}}\right](1.02)^{90/365} = \$0.5931$$

C $S_t = \$0.55$

$T = 90/365$

$t = 30/365$

$T - t = 60/365$

$r = 0.02$

$r^f = 0.05$

$$V_t(0,T) = \frac{\$0.55}{(1.05)^{60/365}} - \frac{\$0.5931}{(1.02)^{60/365}} = -\$0.0456$$

This represents a gain to the short position of \$0.0456 per Swiss franc. In this problem, the US company holds the short forward position.

12 First calculate the fair value or arbitrage-free price of the forward contract:

$S_0 = \$1.0231$

$T = 180/365$

$r = 0.04$

$r^f = 0.05$

$$F(0,T) = \left[\frac{1.0231}{(1.05)^{180/365}}\right](1.04)^{180/365} = \$1.0183$$

The dealer quote for the forward contract is \$1.0225; thus, the forward contract is overpriced. To earn a risk-free profit, you should enter into a forward contract to sell euros forward in six months at \$1.0225. At the same time, buy euros now.

i. Take $\frac{\$1.0231}{(1.05)^{180/365}} = \$0.9988$. Use it to buy $\frac{1}{(1.05)^{180/365}} = 0.9762$ euros.

ii. Enter a forward contract to deliver €1.00 at \$1.0225 in six months.

iii. Invest €0.9762 for six months at 5 percent per year and receive €0.9762 × $1.05^{180/365}$ = €1.00 at the end of six months.

iv. At expiration, deliver the euro and receive \$1.0225. Return over six months is $\frac{\$1.0225}{\$0.9988} - 1 = 0.0237$, or 4.74 percent a year.

This risk-free annual return of 4.74 percent exceeds the US risk-free rate of 4 percent.

13 A The risk to you is that the value of the British pound will rise over the next 30 days and it will require more US dollars to buy the necessary pounds to make payment. To hedge this risk you should enter a forward contract to buy British pounds.

B S_0 = \$1.50

T = 30 / 365

r = 0.055

r^f = 0.045

$$F(0,T) = \left[\frac{\$1.50}{(1.045)^{30/365}}\right](1.055)^{30/365} = \$1.5012$$

C S_t = \$1.53

T = 30/365

t = 10/365

T − t = 20/365

r = 0.055

r^f = 0.045

$$V_t(0,T) = \frac{\$1.53}{1.045^{20/365}} - \frac{\$1.5012}{1.055^{20/365}} = \$0.0295$$

Because you are long, this is a gain of \$0.0295 per British pound.

14 A $r^{fc} = \ln(1.04) = 0.0392$

$r^c = \ln(1.06) = 0.0583$

B S_0 = \$0.6667

T = 90/365

r^{fc} = 0.0392

r^c = 0.0583

$$F(0,T) = \left(\$0.6667 \times e^{-0.0392(90/365)}\right)\left(e^{0.0583(90/365)}\right) = \$0.6698$$

C S_t = \$0.65

T = 90/365

t = 25/365

T − t = 65/365

r^{fc} = 0.0392

r^c = 0.0583

$$V_t(0,T) = \left(\$0.65 \times e^{-0.0392(65/365)}\right) - \left(\$0.6698 \times e^{-0.0583(65/365)}\right)$$
$$= -\$0.0174$$

The value of the contract is −\$0.0174 per Swiss franc.

15 First, calculate the fair value or arbitrage free price of the forward contract:

S_0 = \$0.00812 per yen

T = 90/365

r = 0.045

r^f = 0.02

$$F(0,T) = \left[\frac{\$0.00812}{(1.02)^{90/365}}\right](1.045)^{90/365} = \$0.00817$$

The dealer quote for the forward contract is \$0.00813. Therefore, the forward contract is underpriced. To earn a risk-free profit, you should enter into a forward contract to buy yen in three months at \$0.00813. At the same time, sell yen now.

i. The spot rate of \$0.00812 per yen is equivalent to ¥123.15 per US dollar. Take $\frac{¥123.15}{(1.045)^{90/365}}$ = ¥121.82. Use it to buy $\frac{1}{(1.045)^{90/365}}$ = 0.9892 US dollars.

ii. Enter a forward contract to buy yen at \$0.00813 in three months. One US dollar will buy $\frac{1}{\$0.00813}$ = ¥123.00.

iii. Invest \$0.9892 for three months at 4.5 percent a year and receive \$0.9892 × $1.045^{90/365}$ = \$1.00 at the end of three months.

iv. At expiration, deliver the dollar and receive ¥123. The return over three months is $\frac{¥123.00}{¥121.82} - 1$ = 0.00969, or 3.88 percent a year.

Because we began our transactions in yen, the relevant comparison for the return from our transactions is the Japanese risk-free rate. The 3.88 percent return above exceeds the Japanese risk-free rate of 2 percent. Therefore, we could borrow yen at 2 percent and engage in the above transactions to earn a risk-free return of 3.88 percent that exceeds the rate of borrowing.